Name: _____ Solutions _____

| Problem	Total
Possible	100
Received	

DO NOT OPEN YOUR EXAM UNTIL TOLD TO DO SO.

You may use a 3×5 card of notes. You will not use a calculator.

In answering the exam questions, **except where noted**, do not simplify answers.

For example, leave your answer as $P(5,3)$ or $12!$ or $C(4,3) \cdot C(7,4)$ or $2^5 - 2^3$ or $7 \cdot 6 \cdot 5$ or …
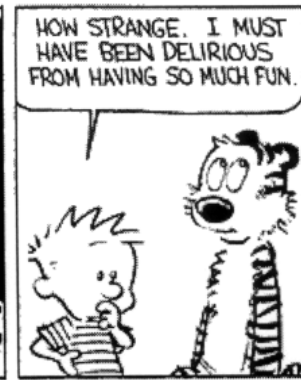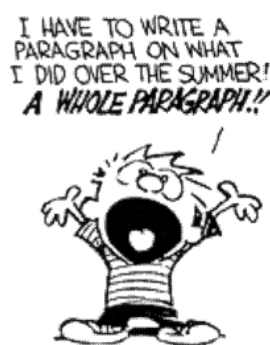
/2 Suppose you have five $1 bills. How many ways can you choose 5 friends from 12 to whom to give $1 each?

$$C(12,5)$$

/2 Suppose you have five bills, one each of $1, $2, $5, $10 and $20. How many ways can you choose 5 friends from 12 to whom to give the $1, $2, $5, $10 and $20 bills (one bill for each friend)?

$$P(12,5) = C(12,5) \cdot 5! = 12 \cdot 11 \cdot 10 \cdot 9 \cdot 8$$

/3 How many five-digit numbers (from 10000 to 99999) are there in which all of the digits are different (e.g. 79823 or 27890 or 54209 or …).

$$9 \cdot 9 \cdot 8 \cdot 7 \cdot 6$$

/3 How many five-digit numbers (from 10000 to 99999) are there in which the digits strictly decrease (e.g. 65,420 or 76,532 or …)?

$C(10,5)$ From the digits 0 1 2 ⋯ 9, choose 5, then put them in decreasing order.

/3 How many license plates are there which consist of three letters and three numbers if repetition is allowed (e.g. GEG747)?

$$26 \cdot 26 \cdot 26 \cdot 10 \cdot 10 \cdot 10$$

/4 How many license plates are there which consist of three letters and three numbers if repetition is NOT allowed, and the letters must be in alphabetical order and the numbers have to be increasing (e.g. EGT247)?

$C(26,3) \cdot C(10,3)$ Choose 3 of the 26 letters and choose 3 of the 10 digits. You don't get to decide the order.

For the next three questions, suppose that we have a package contains 100 light bulbs, of which 20 are defective. A sample of 6 light bulbs is selected at random.

/2 How many different samples (of size 6) are there?

$$C(100,6)$$

/3 How many of those samples contain 2 defective bulbs (and 4 good bulbs)?

$$C(20,2) \cdot C(80,4)$$

/4 How many of the samples contain at least 1 defective bulb?

$$C(100,6) - \underbrace{C(20,0) \cdot C(80,6)}_{\text{no defective}}$$

/4   An art gallery has three paintings by each of four artists. In how many ways can the
12 ~~eighteen~~ paintings be displayed in a row if paintings by the same artist must all be
     grouped together side-by-side?

$$4! \; 3! \, 3! \, 3! \, 3! \quad \text{or} \quad 12 \cdot 2 \cdot 1 \cdot 9 \cdot 2 \cdot 1 \cdot 6 \cdot 2 \cdot 1 \cdot 3 \cdot 2 \cdot 1$$

/3   In how many ways could you select 4 stocks from 9 and 3 bonds from 10?

$$C(9,4) \cdot C(10,3)$$

/5   If you want to have 3 scoops of ice cream and you are choosing from 10 flavors, and
     you can duplicate flavors, how many different options are there? (Note that the order
     of the scoops does NOT matter.)

$$\underset{\text{all same}}{10} \quad + \underset{2/1}{P(10,2)} + \underset{\text{all diff.}}{C(10,3)}$$

See the handout for
why $10 \cdot 10 \cdot 10$ is
not correct.

/3   A club has 10 members. In how many ways can they choose their four officers:
     president, vice president, secretary and treasurer?

$$P(10,4) = C(10,4) \cdot 4! = 10 \cdot 9 \cdot 8 \cdot 7$$

For the next four questions, suppose there is a deck of 70 cards:
- *There are 7 different colors of cards.*
- *Each color has cards from 1 to 10.*

We will choose 6 cards. In how many ways can you choose:

/5   How many hands are there in which you have 3 of one number (of any color),
     2 of another, and 1 of another (e.g. 7 7 7 9 9 2)?

$$\underline{C(10,3) \cdot 3!} \cdot C(7,3) \cdot C(7,2) \cdot C(7,1)$$

or $P(10,3)$ or $10 \cdot 9 \cdot 8$

/3   How many hands are there in which all 6 of your cards are of the same <u>color</u>?

$$\underline{C(7,1)} \cdot \underline{C(10,6)}$$

Choose color   Choose 6 cards of that color.

/2   How many different hands of any sort are there?

$$C(70,6)$$

/3   How many hands are there all of the same number (e.g. 7 7 7 7 7 7)?

$$\underline{C(10,1)} \cdot \underline{C(7,6)}$$

Choose the   Choose 6 colored cards
number   of that number.

3

/2 A basketball player shoots 10 shots. In how many ways could she make 8 of 10 shots?

$$C(10,8)$$

/2 In how many ways can you select 4 shirts from 10 to take on a trip?

$$C(10,4)$$

/2 In how many ways can you select 6 shirts from 10 to take on a trip?

$$C(10,6)$$

/2 In how many ways can you select 4 shirts from 10 to leave home?

$$C(10,4)$$

/3 In how many ways can you select (from 10) and arrange 4 books on a bookshelf?

$$P(10,4)$$

/3 If I have 5 red and 10 green balls, in how many ways could I select 7 balls in which 3 of them are red?

$$C(5,3) \cdot C(10,4)$$

/2 Compute (<u>find the actual number</u>) $C(10,2) = \dfrac{10!}{2!\,8!} = \dfrac{10 \cdot 9}{2 \cdot 1} = 45$

/2 Compute (<u>find the actual number</u>) $P(100,2) = \dfrac{100!}{98!} = 100 \cdot 99 = 9900$

/2 Compute (<u>find the actual number</u>) $C(6,3) = \dfrac{6!}{3!\,3!} = \dfrac{6 \cdot 5 \cdot 4}{3 \cdot 2 \cdot 1} = 20$

4

/3   In how many ways can 12 jurors and 5 alternates be chosen from a group of 25 perspective jurors?

$$\binom{25}{12,5,8} \quad \text{or} \quad C(25,12)\cdot C(13,5)$$

$\uparrow$ not selected

/3   In how many ways could you divide 60 students into three groups of sizes 10, 20 and 30 students?

$$\binom{60}{10,20,30}$$

/4   In how many ways could you divide 60 students into three groups of equal size (20 students in each group)?

$$\binom{60}{20,20,20} \Big/ 3!$$

/2   In how many ways can the Supreme Court (made up of 9 justices) reach a six-to-three decision?

$$C(9,3)$$

/2   How many ways could I arrange six books on a shelf?

$$6!$$

For the final three questions, eight couples (8 men and 8 women) will sit in 16 side-by-side seats. In how many ways can the 16 persons be seated if:

/4   Each couple must be seated together (i.e. each couple is next to each other)?

$$8! \cdot 2^8 \quad \text{or} \quad 16\cdot1\cdot14\cdot1\cdot\,\cdots\,\cdot4\cdot1\cdot2\cdot1$$

Order 8 couples   MW or WM for each couple.

/4   All of the men must be seated next to each other and all of the women must be seated next to each other?

$$2\cdot8!\cdot8! \quad \text{or} \quad 16\cdot7\cdot6\cdot\,\cdots\,\cdot2\cdot1\cdot8\cdot7\cdot\,\cdots\,\cdot2\cdot1$$

$\uparrow$ M or W first?

/4   All of the men must be seated next to each other (but the women do not need to)?

●W●W●W●W●W●W●W●W●

Where to put the M relative to the W.

$$9 \cdot 8! \cdot 8!$$