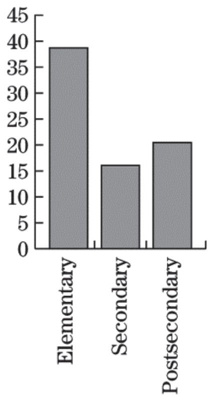
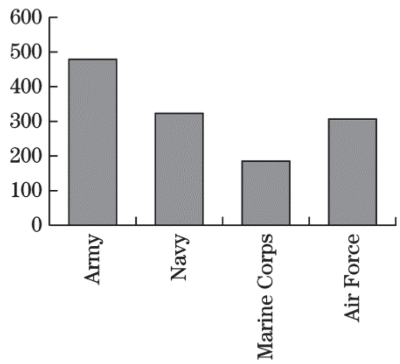
# Chapter 7

**Exercises 7.1**

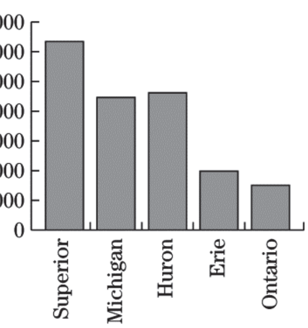
**1.**



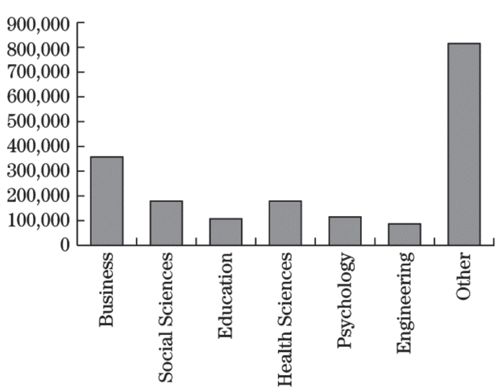

**2.**



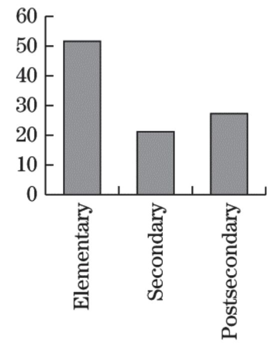

**3.**



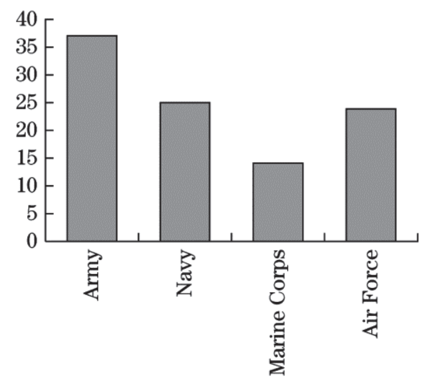

**4.**




**5.**

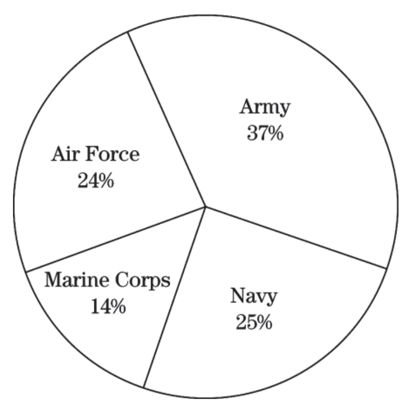



**6.**



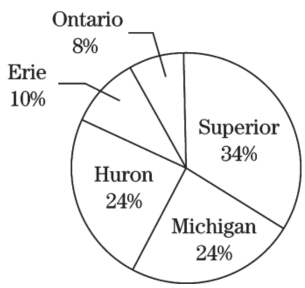

**7.** To find the central angle, multiply the percentage 0.10 by 360 to obtain 36°.

**8.**

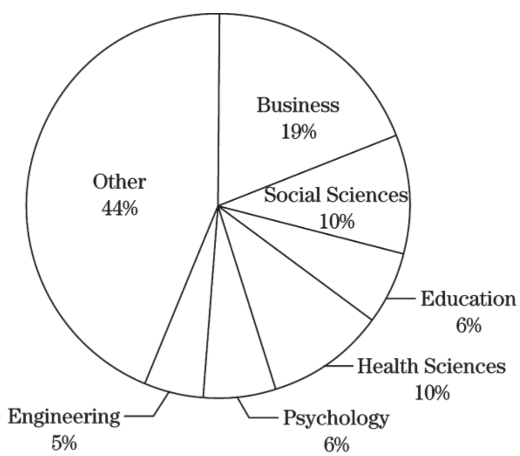


**9.**

| *Lake* | *Percent* | 360° × *Percent* |
|---|---|---|
| Superior | 33.5 | 120.6° |
| Michigan | 23.6 | 85.0° |
| Huron | 24.4 | 87.8° |
| Erie | 10.5 | 37.8° |
| Ontario | 8.0 | 28.8° |




**10.**




**11.** Pr(Master or Doctorate) = 0.421 + 0.191 = 0.612

**12.** Pr(Medical or Law) = 0.112 + 0.041 = 0.153

**13.**

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<tr><td>0</td><td>𝍸 ||</td><td>7</td></tr>
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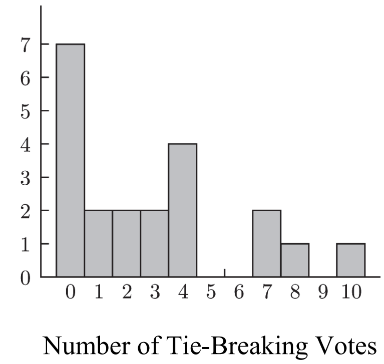


**14.**

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<tr><td>50</td><td>||</td><td>2</td></tr>
<tr><td>51</td><td>||||</td><td>4</td></tr>
<tr><td>52</td><td>||</td><td>2</td></tr>
<tr><td>53</td><td></td><td>0</td></tr>
<tr><td>54</td><td>𝍸</td><td>5</td></tr>
<tr><td>55</td><td>||||</td><td>4</td></tr>
<tr><td>56</td><td>|||</td><td>3</td></tr>
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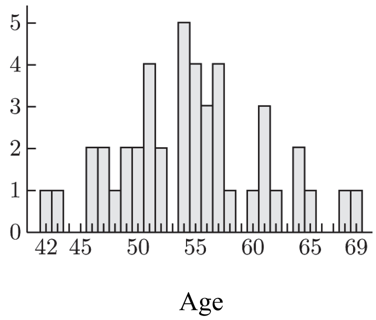
<tr><td>58</td><td>|</td><td>1</td></tr>
<tr><td>59</td><td></td><td>0</td></tr>
<tr><td>60</td><td>|</td><td>1</td></tr>
<tr><td>61</td><td>|||</td><td>3</td></tr>
<tr><td>62</td><td>|</td><td>1</td></tr>
<tr><td>63</td><td></td><td>0</td></tr>
<tr><td>64</td><td>||</td><td>2</td></tr>
<tr><td>65</td><td>|</td><td>1</td></tr>
<tr><td>66</td><td></td><td>0</td></tr>
<tr><td>67</td><td></td><td>0</td></tr>
<tr><td>68</td><td>|</td><td>1</td></tr>
<tr><td>69</td><td>|</td><td>1</td></tr>
</table>

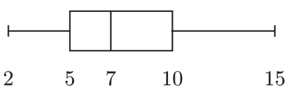
 



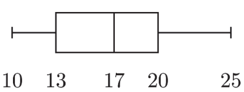

**15.** The median is $\frac{2+3}{2} = 2.5$.

**16.** The median is 4.

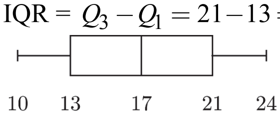
**17.**



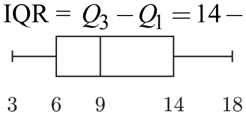

**18.**



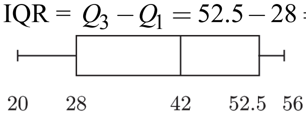

**19.** min = 10, $Q_1 = 13$, $Q_2 = 17$, $Q_3 = 21$, max = 24;
IQR = $Q_3 - Q_1 = 21 - 13 = 8$;



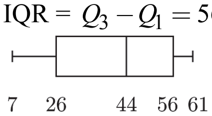

**20.** min = 3, $Q_1 = 6$, $Q_2 = 9$, $Q_3 = 14$, max = 18;
IQR = $Q_3 - Q_1 = 14 - 6 = 8$;




**21.** min = 20, $Q_1 = 28$, $Q_2 = 42$, $Q_3 = 52.5$, max = 56;
IQR = $Q_3 - Q_1 = 52.5 - 28 = 24.5$;




**22.** min = 7, $Q_1 = 26$, $Q_2 = 44$, $Q_3 = 56$, max = 61;
IQR = $Q_3 - Q_1 = 56 - 26 = 30$;




**23.** (A) – (c), (B) – (d), (C) – (a), (D) – (b)

**24.** **a.** 25%

**b.** 25%

**c.** 50%

**d.** 25%

**e.** 90¢

**25.** **a.** min = 200, $Q_1 = 400$, $Q_2 = 600$, $Q_3 = 700$, max = 800

**b.** 25%

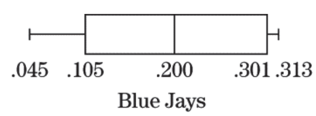
**c.** 25%

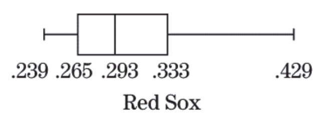
**d.** 50%

**e.** 75%

**26.** The National League has the higher median (.315 vs .306).

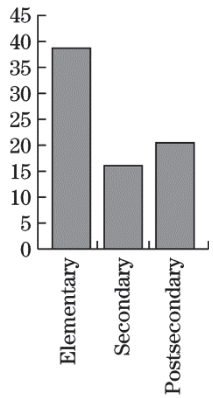
**27.** Blue Jays: 0.188, 0.313, 0.297, 0.304, 0.119, 0.045, 0.091, 0.214, 0.200
Red Sox: 0.239, 0.302, 0.262, 0.333, 0.293, 0.333, 0.267, 0.429, 0.290



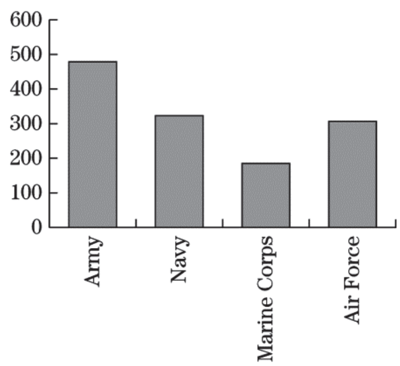




Possible prediction: The Red Sox seem more likely to win because their hitting is more consistent with a higher median.

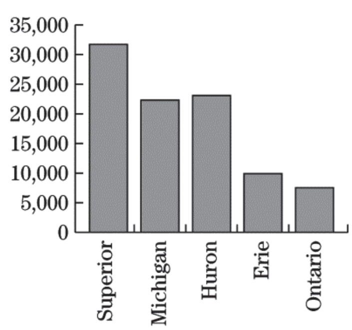
**28.**



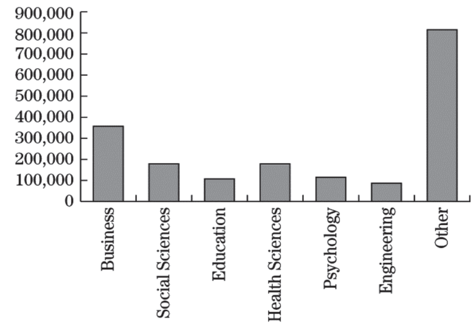

**29.**




**30.**



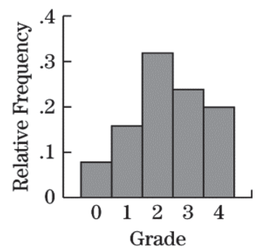

**31.**




**Exercises 7.2**

**1.**

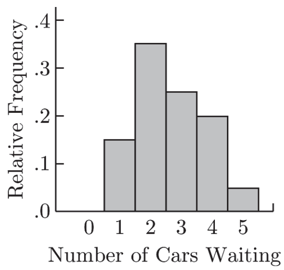
| Grade | Relative Frequency |
|---|---|
| 0 | $\frac{2}{25} = 0.08$ |
| 1 | $\frac{4}{25} = 0.16$ |
| 2 | $\frac{8}{25} = 0.32$ |
| 3 | $\frac{6}{25} = 0.24$ |
| 4 | $\frac{5}{25} = 0.20$ |




**2.**

| Number of cars waiting | Relative Frequency |
|---|---|
| 0 | $\frac{0}{60} = 0$ |
| 1 | $\frac{9}{60} = 0.15$ |
| 2 | $\frac{21}{60} = 0.35$ |
| 3 | $\frac{15}{60} = 0.25$ |
| 4 | $\frac{12}{60} = 0.20$ |
| 5 | $\frac{3}{60} = 0.05$ |




**3.**

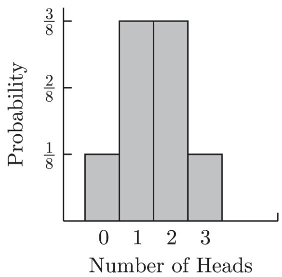
| Number of calls during minute | Relative Frequency |
|---|---|
| 20 | $\frac{3}{60} = 0.05$ |
| 21 | $\frac{3}{60} = 0.05$ |
| 22 | $\frac{0}{60} = 0$ |
| 23 | $\frac{6}{60} = 0.10$ |
| 24 | $\frac{18}{60} = 0.30$ |
| 25 | $\frac{12}{60} = 0.20$ |
| 26 | $\frac{0}{60} = 0$ |
| 27 | $\frac{9}{60} = 0.15$ |
| 28 | $\frac{6}{60} = 0.10$ |
| 29 | $\frac{3}{60} = 0.05$ |

**4.**

| Number produced during hour | Relative Frequency |
|---|---|
| 50 | $\frac{2}{40} = 0.05$ |
| 51 | $\frac{0}{40} = 0$ |
| 52 | $\frac{6}{40} = 0.15$ |
| 53 | $\frac{8}{40} = 0.20$ |
| 54 | $\frac{12}{40} = 0.30$ |
| 55 | $\frac{6}{40} = 0.15$ |
| 56 | $\frac{4}{40} = 0.10$ |
| 57 | $\frac{0}{40} = 0$ |
| 58 | $\frac{0}{40} = 0$ |
| 59 | $\frac{2}{40} = 0.05$ |

**5.** HHH, HHT, HTH, THH,
HTT, THT, TTH, TTT

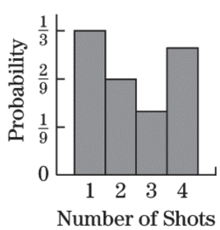
| Number of Heads | Probability |
|---|---|
| 0 | $\frac{\binom{3}{0}}{2^3} = \frac{1}{8}$ |
| 1 | $\frac{\binom{3}{1}}{2^3} = \frac{3}{8}$ |
| 2 | $\frac{\binom{3}{2}}{2^3} = \frac{3}{8}$ |
| 3 | $\frac{\binom{3}{3}}{2^3} = \frac{1}{8}$ |




**6.** Probability of hit $= \frac{1}{3}$

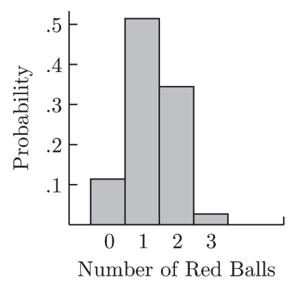
Probability of miss$=\frac{2}{3}$

| Number of Shots | Probability |
|---|---|
| 1 | $\frac{1}{3}$ |
| 2 | $\left(\frac{2}{3}\right)\left(\frac{1}{3}\right)=\frac{2}{9}$ |
| 3 | $\left(\frac{2}{3}\right)^2\left(\frac{1}{3}\right)=\frac{4}{27}$ |
| 4 | $\left(\frac{2}{3}\right)^3=\frac{8}{27}$ |




**7.**

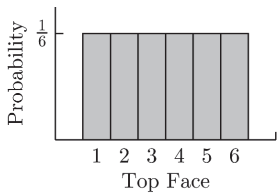
| Number of Red Balls | Probability |
|---|---|
| 0 | $\dfrac{\binom{3}{0}\binom{4}{3}}{\binom{7}{3}}=\dfrac{4}{35}$ |
| 1 | $\dfrac{\binom{3}{1}\binom{4}{2}}{\binom{7}{3}}=\dfrac{18}{35}$ |
| 2 | $\dfrac{\binom{3}{2}\binom{4}{1}}{\binom{7}{3}}=\dfrac{12}{35}$ |
| 3 | $\dfrac{\binom{3}{3}\binom{4}{0}}{\binom{7}{3}}=\dfrac{1}{35}$ |




**8.**

| Top Face | Probability |
|---|---|
| 1 | $\frac{1}{6}$ |
| 2 | $\frac{1}{6}$ |
| 3 | $\frac{1}{6}$ |
| 4 | $\frac{1}{6}$ |
| 5 | $\frac{1}{6}$ |
| 6 | $\frac{1}{6}$ |




**9.**

| No. Red Balls | Player's Earnings | Probability |
|---|---|---|
| 2 | \$5 | $\dfrac{\binom{2}{2}\binom{4}{0}}{\binom{6}{2}}=\dfrac{1}{15}$ |
| 1 | \$1 | $\dfrac{\binom{2}{1}\binom{4}{1}}{\binom{6}{2}}=\dfrac{8}{15}$ |
| 0 | –1\$ | $\dfrac{\binom{2}{0}\binom{4}{2}}{\binom{6}{2}}=\dfrac{6}{15}$ |

**10.**

| Number of tosses | Player's Earnings | Probability |
|---|---|---|
| 1 | –50¢ | $\frac{1}{2}$ |
| 2 | 0¢ | $\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)=\frac{1}{4}$ |
| 3 | 50¢ | $\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)=\frac{1}{8}$ |
| 4 | \$1 | $\left(\frac{1}{2}\right)^3=\frac{1}{8}$ |

**11.** $\Pr(5 \le X \le 7)$
$= \Pr(X = 5) + \Pr(X = 6) + \Pr(X = 7)$
$= .2 + .1 + .3$
$= .6$

**12.** The outcome is between 8 and 12, inclusive.

**13.**

| $k$ | $\Pr(X^2 = k)$ |
|---|---|
| 0 | 0.1 |
| 1 | 0.2 |
| 4 | 0.4 |
| 9 | 0.1 |
| 16 | 0.2 |

**14.**

| $k$ | $\Pr(Y^2 = k)$ |
|---|---|
| 25 | 0.3 |
| 100 | 0.4 |
| 225 | 0.1 |
| 400 | 0.1 |
| 625 | 0.1 |

**15.**

| $k$ | $\Pr(X - 1 = k)$ |
|---|---|
| –1 | 0.1 |
| 0 | 0.2 |
| 1 | 0.4 |
| 2 | 0.1 |
| 3 | 0.2 |

**16.**

| $k$ | $\Pr(Y - 15 = k)$ |
|---|---|
| –10 | 0.3 |
| –5 | 0.4 |
| 0 | 0.1 |
| 5 | 0.1 |
| 10 | 0.1 |

**17.**

| $k$ | $\Pr\left(\frac{1}{5}Y = k\right)$ |
|---|---|
| 1 | 0.3 |
| 2 | 0.4 |
| 3 | 0.1 |
| 4 | 0.1 |
| 5 | 0.1 |

**18.**

| $2X^2 = k$ | $\Pr(2X^2 = k)$ |
|---|---|
| $2(0)^2 = 0$ | 0.1 |
| $2(1)^2 = 2$ | 0.2 |
| $2(2)^2 = 8$ | 0.4 |
| $2(3)^2 = 18$ | 0.1 |
| $2(4)^2 = 32$ | 0.2 |

**19.**

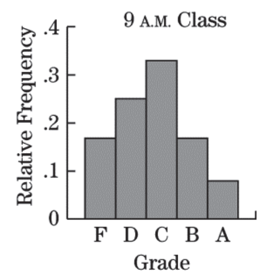
| $(X+1)^2 = k$ | $\Pr((X+1)^2 = k)$ |
|---|---|
| $(0+1)^2 = 1$ | 0.1 |
| $(1+1)^2 = 4$ | 0.2 |
| $(2+1)^2 = 9$ | 0.4 |
| $(3+1)^2 = 16$ | 0.1 |
| $(4+1)^2 = 25$ | 0.2 |

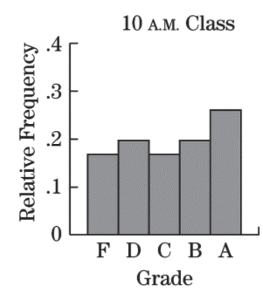
**20.**

| $\left(\frac{1}{5}Y+1\right)^2 = k$ | $\Pr\left(\left(\frac{1}{5}Y+1\right)^2 = k\right)$ |
|---|---|
| $\left(\frac{1}{5}\cdot 5+1\right)^2 = 4$ | 0.3 |
| $\left(\frac{1}{5}\cdot 10+1\right)^2 = 9$ | 0.4 |
| $\left(\frac{1}{5}\cdot 15+1\right)^2 = 16$ | 0.1 |
| $\left(\frac{1}{5}\cdot 20+1\right)^2 = 25$ | 0.1 |
| $\left(\frac{1}{5}\cdot 25+1\right)^2 = 36$ | 0.1 |

**21.**

| | Relative Frequency | |
|---|---|---|
| Grade | 9 A.M. class | 10 A.M. class |
| F | $\frac{10}{60} \approx 0.17$ | $\frac{17}{100} = 0.17$ |
| D | $\frac{15}{60} = 0.25$ | $\frac{20}{100} = 0.20$ |
| C | $\frac{20}{60} \approx 0.33$ | $\frac{17}{100} = 0.17$ |
| B | $\frac{10}{60} \approx 0.17$ | $\frac{20}{100} = 0.20$ |
| A | $\frac{5}{60} \approx 0.08$ | $\frac{26}{100} = 0.26$ |




The 9 A.M. class has the distribution centered on the C grade with relatively few A's. The 10A.M. class has a large percentage of A's and D's with fewer C's.

**22.** **a.** percentage with C or less (9 A.M.):
$$\left(\frac{20+15+10}{60}\right)\times 100\% = 75\%$$

**b.** percentage with C or less (10 A.M.):
$$\left(\frac{17+20+17}{100}\right)\times 100\% = 54\%$$

**c.** percentage with D or F (9 A.M.):
$$\left(\frac{15+10}{60}\right)\times 100\% \approx 41.7\%$$

**d.** percentage with C or more (both classes):
$$\left(\frac{20+10+5+17+20+26}{60+100}\right)\times 100\% = 61.25\%$$

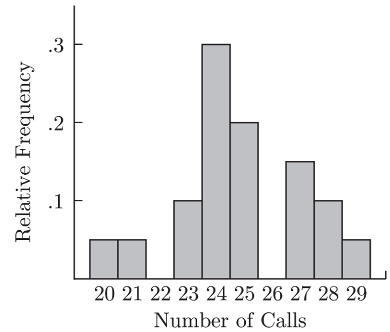
**23.** percentage with C or higher:
$$\left(\frac{8+6+5}{25}\right)\times 100\% = 76\%$$

**24.** percentage of time waiting line has 4 or more cars:
$$\left(\frac{12+3}{60}\right)\times 100\% = 25\%$$

**25. a.** less than 22: 3 + 3 = 6
more than 27: 6 + 3 = 9
combined: $\left(\frac{6+9}{60}\right)\times 100\% = 25\%$

**b.** between 23 and 25:
$$\left(\frac{6+18+12}{60}\right)\times 100\% = 60\%$$

**c.**

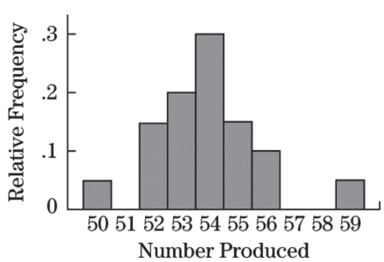



**d.** Estimated average number of calls would be 24 since that number has the highest frequency of occurrence.
It is actually ≈ 25.

**26. a.** $\left(\frac{6+4+2}{40}\right)\times 100\% = 30\%$

**b.** $\left(\frac{8+12+6}{40}\right)\times 100\% = 65\%$

**c.**




**d.** 59

**e.** Estimate an average of 54 items produced

**27.** **a.** $\Pr(U=4)=1-\left(\frac{4}{15}+\frac{2}{15}+\frac{4}{15}+\frac{3}{15}\right)$
$=1-\frac{13}{15}$
$=\frac{2}{15}$

**b.** $\Pr(U\geq 2)=\Pr(U=2)+\Pr(U=3)+\Pr(U=4)$
$=\frac{4}{15}+\frac{3}{15}+\frac{2}{15}$
$=\frac{9}{15}$
$=\frac{3}{5}$

**c.** $\Pr(U\leq 3)=1-\Pr(U=4)$
$=1-\frac{2}{15}$
$=\frac{13}{15}$

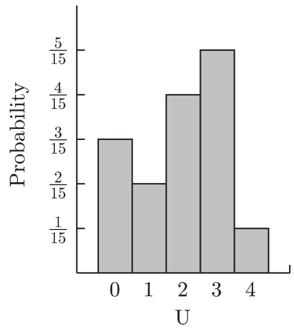
**d.**

| $U+2=K$ | $\Pr(U+2=K)$ |
|---|---|
| $0+2=2$ | $\frac{4}{15}$ |
| $1+2=3$ | $\frac{2}{15}$ |
| $2+2=4$ | $\frac{4}{15}$ |
| $3+2=5$ | $\frac{3}{15}$ |
| $4+2=6$ | $\frac{2}{15}$ |

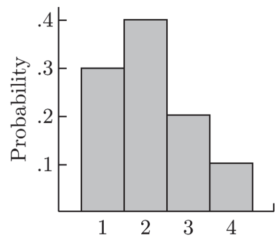
$\Pr(U+2<4)$
$=\Pr(U+2=2)+\Pr(U+2=3)$
$=\frac{4}{15}+\frac{2}{15}$
$=\frac{6}{15}$
$=\frac{2}{5}$

**e.**




**28. a.**




**b.** $\Pr(X = 2) + \Pr(X = 3) = 0.40 + 0.20 = 0.60$

**c.** $\Pr(X \geq 2) = \Pr(X = 2) + \Pr(X = 3) + \Pr(X = 4) = 0.40 + 0.20 + 0.10 = 0.70$

**d.**

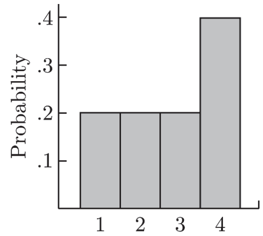
| $X + 2 = k$ | $\Pr(X + 3 = k)$ |
|---|---|
| $1 + 2 = 3$ | 0.30 |
| $2 + 2 = 4$ | 0.40 |
| $3 + 2 = 5$ | 0.20 |
| $4 + 2 = 6$ | 0.10 |

$\Pr(X + 2 \geq 5) = \Pr(X + 2 = 5) + \Pr(X + 2 = 6) = 0.20 + 0.10 = 0.30$

**e.**

| $2X = k$ | $\Pr(2X = k)$ |
|---|---|
| $2(1) = 2$ | 0.30 |
| $2(2) = 4$ | 0.40 |
| $2(3) = 6$ | 0.20 |
| $2(4) = 8$ | 0.10 |

**29. a.**




**b.** $\Pr(Y = 2) + \Pr(Y = 3) = 0.20 + 0.20 = 0.40$

**c.**

| $Y^2 = k$ | $\Pr(Y^2 = k)$ |
|---|---|
| $1^2 = 1$ | 0.20 |
| $2^2 = 4$ | 0.20 |
| $3^2 = 9$ | 0.20 |
| $4^2 = 16$ | 0.40 |

$\Pr(Y^2 \le 9) = \Pr(Y^2 = 1) + \Pr(Y^2 = 4) + \Pr(Y^2 = 9) = 0.20 + 0.20 + 0.20 = 0.60$

**d.** $\Pr(Y \le 10) = \Pr(Y = 1) + P(Y = 2) + P(Y = 3) + P(Y = 4) = 0.20 + 0.20 + 0.20 + 0.40 = 1$

**e.**

| $(Y+2)^2 = k$ | $\Pr\left((Y+2)^2 = k\right)$ |
|---|---|
| $(1+2)^2 = 9$ | 0.20 |
| $(2+2)^2 = 16$ | 0.20 |
| $(3+2)^2 = 25$ | 0.20 |
| $(4+2)^2 = 36$ | 0.40 |

**30.** *Y* has the higher average value since the probability distribution for *Y* has its largest number of occurrences at 4 while *X* has its largest number of occurrences at 2.

**Exercises 7.3**

1. $\binom{5}{3}(0.3)^3(0.7)^2 = (10)(0.027)(0.49) = 0.1323$

2. $\binom{6}{1}(0.4)^1(0.6)^5 = (6)(0.4)(0.07776) = 0.1866$

3. $\binom{4}{3}\left(\frac{1}{3}\right)^3\left(\frac{2}{3}\right)^1 = (4)\left(\frac{1}{27}\right)\left(\frac{2}{3}\right) = \frac{8}{81} \approx 0.0988$

4. $\binom{3}{2}\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^1 = (3)\left(\frac{1}{36}\right)\left(\frac{5}{6}\right) = \frac{5}{72} \approx 0.06944$

5. $\Pr(X=3) = \binom{10}{3}\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^7 = \frac{15}{128} \approx 0.1172$

6. $\Pr(X=0) = \binom{10}{0}\left(\frac{1}{2}\right)^0\left(\frac{1}{2}\right)^{10} = \frac{1}{1024} \approx .0009766$

7. $\Pr(X=7) = \binom{10}{7}\left(\frac{1}{2}\right)^7\left(\frac{1}{2}\right)^3 = \frac{15}{128} \approx 0.1171875$

$\Pr(X=8) = \binom{10}{8}\left(\frac{1}{2}\right)^8\left(\frac{1}{2}\right)^2 = \frac{45}{1024} \approx 0.043945$

$\Pr(X = 7 \text{ or } 8) = 0.117188 + 0.04395 = 0.1611$

8. $\Pr(X=3) = \binom{10}{3}\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^7 = \frac{15}{128} \approx 0.1171875$

$\Pr(X=2) = \binom{10}{2}\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^8 = \frac{45}{1024} \approx 0.043945$

$\Pr(X = 3 \text{ or } 2) = 0.117188 + 0.04395 = 0.1611$

9. $\Pr(\text{At least 1}) = 1 - \Pr(X=0)$
$= 1 - 0.0009766$
$= 0.9990$

10. $\Pr(X=9) = \binom{10}{9}\left(\frac{1}{2}\right)^9\left(\frac{1}{2}\right)^1 = \frac{5}{512} \approx 0.009766$

$\Pr(X=10) = \binom{10}{10}\left(\frac{1}{2}\right)^{10}\left(\frac{1}{2}\right)^0 = \frac{1}{1024} \approx 0.0009766$

$\Pr(\text{At most 7}) = 1 - \Pr(X = 8, 9, 10)$
$= 1 - 0.043945 - 0.009766$
$- 0.0009766$
$= 0.9453$

11. $\Pr(X=4) = \binom{4}{4}\left(\frac{1}{6}\right)^4\left(\frac{5}{6}\right)^0 = \frac{1}{1296} \approx 0.000772$

12. $\Pr(X=2) = \binom{4}{2}\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^2 = \frac{25}{216} \approx 0.1157$

13. $\Pr(X=3) = \binom{4}{3}\left(\frac{1}{6}\right)^3\left(\frac{5}{6}\right)^1 = \frac{5}{324} \approx 0.015432$

$\Pr(X = 2 \text{ or } 3) = 0.11574 + 0.01543 = 0.1312$

14. $\Pr(X=1) = \binom{4}{1}\left(\frac{1}{6}\right)^1\left(\frac{5}{6}\right)^3 = \frac{125}{324} \approx 0.3858$

$\Pr(X = 2 \text{ or } 1) = 0.1157 + 0.3858 = 0.5015$

15. $\Pr(X = 2 \text{ or } 1 \text{ or } 0)$
$= 0.1157 + 0.3858 + 0.4823 = 0.9838$

16. $\Pr(X=4) = \binom{4}{4}\left(\frac{1}{6}\right)^4\left(\frac{5}{6}\right)^0 = \frac{1}{1296} \approx 0.000772$

$\Pr(X = 2 \text{ or } 3 \text{ or } 4)$
$= 0.1157 + 0.0154 + 0.00077 = 0.1319$

17. $\Pr(X=2) = \binom{8}{2}(0.14)^2(0.86)^6 \approx 0.2220$

18. $\Pr(X=0) = \binom{8}{0}(0.14)^0(0.86)^8 \approx 0.2992$

**19.** $\Pr(X=4)=\binom{8}{4}(0.14)^4(0.86)^4\approx 0.01471$

$\Pr(X=5)=\binom{8}{5}(0.14)^5(0.86)^3\approx 0.00192$

$\Pr(X=4\text{ or }5)=0.01471+0.00192=0.01663$

**20.** $\Pr(X=1)=\binom{8}{1}(0.14)^1(0.86)^7\approx 0.3897$

$\Pr(X=2)=\binom{8}{2}(0.14)^2(0.86)^6\approx 0.2220$

$\Pr(X=1\text{ or }2)=0.3897+0.2220=0.6117$

**21.** $\Pr(\text{At least }3)=1-\Pr(X=0\text{ or }1\text{ or }2)$
$=1-0.2992-0.3897-0.2220$
$=0.08908$

**22.** $\Pr(X=7)=\binom{8}{7}(0.14)^7(0.86)^1\approx 0.00000725$

$\Pr(X=8)=\binom{8}{8}(0.14)^8(0.86)^0\approx 0.000000148$

$\Pr(\text{At most }6)=1-\Pr(X=7\text{ or }8)$
$=1-0.00000725-0.000000148$
$=0.999993$

**23.** $\Pr(X=7)=\binom{7}{7}(0.761)^7(0.239)^0\approx 0.1478$

**24.** $\Pr(X=3)=\binom{7}{3}(0.761)^3(0.239)^4\approx 0.0503$

**25.** $\Pr(X=6)=\binom{7}{6}(0.761)^6(0.239)^1\approx 0.3249$

$\Pr(X=6\text{ or }7)=0.3249+0.1478=0.4727$

**26.** $\Pr(X=0)=\binom{7}{0}(0.761)^0(0.239)^7\approx 0.0000445$

$\Pr(X=1)=\binom{7}{1}(0.761)^1(0.239)^6\approx 0.0009928$

$\Pr(X=2)=\binom{7}{2}(0.761)^2(0.239)^5\approx 0.0094837$

$\Pr(X=0\text{ or }1\text{ or }2)$
$=0.0000445+0.0009928+0.0094837$
$=0.01052$

**27.** **a.** $\Pr(X=2)=\binom{4}{2}\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^2=\frac{3}{8}=0.375$

**b.** $\Pr(X=3)=2\binom{4}{3}\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^1=\frac{8}{16}=0.5$

**c.** $\Pr(X=4)=2\binom{4}{4}\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)^0=\frac{2}{16}=0.125$

**28.** **a.** $\Pr(X=15)=\binom{20}{15}(0.6)^{15}(0.4)^5\approx 0.07465$

**b.** $\Pr(X=19)=\binom{20}{19}(0.6)^{19}(0.4)^1\approx 0.000487$

$\Pr(X=20)=\binom{20}{20}(0.6)^{20}(0.4)^0\approx 0.0000366$

$\Pr(\text{Fewer than }19)=1-\Pr(X=19\text{ or }20)$
$=1-0.000487-0.0000366$
$=0.9995$

**29.** $\Pr(X=3)=\binom{5}{3}(0.76)^3(0.24)^2\approx 0.2529$

$\Pr(X=5)=\binom{5}{5}(0.76)^5(0.24)^0\approx 0.2536$

The probability of getting 5 successes is higher.

**30.** Based on the histogram, the probability of getting 4 successes is higher. Out of a group of five students, it is more likely for exactly four of them to have been accepted to their first choice than it is for exactly three of them to have been accepted to their first choice.

**31.** Based on the histogram, the probability of getting 10 successes is higher. Out of a group of

40 cattle, it is more likely that exactly 10 recover than exactly 9 recover.

**32.** $\Pr(X=5)=\binom{40}{5}(0.25)^5(0.75)^{35}\approx 0.0272$

$\Pr(X=15)=\binom{40}{15}(0.25)^{15}(0.75)^{25}\approx 0.0282$

The probability of getting 15 successes is higher.

**33.** The probability that the salesman sells cars to three or four of the customers.

**34.** The proability that at least one of the 10 policyholders files a claim.

**35. a.** $1-0.4602-0.0796=0.4602$

**b.** $0.4602+0.0796=0.5398$

**36. a.** $1-0.9845-0.01302=0.00248$

**b.** $0.9845+0.01302=0.99752$

**37.** The probability of success is .5 therefore the histogram will be symmetrical about the middle value, or 5.

**38.** 0.5; because the histogram is symmetrical, half of the values will be greater than 5 and half will be 5 or less.

**39.** 1; because the sum of all individual probabilities is 1.

**40.** 1; because the sum of all individual probabilities is 1.

**41.** Let "success" = "lives to 100." Then $p = 0.075$, $q = 0.925$, $n = 77$.

$$\begin{aligned}\Pr(X\ge 2)&=1-\Pr(X=0)-\Pr(X=1)\\&=1-(0.925)^{77}\\&\quad-77(0.075)^1(0.925)^{76}\\&\approx 0.9821\end{aligned}$$

**42.** Let "success" = "left-handed." Then $p = 0.1$, $q = 0.9$, $n = 10$.

$$\begin{aligned}\Pr(X\ge 2)&=1-\Pr(X=0)-\Pr(X=1)\\&=1-(0.9)^{10}-10(0.1)^1(0.9)^9\\&\approx 0.2639\end{aligned}$$

**43.** Let "success" = "adverse reaction." Then $p = 0.02$, $q = 0.98$, $n = 56$.

$$\begin{aligned}\Pr(X\ge 3)&=1-\Pr(X=0)-\Pr(X=1)-\Pr(X=2)\\&=1-(0.98)^{56}-\binom{56}{1}(0.02)^1(0.98)^{55}\\&\quad-\binom{56}{2}(0.02)^2(0.98)^{54}\\&\approx 0.1018\end{aligned}$$

**44.** Let "success" be "brand X." Then $p = 0.3$, $q = 0.7$, $n = 9$.

$$\begin{aligned}&\Pr(X>2)\\&=1-\Pr(X\le 2)\\&=1-[\Pr(X=0)+\Pr(X=1)+\Pr(X=2)]\\&=1-\binom{9}{0}(0.3)^0(0.7)^9-\binom{9}{1}(0.3)^1(0.7)^8\\&\quad-\binom{9}{2}(0.3)^2(0.7)^7\\&=1-0.04035-0.15565-0.26683\\&=0.5372\end{aligned}$$

**45.** Let "success" be "a vote for" the candidate. Then $p = 0.6$, $q = 0.4$, $n = 5$.

$$\begin{aligned}&\Pr(X\le 2)\\&=\Pr(X=0)+\Pr(X=1)+\Pr(X=2)\\&=\binom{5}{0}(0.6)^0(0.4)^5+\binom{5}{1}(0.6)^1(0.4)^4+\binom{5}{2}(0.6)^2(0.4)^3\\&=0.01024+0.0768+0.2304\\&\approx 0.3174\end{aligned}$$

**46.** Let "success" be "a guilty vote." Then $p = 0.76$, $q = 0.24$, $n = 12$.

$\Pr(X\ge 10)=\Pr(X=10)+\Pr(X=11)+\Pr(X=12)$

$$\begin{aligned}&=\binom{12}{10}(0.7)^{10}(0.3)^2+\binom{12}{11}(0.7)^{11}(0.3)^1\\&\quad+\binom{12}{12}(0.7)^{12}(0.3)^0\\&=0.16779+0.07118+0.01384\\&=0.2528\end{aligned}$$

**47.** Let "success" = "defective." Then $p = 0.03$, $q = 0.97$, $n = 20$.

$$\Pr(X \geq 2) = 1 - \Pr(X = 0) - \Pr(X = 1)$$
$$= 1 - (0.97)^{20} - 20(0.03)^1(0.97)^{19}$$
$$\approx 0.1198$$

**48.** Let "success" be "defective." Then $p = 0.02$, $q = 0.98$, $n = 300$.

$$\Pr(X \leq 2) = \Pr(X = 0) + \Pr(X = 1) + \Pr(X = 2)$$
$$= \binom{300}{0}(0.02)^0(0.98)^{300} + \binom{300}{1}(0.02)^1(0.98)^{299} + \binom{300}{2}(0.02)^2(0.98)^{298}$$
$$\approx 0.06018$$

**49.**

| | | Mother A | Mother a |
|---|---|---|---|
| Father | A | AA | Aa |
| | a | Aa | aa |

| Child's genes | Probability |
|---|---|
| AA | $\frac{1}{4}$ |
| Aa | $\frac{2}{4}$ |
| aa | $\frac{1}{4}$ |

Let "success" be "aa." Then

$p = \frac{1}{4}$, $q = \frac{3}{4}$, $n = 3$.

$$\Pr(X \geq 1) = 1 - \Pr(X = 0)$$
$$= 1 - \binom{3}{0}\left(\frac{1}{4}\right)^0\left(\frac{3}{4}\right)^3$$
$$\approx 1 - 0.4219$$
$$= 0.5781$$

**50.** The first method has probability $1 - \left(\frac{99}{100}\right)^{100} \approx 0.6340$ of detecting the theft. In the second method, the probability of selecting 4 ingots from a single bin is $\frac{\binom{99}{4}}{\binom{100}{4}} = 0.96,$ so this method has probability $1 - (0.96)^{25} \approx 0.6396$ of detecting the theft, which is slightly better.

**51.** Let "success" be "gets a hit." Then $p = 0.3$, $q = 0.7$, $n = 4$.

$$\Pr(X = 0) = \binom{4}{0}(0.3)^0(0.7)^4 = 0.2401$$
$$\Pr(X = 3) = \binom{4}{3}(0.3)^3(0.7)^1 = 0.0756$$

**52.** Let "success" be "make the free throw." Then $p = 0.9$, $q = 0.1$, $n = 5$.

$$\Pr(X = 2) = \binom{5}{2}(0.9)^2(0.1)^3 = 0.0081$$

**53.** Here $p = 0.82$. The expected value is $\mu = np = 10(0.82) = 8.2$

$$\Pr(X = 8) = C(10,8)(0.82)^8(0.18)^2 = 0.2980$$
$$\Pr(X = 9) = C(10,9)(0.82)^9(0.18)^1 = 0.3017$$

So 9 is the most likely number.

**54.** Let "success" be a bull's-eye. Then $p = 0.64$, $q = 0.36$, $n = 10$.

$$\Pr(X = 6) = \binom{10}{6}(0.64)^6(0.36)^4 \approx 0.2424$$
$$\Pr(X = 7) = \binom{10}{7}(0.64)^7(0.36)^3 \approx 0.2462$$

So 7 bull's-eyes is the most probable number.

**55. a.** For the underdog to win the series in five sets, the underdog must win two of the first four sets and win the fifth set. The probability that this occurs is

$$\binom{4}{2}(1-p)^2p^2(1-p) = \binom{4}{2}(1-p)^3p^2.$$

**b.** Using the same reasoning,

$$\Pr(\text{underdog wins in 3 sets}) = (1-p)^3$$

$$\Pr(\text{underdog wins in 4 sets}) = \binom{3}{2}(1-p)^2 p(1-p) = \binom{3}{2}(1-p)^3 p$$

**c.**

$$\Pr(\text{underdog wins set}) = (1-p)^3\left(1+\binom{3}{1}p+\binom{4}{2}p^2\right) = (1-p)^3(1+3p+6p^2) = (1-.7)^3(1+3(.7)+6(.7)^2) = .16308$$

**d.** Suppose all five sets of the match are played even if one of the players wins three sets before the fifth set. Then the probability that the underdog wins at least three sets is

$$\binom{5}{0}(1-p)^5+\binom{5}{1}p(1-p)^4+\binom{5}{2}p^2(1-p)^3.$$

Substituting $p = .7$ gives a probability $\approx 0.16308$.

**56. a.** For the underdog to win the series in five games, the underdog must win three of the first four games and win the fifth game. The probability that this occurs is

$$\binom{4}{3}(1-p)^3 p(1-p) = \binom{4}{3}(1-p)^4 p.$$

**b.** Using the same reasoning,

$$\Pr(\text{underdog wins in 4 games}) = (1-p)^4$$

$$\Pr(\text{underdog wins in 6 games}) = \binom{5}{3}(1-p)^3 p^2(1-p) = \binom{5}{3}(1-p)^4 p^2$$

$$\Pr(\text{underdog wins in 7 games}) = \binom{6}{3}(1-p)^3 p^3(1-p) = \binom{6}{3}(1-p)^4 p^3$$

**c.** Pr(underdog wins series)

$$= (1-p)^4\left(1+\binom{4}{3}p+\binom{5}{3}p^2+\binom{6}{3}p^3\right) = (1-p)^4(1+4p+10p^2+20p^3) = (1-0.6)^4(1+4(0.6)+10(0.6)^2+20(0.6)^3) = 0.289792$$

**d.** Suppose all seven games of the World Series are played even if one of the teams wins four games before the seventh game. Then the probability that the underdog wins at least four games is

$$\binom{7}{0}(1-p)^7+\binom{7}{1}p(1-p)^6+\binom{7}{2}p^2(1-p)^5+\binom{7}{3}p^3(1-p)^4.$$

Substituting $p = .6$ gives a probability $\approx .289792$.

**57.** 17; because $1-\binom{17}{0}\left(\frac{1}{6}\right)^0\left(\frac{5}{6}\right)^{17} = 0.9549$

**58.** 37; because

$$1-\binom{37}{0}\left(\frac{1}{6}\right)^0\left(\frac{5}{6}\right)^{37}-\binom{37}{1}\left(\frac{1}{6}\right)^1\left(\frac{5}{6}\right)^{36} = 0.9901$$

**59.** 114; because

$$1-\binom{114}{0}(0.00045)^0(0.99955)^{114} = 0.0500$$

**60.** Let "success" = "becomes centenarian." Then $p = 0.075$, $q = 0.925$, $n$ is unknown.

$$\Pr(X\ge 2) = 1-\Pr(X=0)-\Pr(X=1) = 1-(0.925)^n-\binom{n}{1}(0.075)^1(0.925)^{n-1} = 1-(0.925)^n-n(0.075)(0.925)^{n-1}$$

$\Pr(X \ge 2) > 0.9 \Leftrightarrow$

$(0.925)^n + n(0.075)(0.925)^{n-1} < 0.1 \Leftrightarrow$

$(0.925)^{n-1}(0.925+n(0.075)) < 0.1$

The smallest value of $n$ for which this occurs is $n = 51$.

**61.** Using a TI83 graphing calculator input the following steps:

binomcdf(100, .5, 60) – binomcdf(100, .5, 39)

the answer will be approximately 0.9648.

**62.** Using a TI83 graphing calculator input the following steps:
binomcdf(200, $\frac{1}{6}$, 40) − binomcdf(200, $\frac{1}{6}$, 19)
the answer will be approximately 0.9079.

**63.** Using a TI83 graphing calculator input the following steps:
1 − binomcdf(100, 0.6, 59) ≈ 0.5433

**64.** Let "success" = "eventually contracts disease." Then $p = .7$, $q = .3$, $n = 100$.
$\Pr(X \geq 70) = 1 - \text{binomcdf}(100, .7, 69) \approx 0.5491$

**65. a.** Substituting $p = 0.6$ in this formula gives a probability ≈ 0.2898.

**b.** Suppose as in Problem 56(d) above that all $2n - 1$ games are played. Then the probability that the favorite wins at least $n$ games is

$$\binom{2n-1}{n} p^n (1-p)^{n-1} + \binom{2n-1}{n+1} p^{n+1}(1-p)^{n-2} + \text{L} + \binom{2n-1}{2n-1} p^{2n-1}.$$

Some experimenting shows that for $p = .6$, $n = 21$ is the smallest value of $n$ for which this probability exceeds .9. Since the number of games is $2n - 1 = 2(21) - 1 = 41$.

**66.** Using a TI84 graphing calculator input the following steps:
binomcdf(50, 0.45, 25) ≈ 0.8034

**67.** Using a TI84 graphing calculator input the following steps:
1-binomcdf(80, 0.55, 39) ≈ 0.8441

**68.** Using a TI84 graphing calculator input the following steps:
binomcdf(90, 0.40, 55) − binomcdf(90, 0.40, 44)
≈ 0.0346

**69.** Using a TI84 graphing calculator input the following steps:
binomcdf(50, 0.25, 20) − binomcdf(50, 0.25, 9)
≈ 0.8301

**Exercises 7.4**

**1.** Population mean

**2.** Sample mean

**3.** Expected value

**4.** Expected value

**5.** E($X$) = 0(0.25) + 1(0.2) + 2(0.1) + 3(0.25) + 4(0.2) = 1.95

**6.**
$$E(X) = -1(0.1) - \frac{1}{2}(0.4) + 0(0.25) + \frac{1}{2}(0.2) + 1(0.05) = -0.15$$

**7. a.**
$$\text{GPA} = \frac{4+4+4+4+3+3+2+2+2+1}{10} = \frac{29}{10} = 2.9$$

**b.**

| Grade | Relative Frequency |
|---|---|
| 4 | $\frac{4}{10} = 0.4$ |
| 3 | $\frac{2}{10} = 0.2$ |
| 2 | $\frac{3}{10} = 0.3$ |
| 1 | $\frac{1}{10} = 0.1$ |

**c.** $E(X) = 4(0.4) + 3(0.2) + 2(0.3) + 1(0.1)$
$= 2.9$

**8. a.** $\bar{x} = \frac{9.8 + 9.8 + 9.4 + 9.2 + 9.2 + 9.0}{6}$
$= \frac{56.4}{6}$
$= 9.4$

**b.**

| Score | Relative Frequency |
|---|---|
| 9.8 | $\frac{2}{6}$ |
| 9.4 | $\frac{1}{6}$ |
| 9.2 | $\frac{2}{6}$ |
| 9.0 | $\frac{1}{6}$ |

**c.** $\bar{x} = 9.8\left(\frac{2}{6}\right) + 9.4\left(\frac{1}{6}\right) + 9.2\left(\frac{2}{6}\right) + 9.0\left(\frac{1}{6}\right)$
$= 9.4$

**9.** $\bar{x}_A = 0(0.3) + 1(0.3) + 2(0.2) + 3(0.1) + 4(0) + 5(0.1)$
$= 1.5$
$\bar{x}_B = 0(0.2) + 1(0.3) + 2(0.3) + 3(0.1) + 4(0.1) + 5(0)$
$= 1.6$
Group $A$ had fewer cavities.

**10.** $\bar{x}_A = 1000(0.2) + 2000(0.6) + 3000(0.2) = 2000$
$\bar{x}_B = -2000(0.2) + 0(0.2) + 4000(0.6) = 2000$
Both investments have the same expected returns.

**11.** $E(X) = 1(0.2) + 2(0.4) + 3(0.3) + 4(0.1)$
$= 2.3$

**12.** $E(X) = 3(0.2) + 4(0.1) + 5(0.4) + 6(0.2) + 7(0.1)$
$= 4.9$

**13.** $E(X) = 7(0.25) + 8(0.25) + 9(0.25) + 10(0.25)$
$= 8.5$

**14.** $E(X) = 2(.05) + 3(.1) + 4(.2) + 5(.3) + 6(.2) + 7(.1) + 8(.05)$
$= 5$

**15.**

| Earnings | Probability |
|---|---|
| –\$1 | $\frac{37}{38}$ |
| \$35 | $\frac{1}{38}$ |

$E(X) = -1\left(\frac{37}{38}\right) + 35\left(\frac{1}{38}\right) \approx -\$0.0526$

**16.** Let $w$ be the amount won.
$E(x) = -1\left(\frac{36}{38}\right) + w\left(\frac{2}{38}\right) = -\frac{1}{19}$
$w = \frac{-\frac{1}{19} + \frac{36}{38}}{\frac{2}{38}} = \$17$

**17.**

| Earnings | Probability |
|---|---|
| –50¢ | $\frac{2}{6} = \frac{1}{3}$ |
| 0¢ | $\left(\frac{4}{6}\right)\left(\frac{2}{5}\right) = \frac{4}{15}$ |
| 50¢ | $\left(\frac{4}{6}\right)\left(\frac{3}{5}\right)\left(\frac{2}{4}\right) = \frac{1}{5}$ |
| \$1 | $\left(\frac{4}{6}\right)\left(\frac{3}{5}\right)\left(\frac{2}{4}\right)\left(\frac{2}{3}\right) = \frac{2}{15}$ |
| \$1.50 | $\left(\frac{4}{6}\right)\left(\frac{3}{5}\right)\left(\frac{2}{4}\right)\left(\frac{1}{3}\right)\left(\frac{2}{2}\right) = \frac{1}{15}$ |

$E(X) = -.5\left(\frac{1}{3}\right) + 0\left(\frac{4}{15}\right) + .5\left(\frac{1}{5}\right) + 1\left(\frac{2}{15}\right) + 1.5\left(\frac{1}{15}\right)$
$\approx \$.1667$

**18.**

| Winnings | Probability |
|---|---|
| \$2 | $\left(\frac{2}{8}\right)\left(\frac{1}{7}\right) = \frac{1}{28}$ |
| \$1 | $\left(\frac{2}{8}\right)\left(\frac{6}{8}\right) + \left(\frac{6}{8}\right)\left(\frac{2}{7}\right) = \frac{12}{28}$ |
| \$0 | $\left(\frac{6}{8}\right)\left(\frac{5}{7}\right) = \frac{15}{28}$ |

$E(X) = 2\left(\frac{1}{28}\right) + 1\left(\frac{12}{28}\right) + 0\left(\frac{15}{28}\right) = 0.5$
The player should pay 50¢ per play to break even.

**19.** Let $x$ be the cost of the policy.
$E(X) = (-x)(0.9) + (10{,}000 - x)(0.1) = -x + 1000$
The expected value is zero if $x = 1000$.
He should be willing to pay up to $1000.

**20.** The probability the man dies is 0.1.
The probability the woman dies is 0.05.
Assuming independent life spans, the probability both the man and the woman die in the next 5 years is (0.1)(0.05) = 0.005. Therefore, the probability that only the man dies is (0.1)(0.95) = 0.095 and the probability that only the woman dies is (0.9)(0.05) = 0.045. The probability they both live is (0.9)(0.95) = 0.855.
Let $x$ be the cost of the policy.
$$\begin{aligned}E(X) &= (-x)(0.855) + (10{,}000 - x)(0.095) + (10{,}000 - x)(0.045) + (15{,}000 - x)(0.005)\\ &= -x + 1475.\end{aligned}$$
The expected value is zero if $x = 1475$. They should be willing to pay up to $1475.

**21.** $\dfrac{5\times 0.300 + 4\times 0.350}{9} \approx 0.322$

**22.** $\dfrac{3\times 91 + 5\times 87}{8} = \dfrac{708}{8} = 88.5$

**23.**

| Recorded Value | Probability |
|---|---|
| 1 | $\frac{1}{36}$ |
| 2 | $\frac{3}{36}$ |
| 3 | $\frac{5}{36}$ |
| 4 | $\frac{7}{36}$ |
| 5 | $\frac{9}{36}$ |
| 6 | $\frac{11}{36}$ |

$$E(X) = 1\left(\frac{1}{36}\right) + 2\left(\frac{3}{36}\right) + 3\left(\frac{5}{36}\right) + 4\left(\frac{7}{36}\right) + 5\left(\frac{9}{36}\right) + 6\left(\frac{11}{36}\right) \approx 4.47$$

**24.**

| Smaller Value | Probability |
|---|---|
| 1 | $\frac{11}{36}$ |
| 2 | $\frac{9}{36}$ |
| 3 | $\frac{7}{36}$ |
| 4 | $\frac{5}{36}$ |
| 5 | $\frac{3}{36}$ |
| 6 | $\frac{1}{36}$ |

$$\mathrm{E}(X)=1\left(\frac{11}{36}\right)+2\left(\frac{9}{36}\right)+3\left(\frac{7}{36}\right)+4\left(\frac{5}{36}\right)+5\left(\frac{3}{36}\right)+6\left(\frac{1}{36}\right)=\frac{91}{36}\approx 2.528$$

**25.** $\mathrm{E}(X)=-1\left(\frac{125}{216}\right)+1\left(\frac{75}{216}\right)+3\left(\frac{15}{216}\right)+5\left(\frac{1}{216}\right)=\frac{0}{216}=0$

**26.**

| House Winnings | Probability |
|---|---|
| 0 | $\frac{5}{9}$ |
| 1 | $\frac{5}{12}$ |
| 2 | $\frac{1}{36}$ |

$$\mathrm{E}(X)=0\left(\frac{5}{9}\right)+1\left(\frac{5}{12}\right)+2\left(\frac{1}{36}\right)=\frac{17}{36}\approx 47.22 \text{ cents}$$

**27.** $\mathrm{E}(X)=15(0.205)=3.075$

**28.** $\mathrm{E}(X)=30(0.71)=21.3$

**29.** The expected number of times that a 5 or a 6 will appear is $\mu=np=30\left(\frac{1}{3}\right)=10$

**30.** The expected number of hits is $\mu=np=40(0.275)=11$

**31.** The expected value of points when taking one three point shot is $\mu = 3(0.40) = 1.2$ points. The expected value of taking three free-throws when the success probability is 60% is $\mu = 3(0.60) = 1.8$, which is the greater expected value.

**32.** To calculate the probability of success for a binomial random variable with 20 trials whose expected value is 3, use the formula: $\mu=np$. Substitute 3 for $\mu$ and 20 for $n$: $3 = 20p$. Thus, $p = 0.15$.

**33.**

| Number of Republicans | Probability |
|---|---|
| 0 | $\frac{10}{84}$ |
| 1 | $\frac{40}{84}$ |
| 2 | $\frac{30}{84}$ |
| 3 | $\frac{4}{84}$ |

$$\text{E(Rep.)} = 0\left(\frac{10}{84}\right)+1\left(\frac{40}{84}\right)+2\left(\frac{30}{84}\right)+3\left(\frac{4}{84}\right)=\frac{4}{3}\approx 1.333$$

$$\text{E(Dem.)} = 3-E(\text{Rep.}) = 3-\frac{4}{3}=\frac{5}{3}\approx 1.667$$

**34.**

| Number of Red | Probability |
|---|---|
| 0 | $\frac{1}{210}$ |
| 1 | $\frac{24}{210}$ |
| 2 | $\frac{90}{210}$ |
| 3 | $\frac{80}{210}$ |
| 4 | $\frac{15}{210}$ |

$$\text{E(Red)} = 0\left(\frac{1}{210}\right)+1\left(\frac{24}{210}\right)+2\left(\frac{90}{210}\right)+3\left(\frac{80}{210}\right)+4\left(\frac{15}{210}\right)$$
$$=\frac{12}{5}=2.4$$

$$\text{E(Green)} = 4-E(\text{Red}) = 4-\frac{12}{5}=\frac{3}{5}=1.6$$

**35.** No Replacing

| Number of Red | Probability |
|---|---|
| 0 | $\frac{1}{35}$ |
| 1 | $\frac{12}{35}$ |
| 2 | $\frac{18}{35}$ |
| 3 | $\frac{4}{35}$ |

$$\text{E(Red)} = 0\left(\frac{1}{35}\right)+1\left(\frac{12}{35}\right)+2\left(\frac{18}{35}\right)+3\left(\frac{4}{35}\right)$$
$$=\frac{12}{7}\approx 1.714$$

Replacing

| Number of Red | Probability |
|---|---|
| 0 | $\frac{27}{343}$ |
| 1 | $\frac{108}{343}$ |
| 2 | $\frac{144}{343}$ |
| 3 | $\frac{64}{343}$ |

$$\text{E(Red)} = 0\left(\frac{27}{343}\right)+1\left(\frac{108}{343}\right)+2\left(\frac{144}{343}\right)+3\left(\frac{64}{343}\right)$$
$$=\frac{12}{7}\approx 1.714$$

**36.** No Replacing

| Number of Hearts | Probability |
|---|---|
| 0 | $\frac{741}{1326}$ |
| 1 | $\frac{507}{1326}$ |
| 2 | $\frac{78}{1326}$ |

$$\begin{aligned}\text{E(Heart)} &= 0\left(\frac{741}{1326}\right)+1\left(\frac{507}{1326}\right)+2\left(\frac{78}{1326}\right)\\ &= \frac{1}{2}=0.5\end{aligned}$$

| Replacing Number of Hearts | Probability |
|---|---|
| 0 | $\frac{9}{16}$ |
| 1 | $\frac{3}{8}$ |
| 2 | $\frac{1}{16}$ |

$$\begin{aligned}\text{E(Heart)} &= 0\left(\frac{9}{16}\right)+1\left(\frac{3}{8}\right)+2\left(\frac{1}{16}\right)\\ &= \frac{1}{2}=0.5\end{aligned}$$

**37.** Let $x$ = chance of rain.

$$\begin{aligned}-8000+40,000x &= 0\\ x &= 0.20 \rightarrow 20\%\end{aligned}$$

**38.** Let $x$ = probability stolen.

$$\begin{aligned}-150+75,000x &= -250+100,000x\\ 100 &= 25,000x\\ x &= 0.004\end{aligned}$$

**39.** $\dfrac{7x+4y}{x+y}$

Answer (b) is correct.

**40.** $\dfrac{kx+y}{k+1}$

Answer (b) is correct.

**41.** Solve $\dfrac{16\cdot 54+14x}{30}=56.1$

$$\begin{aligned}14x+864 &= 1683\\ 14x &= 819\\ x &= 58.5°\end{aligned}$$

**42.** Solve $\dfrac{83+92+89+3x}{6}=90$

$$\begin{aligned}3x+264 &= 540\\ 3x &= 276\\ x &= 92\end{aligned}$$

**43.** $\dfrac{5+6+x}{3}=\dfrac{2+7+9}{3}$

$$\begin{aligned}11+x &= 18\\ x &= 7\end{aligned}$$

**44.**

$$\begin{aligned}-2(0.1)+-1(0.2)+0(0.6)+x(0.1) &= 0\\ -0.2-0.2+0+0.1x &= 0\\ -0.4+0.1x &= 0\\ 0.1x &= 0.4\\ x &= 4\end{aligned}$$

**45.** Solve $\dfrac{12\cdot 5+24x}{5+x}=19$

$$\begin{aligned}19x+95 &= 60+24x\\ -5x &= -35\\ x &= 7\end{aligned}$$

**46.** Let $x$ = revenue for third week

$2x$ = revenue for first week

$\frac{1}{2}x$ = revenue for second week.

$$\begin{aligned}\frac{x+2x+\frac{1}{2}x}{3} &= 14,000\\ \frac{7}{2}x &= 42,000\\ x &= \$12,000\\ 2x &= \$24,000\end{aligned}$$

**47.** Let $x$ = number of cases.

$$\begin{aligned}20\left(\frac{1}{2}x\right)+30\left(\frac{1}{2}x\right) &= 75,000\\ 25x &= 75,000\\ x &= 3000 \text{ cases}\end{aligned}$$

**48.** Let $x$ = number of magazines.

$$\begin{aligned}2.00\left(\frac{1}{2}x\right)+2.50\left(\frac{1}{2}x\right) &= 135.00\\ 2.25x &= 135.00\\ x &= 60 \text{ magazines}\end{aligned}$$

**Exercises 7.5**

**1.** $m = 70(0.5) + 71(0.2) + 72(0.1) + 73(0.2) = 71$

$$\begin{aligned}\sigma^2 &= (70-71)^2(0.5)+(71-71)^2(0.2)+(72-71)^2(0.1)+(73-71)^2(0.2)\\ &= 0.5+0+0.1+0.8\\ &= 1.4\end{aligned}$$

**2.** $\mu = -1\left(\frac{1}{8}\right)-\frac{1}{2}\left(\frac{3}{8}\right)+0\left(\frac{1}{8}\right)+\frac{1}{2}\left(\frac{1}{8}\right)+1\left(\frac{2}{8}\right)=0$

$$\begin{aligned}\sigma^2 &= (-1-0)^2\left(\frac{1}{8}\right)+\left(-\frac{1}{2}-0\right)^2\left(\frac{3}{8}\right)+(0-0)^2\left(\frac{1}{8}\right)+\left(\frac{1}{2}-0\right)^2\left(\frac{1}{8}\right)+(1-0)^2\left(\frac{2}{8}\right)\\ &= \frac{1}{8}+\frac{3}{32}+0+\frac{1}{32}+\frac{2}{8}\\ &= \frac{1}{2}\end{aligned}$$

**3.** *B*

**4.** *C*

**5. a.** $\mu_A = -10(0.2)+20(0.2)+25(0.6)=17$

$\mu_B = 0(0.3)+10(0.4)+30(0.3)=13$

$\sigma_A^2 = (-10-17)^2(0.2)+(20-17)^2(0.2)+(25-17)^2(0.6)=145.8+1.8+38.4=186$

$\sigma_B^2 = (0-13)^2(0.3)+(10-13)^2(0.4)+(30-13)^2(0.3)=50.7+3.6+86.7=141$

**b.** Investment A

**c.** Investment B

**6.**

| Golfer A | | Golfer B | |
|---|---|---|---|
| Score | Frequency | Score | Frequency |
| 39 | 2 | 40 | 3 |
| 40 | 6 | 41 | 4 |
| 41 | 7 | 42 | 5 |
| 42 | 1 | 43 | 6 |
| 43 | 3 | 44 | 2 |
| 44 | 1 | | |

**a.** 
$$\begin{aligned}\bar{X}_A &= \frac{39(2)+40(6)+41(7)+42(1)+43(3)+44(1)}{20}\\ &= \frac{820}{20}\\ &= 41\end{aligned}$$

$$s_A^2 = \frac{1}{19}[(39-41)^2(2)+(40-41)^2(6)+(41-41)^2(7)+(42-41)^2(1)+(43-41)^2(3)+(44-41)^2(1)]$$

$$
\begin{aligned}
&= \frac{1}{19}[8+6+0+1+12+9] \\
&= \frac{36}{19} \\
&\approx 1.895 \\
\overline{X}_B &= \frac{40(3)+41(4)+42(5)+43(6)+44(2)}{20} \\
&= \frac{840}{20} \\
&= 42 \\
s_B^2 &= \frac{1}{19}[(40-42)^2(3)+(41-42)^2(4)+(42-42)^2(5)+(43-42)^2(6)+(44-42)^2(2)] \\
&= \frac{1}{19}[12+4+0+6+8] \\
&= \frac{30}{19} \\
&\approx 1.579
\end{aligned}
$$

**b.** Golfer A is better.

**c.** Golfer B is more consistent.

**7. a.** $\mu_A = 100(0.1)+101(0.2)+102(0.3)+103(0)+104(0)+105(0.2)+106(0.2)=103$

$\sigma_A^2 = (100-103)^2(0.1)+(101-103)^2(0.2)+\cdots+(106-103)^2(0.2)=4.6$

$\mu_B = 100(0)+101(0.2)+102(0)+103(0.2)+104(0.1)+105(0.2)+106(0.3)=104$

$\sigma_B^2 = (100-104)^2(0)+(101-104)^2(0.2)+\cdots+(106-104)^2(0.3)=3.4$

**b.** Business B

**c.** Business B

**8. a.**

| Student A | | Student B | |
|---|---|---|---|
| Grade | Relative Frequency | Grade | Relative Frequency |
| 4 | 0.3 | 4 | 0.6 |
| 3 | 0.3 | 3 | 0.1 |
| 2 | 0.3 | 2 | 0 |
| 1 | 0.1 | 1 | 0.3 |

$\mu_A = 4(0.3)+3(0.3)+2(0.3)+1(0.1)=2.8$

$\sigma_A^2 = (4-2.8)^2(0.3)+(3-2.8)^2(0.3)+(2-2.8)^2(0.3)+(1-2.8)^2(0.1)=0.96$

$\mu_B = 4(0.6)+3(0.1)+2(0)+1(0.3)=3$

$\sigma_B^2 = (4-3)^2(0.6)+(3-3)^2(0.1)+(2-3)^2(0)+(1-3)^2(0.3)=1.8$

**b.** Student B

**c.** Student A

**9.** The number of heads is a binomial random variable $X$ with $n = 10, p = 0.5$ so $\mu_X = 10\times 0.5 = 5,$
$\sigma_X = \sqrt{10\times 0.5\times 0.5} \approx 1.581.$

**10.** The number of times the sum seven appears is a binomial random variable $X$ with $n = 720,\ p = \frac{1}{6}$ so

$\mu_X = 720\times\frac{1}{6} = 120,\ \sigma_X = \sqrt{720\times\frac{1}{6}\times\frac{5}{6}} = 10.$

**11.** The number of smart thermostats is a binomial random variable $X$ with $n = 200, p = 0.015$ so
$\mu_X = 200\times 0.015 = 3,\ \sigma_X = \sqrt{200\times 0.015\times 0.985} \approx 1.719.$

**12.** The number of successful free throws is a binomial random variable $X$ with $n = 20,\ p = \frac{3}{5}$ so $\mu_X = 20\times\frac{3}{5} = 12,$

$\sigma_X = \sqrt{20\times\frac{3}{5}\times\frac{2}{5}} \approx 2.191.$

**13. a.** $35 - c = 25$ and $35 + c = 45$ $c = 10.$

Probability $\geq 1 - \frac{5^2}{10^2} = 1 - \frac{25}{100} = .75$

**b.** $35 - c = 20$ and $35 + c = 50$ $c = 15$

Probability $\geq 1 - \frac{5^2}{15^2} \approx 0.89$

**c.** $35 - c = 29$ and $35 + c = 41$ $c = 6$

Probability $\geq 1 - \frac{5^2}{6^2} \approx 0.31$

**14. a.** $8 - c = 6$ and $8 + c = 10$ $c = 2$

Probability $\geq 1 - \frac{0.4^2}{2^2} = 0.96$

**b.** $8 - c = 7.2$ and $8 + c = 8.8$ . $c = .8$

Probability $\geq 1 - \frac{0.4^2}{0.8^2} = 0.75$

**c.** $8 - c = 7.5$ and $8 + c = 8.5$ . $c = .5$

Probability $\geq 1 - \frac{0.4^2}{0.5^2} = 0.36$

**15.** $\mu = 3000, \sigma = 250$
$3000 - c = 2000$ and $3000 + c = 4000$ $c = 1000$

Probability $\geq 1 - \frac{250^2}{1000^2} = 0.9375$

Number of bulbs to replace: $\geq 5000(0.9375) \approx 4688$

**16.** $\mu = 15, \sigma = 10$

$15 - c = 0$ and $15 + c = 30 \qquad c = 15$

$$\text{Probability} \geq 1 - \frac{10^2}{15^2} = \frac{125}{225} = \frac{5}{9}$$

$$\text{Number of batches} \geq 100\left(\frac{5}{9}\right) \approx 56$$

**17.** $\text{Probability} = 1 - \frac{6^2}{c^2} = \frac{7}{16}$

$$16c^2 - 576 = 7c^2$$
$$9c^2 = 576$$
$$c = \sqrt{64} = 8$$

**18.** $1 - \frac{0.2^2}{c^2} = \frac{15}{16}$

$$16c^2 - 0.64 = 15c^2$$
$$c^2 = 0.64$$
$$c = 0.8$$

**19.** **a.** $\mu = 2\left(\frac{1}{36}\right) + 3\left(\frac{2}{36}\right) + \cdots + 12\left(\frac{1}{36}\right) = 7$

$$\sigma^2 = (2-7)^2\left(\frac{1}{36}\right) + \cdots + (12-7)^2\left(\frac{1}{36}\right)$$
$$= \frac{210}{36}$$
$$= \frac{35}{6}$$

**b.** $\Pr(4 \leq X \leq 10)$

$$= \frac{3}{36} + \frac{4}{36} + \frac{5}{36} + \frac{6}{36} + \frac{5}{36} + \frac{4}{36} + \frac{3}{36}$$
$$= \frac{30}{36}$$
$$= \frac{5}{6}$$

**c.** $7 - c = 4$ and $7 + c = 10 \qquad c = 3$

$$\text{Probability} \geq 1 - \frac{\frac{35}{6}}{3^2} = \frac{19}{54}$$

**20.** **a.** $\Pr(0 \leq X \leq 4) \approx 0.112 + 0.269 + 0.296 + 0.197 + 0.089 = 0.963$

**b.** $2 - c = 0$ and $2 + c = 4 \qquad c = 2$

$$\text{Probability} \geq 1 - \frac{\frac{5}{3}}{2^2} \approx 0.583$$

**21.** $E(X) = (-2 - 1 + 0 + 1 + 2)(.2) = 0$

$E(X^2) = (4+1+0+1+4)(.2) = 2$

$\text{Var}(X) = E(X^2) - E(X)^2 = 2 - 0 = 2$

**22.**

| $X - 70$ | Probability |
|---|---|
| 0 | 0.5 |
| 1 | 0.2 |
| 2 | 0.1 |
| 3 | 0.2 |

$\mu = 0(0.5) + 1(0.2) + 2(0.1) + 3(0.2) = 1$

$\sigma^2 = (0-1)^2(0.5) + (1-1)^2(0.2) + (2-1)^2(0.1) + (3-1)^2(0.2) = 0.5 + 0 + 0.1 + 0.8 = 1.4$

**23.**

| $2X$ | Probability |
|---|---|
| –2 | $\frac{1}{8}$ |
| –1 | $\frac{3}{8}$ |
| 0 | $\frac{1}{8}$ |
| 1 | $\frac{1}{8}$ |
| 2 | $\frac{2}{8}$ |

$\mu = -2\left(\frac{1}{8}\right) - 1\left(\frac{3}{8}\right) + \cdots + 2\left(\frac{2}{8}\right) = 0$

$\sigma_{2X}^2 = (-2-0)^2\left(\frac{1}{8}\right) + \cdots + (2-0)^2\left(\frac{2}{8}\right) = 2$

$\sigma_{2X}^2 = 4\sigma_X^2 = 4\left(\frac{1}{2}\right) = 2$

**24.** If the same value *a* is subtracted from each value, all value are decreased by that value so it should be intuitive that the expected value of the random variable $X - a$ would be equal to the expected value of the random variable *X* minus the constant a. Also, if the same value *a* is multiplied by every value, the expected value of a random variable times a constant should be equal to the constant times the expected value of the random variable.

**25.** $\mu = \dfrac{60{,}168 + 59{,}770 + \cdots + 46{,}817}{10} \approx 53{,}227$

$\sigma^2 \approx \dfrac{(60{,}168 - 53{,}227)^2 + \cdots + (46{,}817 - 53{,}227)^2}{10}$

$\sigma \approx 4453.54$

The schools within one standard deviation of the mean are University of Florida, Michigan State University, University of Texas at Austin, University of Minnesota, Texas A & M University, and Florida International University

**26.** $\mu = \frac{67,255 + 66,383 + \cdots + 64,325}{8} \approx \$65,427$

$\sigma^2 = \frac{(67,255 - 65,427)^2 + \cdots + (64,325 - 65,427)^2}{8}$

$\sigma \approx \$976.57$

The only school with costs at least one standard deviation greater than the mean is Harvey Mudd College.

**27.** a. $\mu_A = \frac{(3)(5) + (4)(7) + (5)(8) + (6)(2) + (7)(1) + (8)(2)}{25} = 4.72$

$\sigma_A^2 = \frac{(3 - 4.72)^2(5) + \cdots + (8 - 4.72)^2(2)}{25} = 1.9616$

$\sigma_A \approx 1.40$

$\mu_B = \frac{(3)(5) + (4)(10) + (5)(3) + (6)(3) + (7)(0) + (8)(4)}{25} = 4.8$

$\sigma_B^2 = \frac{(3 - 4.8)^2(5) + \cdots + (8 - 4.8)^2(4)}{25} = 2.72$

$\sigma_B \approx 1.65$

**b.** University B

**c.** University A

**28.** a. $\mu_A = (-5)(0.23) + (-1)(0.32) + (1)(0.35) + (5)(0.07) + (10)(0.03) = -0.47$

$\sigma_A^2 = (-5 + 0.47)^2(0.23) + \cdots + (10 + 0.47)^2(0.03) = 10.9491$

$\sigma_A \approx 3.31$

$\mu_B = (-5)(0.32) + (-1)(0.10) + (1)(0.40) + (5)(0.13) + (10)(0.05) = -0.15$

$\sigma_B^2 = (-5 + 0.15)^2(0.32) + \cdots + (10 + 0.15)^2(0.05) = 16.7275$

$\sigma_B \approx 4.09$

**b.** Game B

**c.** Game A

**Exercises 7.6**

**1.** $A(1.25) = 0.8944$

**2.** $\Pr(-.75 \le Z \le 1) = A(1) - A(-.75)$
$= 0.8413 - 0.2266$
$= 0.6147$

**3.** $1 - A(.25) = 1 - 0.5987 = 0.4013$

**4.** $\Pr(Z \le -1) + \Pr(Z \ge 1) = A(-1) + (1 - A(1))$
$= 0.1587 + (1 - 0.8413)$
$= 0.3174$

**5.** $A(1.5) - A(.5) = 0.9332 - 0.6915 = 0.2417$

**6.** $\Pr(Z \le -1) = A(-1) = 0.1587$

**7.** $A(-0.5) + (1 - A(0.5)) = 0.3085 + (1 - 0.6915)$
$= 0.6170$

**8.** $\Pr(Z \ge -1.25) = 1 - A(-1.25) = 1 - 0.1056 = 0.8944$

**9.** $\Pr(Z \ge z) = 0.0401$
$A(z) = 1 - 0.0401 = 0.9599$
$z = 1.75$

**10.** $\Pr(Z \le -z) + \Pr(Z \ge z) = 2A(-z) = 0.0456$
$$A(-z) = \frac{0.0456}{2} = 0.0228$$
$-z = -2.00$
$z = 2.00$

**11.** $\Pr(-z \le Z \le z) = 0.5468$
$$A(-z) = \frac{1 - 0.5468}{2} = 0.2266$$
$-z = -0.75$
$z = 0.75$

**12.** $\Pr(Z \ge -z) = 1 - A(-z) = 0.6915$
$A(-z) = 1 - 0.6915 = 0.3085$
$z = 0.50$

**13.** The 80th percentile of the standard normal distribution is approximately 0.84 (use table or InvNorm(0.80) on TI 83)

**14.** The 55th percentile of the standard normal distribution is approximately 0.13.

**15.** $\mu = 6, \sigma \approx 2$

**16.** $\mu = 80, \sigma \approx 10$

**17.** $\mu = 9, \sigma \approx 1$

**18.** $\mu = 3.0, \sigma \approx 0.10$

**19.** $$\frac{4-8}{\frac{3}{4}} = -\frac{4}{1}\cdot\frac{4}{3} = -\frac{16}{3}$$

**20.** $$\frac{9\frac{1}{4}-8}{\frac{3}{4}} = \frac{5}{4}\cdot\frac{4}{3} = \frac{5}{3}$$

**21.** $$\frac{x-8}{\frac{3}{4}} = 10$$
$$x = \frac{30}{4} + 8 = \frac{62}{4} = \frac{31}{2} = 15\frac{1}{2}$$

**22.** $$\frac{x-8}{\frac{3}{4}} = -2$$
$$x = -\frac{3}{2} + 8 = \frac{13}{2} = 6\frac{1}{2}$$

**23.** $$\begin{aligned}\Pr(X \ge 9) &= \Pr\left(Z \ge \frac{9-10}{\frac{1}{2}}\right)\\ &= \Pr(Z \ge -2)\\ &= 1 - \Pr(Z \le -2)\\ &= 1 - 0.0228\\ &= 0.9772\end{aligned}$$

**24.** $$\begin{aligned}\Pr(X \le 32) &= \Pr\left(Z \le \frac{32-30}{4}\right)\\ &= \Pr(Z \le 0.5)\\ &= A(0.5)\\ &= 0.6915\end{aligned}$$

**25.** $$\begin{aligned}\Pr(6 \le X \le 10) &= \Pr\left(\frac{6-7}{2} \le Z \le \frac{10-7}{2}\right)\\ &= \Pr(-0.50 \le Z \le 1.50)\\ &= 0.9332 - 0.3085\\ &= 0.6247\end{aligned}$$

**26.** $\Pr(X < 3.5) + \Pr(X > 4.5)$
$= 1 - \Pr(3.5 \le X \le 4.5)$
$$= 1 - \left[A\left(\frac{4.5-4}{.4}\right) - A\left(\frac{3.5-4}{.4}\right)\right]$$
$$= 1 - [A(1.25) - A(-1.25)]$$
$$= 1 - (0.8944 - 0.1056)$$
$$= 0.2112$$

**27.** $\Pr(-2 \le Z \le 2) = A(2) - A(-2)$
$$= 0.9772 - 0.0228$$
$$= 0.9544$$

**28.** $\Pr(-2.5 \le Z \le 2.5) = A(2.5) - A(-2.5)$
$$= 0.9938 - 0.0062$$
$$= 0.9876$$

**29.** From Table 2 we see that $\Pr(Z \le 2) = .9772$ for standard normal $Z$. Solve $\frac{6-5}{\sigma} = 2$: $2\sigma = 1$, $\sigma = .5$.

**30.** Since $\Pr(14.5 \le X) = .0013$ we have $\Pr(X \le 14.5) = 1 - .0013 = .9987$. From Table 2 we see that $\Pr(Z \le 3) = .9987$ for standard normal $Z$. Solve $\frac{14.5-10}{\sigma} = 3$: $3\sigma = 4.5$, $\sigma = 1.5$.

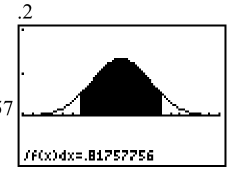
**31.** $\mu = 3.3$, $\sigma \approx .2$
$$\Pr(X \ge 4) = \Pr\left(Z \ge \frac{4-3.3}{.2}\right)$$
$$= \Pr(Z \ge 3.5)$$
$$= 1 - \Pr(Z \le 3.5)$$
$$= 1 - 0.9998$$
$$= 0.0002$$

**32.** $\mu = 16\frac{3}{4}$, $\sigma = \frac{1}{4}$
$$\Pr(X < 16) = \Pr\left(Z < \frac{16 - 16\frac{3}{4}}{\frac{1}{4}}\right)$$
$$= \Pr(Z < -3) = 0.0013$$

**33.** $\mu = 6$, $\sigma = 0.04$
$\Pr(5.95 \le X \le 6.05)$
$$= \Pr\left(\frac{5.95-6}{0.04} x \le Z \le \frac{6.05-6}{0.04}\right)$$
$$= \Pr(-1.25 \le Z \le 1.25)$$
$$= 0.8944 - 0.1056$$
$$= 0.7888$$

**34.** $\mu = 67$, $\sigma = 3$

.2

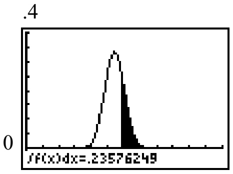
57  77


–.1

$\Pr(63 \le X \le 71) \approx 0.8176$

**35.** $\mu = 5.4$, $\sigma = .6$

.4

0  10


–.1

$\Pr(X > 5.832) \approx 0.2358$

**36. a.** $\mu = 100$, $\sigma = 16$
$$\Pr(X \ge 140) = \Pr\left(Z \ge \frac{140-100}{16}\right)$$
$$= \Pr(Z \ge 2.5)$$
$$= 1 - \Pr(Z \le 2.5)$$
$$= 1 - 0.9938$$
$$= 0.0062$$
$$= 0.62\%$$

**b.** $x_{90} = 100 + 16z_{90} = 100 + 16 \times 1.28 \approx 120$

**37.** $\mu = 7500$, $\sigma = 1000$
$\Pr(X > 9750)$
$$= \Pr\left(Z > \frac{9750-7500}{1000}\right)$$
$$= \Pr(Z > 2.25)$$
$$= 1 - \Pr(Z \le 2.25)$$
$$= 1 - 0.9878$$
$$= 0.0122$$

**38.** $\mu = 1200,\ \sigma = 160$

$$\Pr(X < 1000) = \Pr\left(Z < \frac{1000-1200}{160}\right)$$
$$= \Pr(Z < -1.25)$$
$$= 0.1056$$

**39.** $\mu = 520,\ \sigma = 75$

**a.** $x_{90} = 520 + 75z_{90}$
$= 520 + 75 \times 1.28$
$= 616$

**b.** $\Pr(-z \le Z \le z) = 0.90$
$\Pr(Z \le -z) = 0.05 \Rightarrow z_{05} \approx -1.65$
$$\frac{x-\mu}{\sigma} = \frac{x-520}{75} = -1.65$$
$\Rightarrow x_{05} = 396.25 \approx 396$
$$\frac{x-\mu}{\sigma} = \frac{x-520}{75} = 1.65$$
$\Rightarrow x_{95} = 643.75 \approx 644$
Between 396 and 644

**c.** $x_{98} = 520 + 75z_{98} = 520 + 75 \times 2.05 = 674$

**40.** $\mu = 300,\ \sigma = 50$
$\Pr(Z \le z) = 0.95 \Rightarrow z_{95} \approx 1.64$
$$\frac{x-\mu}{50} = \frac{x-300}{50} = 1.64 \Rightarrow x_{95} = 382$$
They need 382 bags.

**41.** $\mu = 30{,}000,\ \sigma = 5000$
$\Pr(Z \le z) = .02 \Rightarrow z_{02} \approx -2.05$
$$\frac{x-\mu}{\sigma} = \frac{x-30{,}000}{5000} = -2.05 \Rightarrow x_{02} = 19{,}750$$
19,750 miles

**42.** $\mu = ?,\ \sigma = 0.25$

**a.** $\Pr(Z > z) = 0.005 \Rightarrow z_{99.5} \approx 2.60$
$$\frac{x-\mu}{\sigma} = \frac{6-\mu}{0.25} = 2.60$$
$\Rightarrow \mu \approx 5.35$ ounces

**b.** $\Pr(Z > z) = 0.99 \Rightarrow z_{01} \approx -2.35$
$$\frac{x-\mu}{\sigma} = \frac{x-5.35}{0.25} = -2.35$$
$\Rightarrow x_{01} \approx 4.76$ ounces

**43.** True; As $\sigma$. increases, the normal curve flattens out.

**44.** $\mu = 4,\ \sigma = .5$

**a.** $\Pr(3 \le X \le 5)$
$4 - c = 3$ and $4 + c = 5$ $\quad c = 1$
Probability: $1 - \frac{0.5^2}{1^2} = 0.75$ so
$\Pr(3 \le X \le 5) \ge 0.75.$

**b.** $$\Pr(3 \le X \le 5) = \Pr\left(\frac{3-4}{0.5} \le Z \le \frac{5-4}{0.5}\right)$$
$= \Pr(-2 \le Z \le 2)$
$= 0.9772 - 0.0228$
$= 0.9544$

**c.** The probability found using the Chebychev inequality is an estimate and can be used for any random variable. The probability found in (b) assumes the random variable is normally distributed and thus the extra information about the shape of the distribution of the random variable provides a more accurate estimate of the probability.

**45.** normalcdf(1,5,3,2)

NORMDIST (5,3,2,TRUE) – NORMDIST (1,3,2,TRUE)

$P(1 < x < 5)$ for $x$ normal (3,2)

**46.** normalcdf(9.625,30,7.75,1.25)

1 – NORMDIST (9.625,7.75,1.25,TRUE)

$P(x > 9.625)$ for $x$ normal (7.75,1.25)

**47.** normalcdf(0,1000,1200,100)

NORMDIST (1000,1200,100,TRUE)

$P(x < 1000)$ for $x$ normal (1200,100)

**48.** invNorm(.99,1200,100)

NORMINV (0.99,1200,100)

inverse normal probability 0.99, mean=1200, std dev=100

**Exercises 7.7**

**1.** $n = 25,\ p = \frac{1}{5}$

$\mu = np = 25\left(\frac{1}{5}\right) = 5,$

$\sigma = \sqrt{npq} = \sqrt{25\left(\frac{1}{5}\right)\left(\frac{4}{5}\right)} = 2$

**a.** $\Pr(X = 5) \approx \Pr\left(\frac{4.5-5}{2} \le Z \le \frac{5.5-5}{2}\right)$
$= \Pr(-.25 \le Z \le .25)$
$= 0.5987 - 0.4013$
$= 0.1974$

**b.** $\Pr(3 \le X \le 7)$
$\approx \Pr\left(\frac{2.5-5}{2} \le Z \le \frac{7.5-5}{2}\right)$
$= \Pr(-1.25 \le Z \le 1.25)$
$= 0.8944 - 0.1056$
$= 0.7888$

**c.** $\Pr(X < 10) \approx \Pr\left(Z \le \frac{9.5-5}{2}\right)$
$= \Pr(Z \le 2.25)$
$= 0.9878$

**2.** $n = 18,\ p = \frac{2}{3}$

$\mu = np = 18\left(\frac{2}{3}\right) = 12,$

$\sigma = \sqrt{npq} = \sqrt{18\left(\frac{2}{3}\right)\left(\frac{1}{3}\right)} = 2$

**a.** $\Pr(X = 10) \approx \Pr\left(\frac{9.5-12}{2} \le Z \le \frac{10.5-12}{2}\right)$
$= \Pr(-1.25 \le Z \le -0.75)$
$= 0.2266 - 0.1056$
$= 0.1210$

**b.** $\Pr(8 \le X \le 16)$
$\approx \Pr\left(\frac{7.5-12}{2} \le Z \le \frac{16.5-12}{2}\right)$
$= \Pr(-2.25 \le Z \le 2.25)$
$= 0.9878 - 0.0122$
$= 0.9756$

**c.** $\Pr(X > 12) \approx \Pr\left(Z \ge \frac{12.5-12}{2}\right)$
$= \Pr(Z \ge .25)$
$= 1 - 0.5987$
$= 0.4013$

**3.** $n = 20,\ p = \frac{1}{6}$

$\mu = 20\left(\frac{1}{6}\right) = \frac{10}{3},\ \sigma = \sqrt{20\left(\frac{1}{6}\right)\left(\frac{5}{6}\right)} = \frac{5}{3}$

$\Pr(X \ge 8) \approx \Pr\left(Z \ge \frac{7.5 - \frac{10}{3}}{\frac{5}{3}}\right)$
$= \Pr(Z \ge 2.5)$
$= 1 - 0.9938$
$= 0.0062$

**4.** $n = 16,\ p = \frac{1}{2}$

$\mu = 16\left(\frac{1}{2}\right) = 8,\ \sigma = \sqrt{16\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)} = 2$

$\Pr(X \ge 12) \approx \Pr\left(Z \ge \frac{11.5-8}{2}\right)$
$= \Pr(Z \ge 1.75)$
$= 1 - 0.9599$
$= 0.0401$

**5.** $n = 100,\ p = \frac{1}{2}$

$\mu = 100\left(\frac{1}{2}\right) = 50,\ \sigma = \sqrt{100\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)} = 5$

$\Pr(X \ge 63) \approx \Pr\left(Z \ge \frac{62.5-50}{5}\right)$
$= \Pr(Z \ge 2.5)$
$= 1 - 0.9938$
$= 0.0062$

**6.** $n = 90,\ p = \frac{9}{19}$

$\mu = 90\left(\frac{9}{19}\right) = \frac{810}{19},\ \sigma = \sqrt{90\left(\frac{9}{19}\right)\left(\frac{10}{19}\right)} = \frac{90}{19}$

$$\begin{aligned}\Pr(X > 45) &\approx \Pr\left(Z \ge \frac{45.5 - \frac{810}{19}}{\frac{90}{19}}\right)\\ &\approx \Pr(Z \ge 0.61)\\ &\approx \Pr(Z \ge 0.60)\\ &= 1 - 0.7257\\ &= 0.2743\end{aligned}$$

**7.** $n = 75,\ p = \frac{3}{4}$

$\mu = 75\left(\frac{3}{4}\right) = 56.25, \quad \sigma = \sqrt{75\left(\frac{3}{4}\right)\left(\frac{1}{4}\right)} = 3.75$

$$\begin{aligned}\Pr(X \ge 68) &\approx \Pr\left(Z \ge \frac{67.5 - 56.25}{3.75}\right)\\ &= \Pr(Z \ge 3)\\ &= 1 - 0.9987\\ &= 0.0013\end{aligned}$$

**8.** $n = 54,\ p = \frac{2}{5}$

$\mu = 54\left(\frac{2}{5}\right) = 21.6,\ \sigma = \sqrt{54\left(\frac{2}{5}\right)\left(\frac{3}{5}\right)} = 3.6$

$$\begin{aligned}\Pr(X < 14) &\approx \Pr\left(Z \le \frac{13.5 - 21.6}{3.6}\right)\\ &= \Pr(Z \le -2.25)\\ &= 0.0122\end{aligned}$$

**9.** $n = 20, p = 0.310$

$\mu = 20(0.310) = 6.2,$

$\sigma = \sqrt{20(0.31)(0.69)} \approx 2.068$

$$\begin{aligned}\Pr(X \ge 6) &\approx \Pr\left(Z \ge \frac{5.5 - 6.2}{2.068}\right)\\ &\approx \Pr(Z \ge -0.34)\\ &\approx \Pr(Z \ge -0.35)\\ &= 1 - 0.3632\\ &= 0.6368\end{aligned}$$

**10.** $n = 1000, p = 0.25$

$\mu = 1000(0.25) = 250,\ \sigma = \sqrt{1000(0.25)(0.75)} \approx 13.693$

$$\begin{aligned}\Pr(X \ge 290) &\approx \Pr\left(Z \ge \frac{289.5 - 250}{13.693}\right)\\ &\approx \Pr(Z \ge 2.88)\\ &\approx \Pr(Z \ge 2.90)\\ &= 1 - 0.9981\\ &= 0.0019\end{aligned}$$

Yes, the new campaign reached more of the target audience than would have been expected from the old campaign.

**11.** $n = 1000, p = 0.02$

$\mu = 1000(0.02) = 20,$

$\sigma = \sqrt{1000(0.02)(0.98)} \approx 4.427$

$$\begin{aligned}\Pr(X < 15) &\approx \Pr\left(Z \le \frac{14.5 - 20}{4.427}\right)\\ &\approx \Pr(Z \le -1.24)\\ &\approx \Pr(Z \le -1.25)\\ &= 0.1056\end{aligned}$$

**12.** $E(X) = \mu = np = 70(0.20) = 14$

$\sigma = \sqrt{70(0.20)(0.80)} \approx 3.347$

$$\begin{aligned}\Pr(X = 14) &\approx \Pr\left(\frac{13.5 - 14}{3.347} \le Z \le \frac{14.5 - 14}{3.347}\right)\\ &\approx \Pr(-0.15 \le Z \le 0.15)\\ &= 0.5596 - 0.4404\\ &= 0.1192\end{aligned}$$

**13.** probability of failure = (0.01)(0.02)(0.01) = 0.000002

$n = 1{,}000{,}000,$

$E(X) = \mu = 1{,}000{,}000(0.000002) = 2$

$\sigma = \sqrt{1{,}000{,}000(0.000002)(0.999998)} \approx 1.414$

$$\begin{aligned}\Pr(X > 3) &\approx \Pr\left(Z \ge \frac{3.5 - 2}{1.414}\right)\\ &\approx \Pr(Z \ge 1.06)\\ &\approx \Pr(Z \ge 1.05)\\ &= 1 - 0.8531\\ &= 0.1469\end{aligned}$$

**14.** Estimated probability: $\frac{175}{250} = 0.7$

$n = 250, p = 0.75$
$\mu = 250(0.75) = 187.5,$
$\sigma = \sqrt{250(0.75)(0.25)} \approx 6.847$

$$\Pr(X \le 175) \approx \Pr\left(Z \le \frac{175.5 - 187.5}{6.847}\right)$$
$$\approx \Pr(Z \le -1.75)$$
$$= 0.0401$$

**15.** $n = 100, p = 0.35$
$\mu = 100(0.35) = 35,$
$\sigma = \sqrt{100(0.35)(0.65)} \approx 4.770$
$\Pr(30 \le X \le 40)$

$$= \Pr\left(\frac{29.5 - 35}{4.77} \le Z \le \frac{40.5 - 35}{4.77}\right)$$
$$\approx \Pr(-1.15 \le Z \le 1.15)$$
$$= 0.8749 - 0.1251$$
$$= 0.7498$$

**16.** $n = 150, p = 0.14$
$\mu = 150(0.14) = 21,$
$\sigma = \sqrt{150(0.14)(0.86)} \approx 4.250$

Passengers will have to be bumped if fewer than 10 passengers cancel.

$$\Pr(X \le 9) = \Pr\left(Z \le \frac{9.5 - 21}{4.25}\right)$$
$$\approx \Pr(Z \le -2.71)$$
$$\approx \Pr(Z \le -2.70)$$
$$= 0.0035$$

**17.** $n = 1000, p = 0.03$
$\mu = 1000(0.03) = 30,$
$\sigma = \sqrt{1000(0.03)(0.97)} \approx 5.394$

$$\Pr(X \ge 29) \approx \Pr\left(Z \ge \frac{28.5 - 30}{5.394}\right)$$
$$\approx \Pr(Z \ge -0.278)$$
$$\approx \Pr(Z \ge -0.28)$$
$$= 1 - 0.3897$$
$$= 0.61$$

**18.** $n = 100, \; p = \frac{1}{2}$

$$\mu = 100 \cdot \frac{1}{2} = 50, \quad \sigma = \sqrt{100 \cdot \frac{1}{2} \cdot \frac{1}{2}} = 5$$

$$\Pr(X > 65) \approx \Pr\left(Z \ge \frac{65.5 - 50}{5}\right)$$
$$= \Pr(Z \ge 3.1)$$
$$= 1 - 0.9990$$
$$= 0.001$$

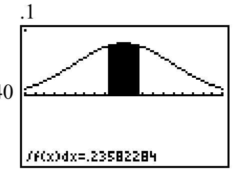
The probability that more than 65 tails occur is the same, so the probability that more than 65 heads or tails occur is $2 \times .001 = .002$.

**19.** $n = 100, \; p = \frac{1}{2}$

Exact: $\Pr(49 \le X \le 51) \approx .2356$
Normal Approximation:

$$\mu = 100\left(\frac{1}{2}\right) = 50, \; \sigma = \sqrt{100\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)} = 5$$

.1

40  60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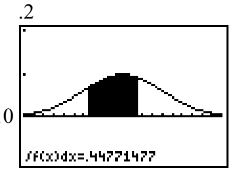
$\Pr(48.5 \le X \le 51.5) \approx 0.2358$

**20.** $n = 120, \; p = \frac{1}{6}$

Exact: $\Pr(17 \le X \le 21) \approx 0.4544$
Normal Approximation:

$$\mu = 120\left(\frac{1}{6}\right) = 20, \; \sigma = \sqrt{120\left(\frac{1}{6}\right)\left(\frac{5}{6}\right)} \approx 4.082$$

.2

10  30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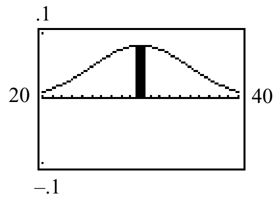
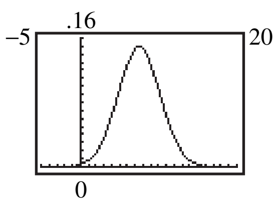
$\Pr(16.5 \le X \le 21.5) \approx 0.4477$

**21.** $n = 150, p = 0.2$
Exact: $\Pr(X = 30) \approx 0.0812$
Normal Approximation: $\mu = 150(0.2) = 30$,
$\sigma = \sqrt{150(0.2)(0.8)} \approx 4.899$




$\Pr(29.5 \le X \le 30.5) \approx 0.0813$

**22.** $n = 150, p = 0.05$
Exact: $\Pr(X \le 5) \approx 0.2344$
Normal Approximation:
$\mu = 150(0.05) = 7.5$,
$\sigma = \sqrt{150(0.05)(0.95)} \approx 2.669$




$\Pr(X \le 5) \approx 0.2269$

**23.** 0.0410

1 – binomcdf (300, 0.02, 10)

1 – BINOMDIST (10, 300, 0.02, 1)

binomial probabilities $n = 300, p = 0.02$,

endpoint = 10

**24.** **0**.0262

1 – binomcdf (40, 0.25, 15)

1 – BINOMDIST (15, 40, 0.25, 1)

binomial probabilities $n = 40, p = 0.25$,

endpoint = 15

**25.** 0.0796

binompdf (100,0.5,50)

BINOMDIST (50, 100, 0.5, 0)

binomial probabilities $n = 100, p = 0.5$,

endpoint = 50

**26.** **0**.2197

1 - binomcdf (100,1/6,19);

1 – BINOMDIST (19, 100, 1/6, 1)

binomial probabilities $n = 100, p = 1/6$,

endpoint = 19

**Chapter 7 Fundamental Concept Check**

**1.** *Bar Charts* and *pie charts* provide graphical ways of displaying qualitative data. A *histogram* is a graphical way of presenting a frequency distribution. A *box plot* is a graphical way of presenting a five–number summary of a collection of data.

**2.** The following definitions apply to a set of numbers. The *median* is the middle value when the numbers are ordered. The *first quartile* is a number for which roughly 25% of the numbers are less than that number; same as the $25^{th}$ percentile. The *third quartile* is a number for which roughly 75% of the numbers are less than that that number, same as the $75^{th}$ percentile. The *interquartile range* is the difference between the third quartile and the first quartile. The five–number summary consist of the minimum value, first quartile, median, third quartile, and maximum value.

**3.** A *frequency distribution* for a collection of numerical data is a table listing each number in the collection and the number of times it appears. A *relative frequency distribution* for a collection of numerical data is a table listing each number in the collection and the percentage of times it appears. A *probability distribution* is a table displaying the outcomes of an experiment and their probabilities.

**4.** Consider a table for the distribution. To construct a histogram for the table, draw a coordinate system, write the numbers from the left column of the table below the $x$–axis, and above each number draw a rectangle having height given in the second column of the table.

**5.** A *random variable* is a variable that assigns a number to each outcome of an experiment.

**6.** A *probability distribution* is a table whose first column lists the possible values of the random variable and whose second column gives the probabilities associated with each value.

**7.** A *binomial random variable* arises from observing the number of successes in an experiment consisting of a sequence of independent binomial trials.

**8.** $\binom{n}{k} p^k (1-p)^{n-k}$, where $p$ is the probability of success on each binomial trial.

**9.** The three values give a general impression of the behavior of the random variable. The *expected value* gives the average you would expect when repeating the experiment many times. The *variance* gives you an idea of how closely concentrated the outcomes are likely to be around the expected value. The *standard deviation* serves the same purpose as the variance, but has the same unit of measure as the random variable.

**10.** The *Chebychev Inequality* gives a lower bound on the likelihood that the outcome of a random variable is within a specified distance from the expected value.

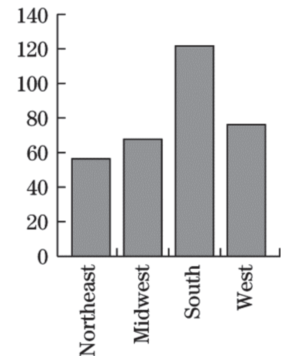
**11.** A *normal random variable* is a random variable whose probabilities are determined by calculating areas under a bell–shaped curve that is described by its mean $\mu$ and standard deviation $\sigma$.

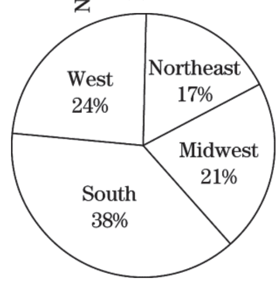
**12.** The outcomes $S$ for which $\Pr(X \le S) = p\%$.

**13.** Probabilities associated with a *binomial random variable* having parameters $n$ and $p$ can be approximated with the use of a normal curve having $\mu = np$ and $\sigma = \sqrt{np(1-p)}$.
$\Pr(a \le X \le b)$ is approximately the area under the normal curve from $x = a - \frac{1}{2}$ to $x = b + \frac{1}{2}$.

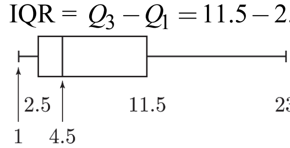
**Chapter 7 Review Exercises**

**1.**




**2.** min = 1,
$Q_1 = 2.5,\ Q_2 = 4.5, Q_3 = 11.5, \text{max} = 23;$
IQR = $Q_3 - Q_1 = 11.5 - 2.5 = 9$;




**3.**

| Number Waiting in Line | Relative Frequency |
|---|---|
| 0 | 0.04 |
| 1 | 0.10 |
| 2 | 0.18 |
| 3 | 0.26 |
| 4 | 0.22 |
| 5 | 0.14 |
| 6 | 0.06 |

Pr(at most 3 customers in line)
$= 0.04 + 0.10 + 0.18 + 0.26$
$= 0.58$

**4. a.** Possible outcomes are HH, HT, TH, TT

| Number of Heads, $k$ | $\Pr(X = k)$ |
|---|---|
| 0 | 0.25 |
| 1 | 0.50 |
| 2 | 0.25 |

**b.**

| $k$ | $\Pr(2X + 5 = k)$ |
|---|---|
| 5 | 0.25 |
| 7 | 0.50 |
| 9 | 0.25 |

**5.** $n = 3,\ p = \frac{1}{3}$

**a.**

| $k$ | $\Pr(X = k)$ |
|---|---|
| 0 | $\binom{3}{0}\left(\frac{1}{3}\right)^0\left(\frac{2}{3}\right)^3 = \frac{8}{27}$ |
| 1 | $\binom{3}{1}\left(\frac{1}{3}\right)^1\left(\frac{2}{3}\right)^2 = \frac{12}{27}$ |
| 2 | $\binom{3}{2}\left(\frac{1}{3}\right)^2\left(\frac{2}{3}\right)^1 = \frac{6}{27}$ |
| 3 | $\binom{3}{3}\left(\frac{1}{3}\right)^3\left(\frac{2}{3}\right)^0 = \frac{1}{27}$ |

**b.** $\mu = 0\left(\frac{8}{27}\right) + 1\left(\frac{12}{27}\right) + 2\left(\frac{6}{27}\right) + 3\left(\frac{1}{27}\right) = 1$

$$\sigma^2 = (0-1)^2\left(\frac{8}{27}\right) + (1-1)^2\left(\frac{12}{27}\right) + (2-1)^2\left(\frac{6}{27}\right) + (3-1)^2\left(\frac{1}{27}\right)$$

$$= \frac{2}{3}$$

**6.** $n = 4, p = .3$

$\Pr(X = 2) = \binom{4}{2}(0.3)^2(0.7)^2 = 0.2646$

**7.** The student has a .6 probability of guessing correctly on the six questions with answer *true* and a 0.4 probability of guessing correctly on the four questions with answer *false*. Therefore the student's expected score is $6(0.6) + 4(0.4) = 5.2$ correct answers which gives 52 points or 52%.
A better strategy is to choose true for all the questions which guarantees a score of 60%.

**8. a.** $\Pr(\text{get 7 twice}) = \binom{12}{2}\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^{10} \approx 0.2961$

**b.** $\Pr(\text{get 7 at least twice})$
$= 1 - \Pr(\text{get 7 zero or one time})$
$= 1 - \binom{12}{0}\left(\frac{1}{6}\right)^0\left(\frac{5}{6}\right)^{12} - \binom{12}{1}\left(\frac{1}{6}\right)^1\left(\frac{5}{6}\right)^{11}$
$\approx 0.6187$

**c.** The expected number of 7's is $12 \cdot \frac{1}{6} = 2.$

**9.** $\mu = 0(0.2) + 1(0.3) + 5(0.1) + 10(0.4) = 4.8$
$\sigma^2 = (0-4.8)^2(0.2) + (1-4.8)^2(0.3) + (5-4.8)^2(0.1) + (10-4.8)^2(0.4)$
$= 19.76$

**10.** Let $X$ be the number of red balls.

| $k$ | $\Pr(X = k)$ |
|---|---|
| 0 | $\frac{\binom{4}{0}\binom{4}{4}}{\binom{8}{4}} = \frac{1}{70}$ |
| 1 | $\frac{\binom{4}{1}\binom{4}{3}}{\binom{8}{4}} = \frac{16}{70}$ |
| 2 | $\frac{\binom{4}{2}\binom{4}{2}}{\binom{8}{4}} = \frac{36}{70}$ |
| 3 | $\frac{\binom{4}{3}\binom{4}{1}}{\binom{8}{4}} = \frac{16}{70}$ |

4 $\dfrac{\binom{4}{4}\binom{4}{0}}{\binom{8}{4}} = \dfrac{1}{70}$

$$\mu = 0\left(\frac{1}{70}\right) + 1\left(\frac{16}{70}\right) + 2\left(\frac{36}{70}\right) + 3\left(\frac{16}{70}\right) + 4\left(\frac{1}{70}\right) = 2$$

$$\sigma^2 = (0-2)^2\left(\frac{1}{70}\right) + (1-2)^2\left(\frac{16}{70}\right) + (2-2)^2\left(\frac{36}{70}\right) + (3-2)^2\left(\frac{16}{70}\right) + (4-2)^2\left(\frac{1}{70}\right)$$
$$= \frac{4}{7}$$

**11.** $X$ has mean
$\mu = (-2)(0.3) + 0(0.1) + 1(0.4) + 3(0.2) = 0.4,$
variance
$$\sigma^2 = (-2-0.4)^2(0.3) + (0-0.4)^2(0.1) + (1-0.4)^2(0.4) + (3-0.4)^2(0.2)$$
$$= 3.24,$$
and standard deviation
$\sigma = \sqrt{3.24} = 1.8.$

**12.** When a pair of fair dice is rolled, the probabilities that the result is 7 or 11 are $\frac{1}{6}$ and $\frac{1}{18}$ respectively. Hence Lucy's expected winnings are $(-10)\frac{2}{9} + 3 \cdot \frac{7}{9} = \frac{1}{9} \approx .11,$ or 11 cents per roll.

**13.** $\mu = 10,\ \sigma = \frac{1}{3}$
$10 - c = 9$ and $10 + c = 11$ $\quad c = 1$
Probability: $\geq 1 - \dfrac{\left(\frac{1}{3}\right)^2}{1^2} = \dfrac{8}{9}$

**14.** $\mu = 50, \sigma = 8$
$50 - c = 38$ and $50 + c = 62$ $\quad c = 12$
Probability $\geq 1 - \dfrac{8^2}{12^2} = \dfrac{5}{9}$

**15.** $$\Pr(6.5 \leq X \leq 11) = \Pr\left(\frac{6.5-5}{3} \leq Z \leq \frac{11-5}{3}\right)$$
$$= A(2) - A(0.5)$$
$$= 0.9772 - 0.6915$$
$$= 0.2857$$

**16.** $\Pr(Z \geq 0.75) = 1 - 0.7734 = 0.2266$

**17.** $\mu = 5.75, \sigma = 0.2$

$$\Pr(X \ge 6) = \Pr\left(Z \ge \frac{6-5.75}{0.2}\right)$$
$$= \Pr(Z \ge 1.25)$$
$$= 1 - 0.8944$$
$$= 0.1056$$

10.56%

**18.** $\Pr(Z \ge z) = 0.7734$
$\Pr(Z < z) = 1 - 0.7734 = 0.2266$
$z = -0.75$

**19.** $\mu = 80, \sigma = 15$
$\Pr(80 - h \le X \le 80 + h) = 0.8664$
$\frac{1-0.8664}{2} = 0.0668 \Rightarrow$ (area left of $80 - h$)
$\Pr(Z \le z) = 0.0668$ when $z = -1.5$
$\Pr(-1.5 \le Z \le 1.5) = 0.8664$
Therefore, $\frac{x-\mu}{\sigma} = -1.5$ and $\frac{x+\mu}{\sigma} = 1.5.$
$\frac{(80-h)-80}{15} = -1.5$ and $\frac{(80+h)-80}{15} = 1.5$
$h = 22.5$

**20. a.** $$\Pr(133 \le X) \approx \Pr\left(\frac{132.5-100}{15} \le Z\right)$$
$$\approx \Pr(2.167 \le Z)$$
$$\approx \Pr(2.20 \le Z)$$
$$= 1 - 0.9861$$
$$= 0.0139$$
$$= 1.39\%$$

**b.** $$x_{95} = 100 + 15z_{95}$$
$$= 100 + 15 \cdot 1.65$$
$$= 124.75$$

**21.** $n = 54, \; p = \frac{2}{5}$

$$\mu = 54\left(\frac{2}{5}\right) = 21.6$$
$$\sigma = \sqrt{54\left(\frac{2}{5}\right)\left(\frac{3}{5}\right)} = 3.6$$
$$\Pr(X \le 13) \approx \Pr\left(Z \le \frac{13.5-21.6}{3.6}\right) = \Pr(Z \le -2.25) = 0.0122$$

**22.** $n = 75,\ p = \frac{1}{4}$

$\mu = 75\left(\frac{1}{4}\right) = 18.75$

$\sigma = \sqrt{75\left(\frac{1}{4}\right)\left(\frac{3}{4}\right)} = 3.75$

$\Pr(8 \le X \le 22) \approx \Pr\left(\frac{7.5 - 18.75}{3.75} \le Z \le \frac{22.5 - 18.75}{3.75}\right) = \Pr(-3 \le Z \le 1) = 0.8413 - 0.0013 = 0.84$

**Conceptual Exercises**

**23.** **a.** scoring in the third quartile is not very good: 100, 40, 40, 40,

**b.** scoring in the third quartile corresponds to a perfect grade: 100, 100, 90, 80, 70

**24.** **a.** The mean and median are equal: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 : The mean is 5.5; the median is 5.5

**b.** the mean is less than the median: 1, 1, 1, 1, 4, 5, 6, 7, 8, 9 : The mean is 4.3; the median is 4.5

**c.** the median is less than the mean 1, 2, 3, 4, 5, 6, 10, 12, 14, 100. The median is 5.5; the mean is 15.7

**25.** A population mean is the average of all the data in the entire population. When a sample is taken from a population, the sample mean is the average of all the data in that particular sample. Sample means vary whereas the population mean is fixed.

**26.** A sample mean is taken from the population and is the average of all the data values in a particular sample, which is a subset of the population.

**27.** Yes; in general, if we add a constant to each number in a set, then the mean will increase by that constant.

**28.** Yes; in general, if we multiply each number in a set by some constant, then the standard deviation will be multiplied by that constant.

**29.** The binomial probability distribution applies when there is a fixed number of independent trials when the probability of success is constant. The outcome of each trial is classified as either a "success" or a "failure".

**30.** Repeated trials that do not produce a binomial distribution: 1) tossing a coin until a head appears. 2) Having children until a girl is born.

**Chapter 7 Project**

**1.** Answers may vary, but the correct intuition is that the probability that both balls are red is higher if the first ball is replaced.

**2.** $\Pr(\text{both red}) = \frac{4}{10} \cdot \frac{4}{10} = 0.16$ with replacement.

**3.** $\Pr(\text{both red}) = \frac{4}{10} \cdot \frac{3}{9} = 0.1333$ without replacement.

**4.** Answers may vary depending on guess in question 1.

**5.** Answers may vary, but the correct intuition is that the expected number of red balls is the same with or without replacement.

**6.** With replacement, the expected number of red balls is $2 \cdot \frac{4}{10} = 0.8.$

**7.** Without replacement, Pr(RR) = 0.1333,

$\Pr(\text{RW}) = \frac{4}{10} \cdot \frac{6}{9} = 0.2667$ and

$\Pr(\text{WR}) = \frac{6}{10} \cdot \frac{4}{9} = 0.2667.$ The expected number of red balls is
2(0.1333) + 1(0.2667) + 1(0.2667) = 0.8.

**8.** Answers may vary depending on guess in question 5.

**9.** **a.** 40%

**b.** 40%

**c.** No

**d.** Repeating the experiment of selecting 2 balls from the urn many times and finding the average number of red balls is similar to removing 2 tablespoons of the red and white mixture. As shown in parts b and c, the expected number or "amount" of red balls does not depend on whether the first tablespoon is replaced.