Chapter 6

Exercises 6.1

1. a. The set of all possible pairs: {RS, RT, RU, RV, ST, SU, SV, TU, TV, UV}

b. The set of pairs containing R: {RS, RT, RU, RV}

c. The set of pairs containing neither R nor S: {TU, TV, UV}

2. a. {M, I, S, P}

b. {I}

3. a. {HH, HT, TH, TT}

b. {HH, HT}

4. a. {(1, 1), (1, 2), (1, 3), (1, 4), (2, 1), (2, 2), (2, 3), (2, 4), (3, 1), (3, 2), (3, 3), (3, 4), (4, 1), (4, 2), (4, 3), (4, 4)}

b. (i) {(2, 2), (2, 4), (4, 2), (4, 4)}
(ii) {(1, 2), (1, 2), (1, 3), (1, 4), (2, 1), (2, 3), (3, 1), (3, 2), (3, 3), (3, 4), (4, 1), (4, 3)}
(iii) {(3, 3), (3, 4), (4, 3), (4, 4)}
(iv) {(3, 4), (4, 3)}
(v) {(2, 4), (3, 3), (3, 4), (4, 2), (4, 3), (4, 4)}
(vi) {(1, 1), (2, 2), (3, 3), (4, 4)}
(vii) {(1, 2), (1, 3), (2, 1), (2, 2), (2, 4), (3, 1), (3, 3), (3, 4), (4, 2), (4, 3)}
(viii) {(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3)}

5. a. {(I, red), (I, white), (II, red), (II, white)}

b. All combinations with I: {(I, red), (I, white)}

6. a. {HHHH, HHHT, HHTH, HHTT, HTHH, HTHT, HTTH, HTTT, THHH, THHT, THTH, THTT, TTHH, TTHT, TTTH, TTTT}

b. All events with 3 or 4 H's: {HHHH, HHHT, HHTH, HTHH, THHH}

c. {HHHH, HHHT, HHTH, HHTT, HTHH, HTHT, HTTH, HTTT}

d. {HHHH, HHHT, HHTH, HTHH}

7. a. S = {all positive numbers of minutes}

b. $E \cap F$ = "more than 5 but less than 8 minutes"
$E \cap G = \varnothing$ (There's no time longer than 5 minutes but less than 4 minutes.)
$E' =$ "5 minutes or less"
$F' =$ "8 minutes or more"
$E' \cap F = E' =$ "5 minutes or less"
$E' \cap F \cap G = G =$ "less than 4 minutes"
$E \cup F = S$

8. $E \cap F$ = "More than \$50,000 but less than \$75,000"
$E' =$ "\$50,000 or less"
$F' =$ "\$75,000 or more"

9. a. Eight possible combinations: {(Fr, Lib), (Fr, Con), (So, Lib), (So, Con), (Jr, Lib), (Jr, Con), (Sr, Lib), (Sr, Con)}

b. All combinations with Con: {(Fr, Con), (So, Con), (Jr, Con), (Sr, Con)}

c. {(Jr, Lib)}

d. All combinations with neither Fr nor Con: {(So, Lib), (Jr, Lib), (Sr, Lib)}

10. a. (i) No; there can be a red Chevrolet.
(ii) Yes; if a Ford is green, it's not red.
(iii) Yes; if a car is a Ford, it's not a Chevrolet.
(iv) No; the car can be a red Chrysler
(v) No; the car can be a black Chevrolet.
(vi) Yes; if a car is a green Ford, it's neither black nor a Chrysler.
(vii) No; there can be a green Ford that's not red.
(viii) No; the car could be a red Ford, for example.

b. (i) "The car is a red Chevrolet."
(ii) "The car is red or a Chevrolet."
(iii) "The car is not red."
(iv) "The car is not a Chevrolet."
(v) "The car is not a green Ford."
(vi) "The car is neither black nor a Chrysler."
(vii) "The car is red or is a green Ford."
(viii) null set
(ix) "The car is a red Chrysler."
(x) "The car is red, black, or a Chrysler."
(xi) null set
(xii) "The car is neither red nor a Chevrolet."
(xiii) "The car is neither red nor a green Ford."

11. a. No; $E \cap F = \{2\}$

b. Yes; $F \cap G = \varnothing$

12. They are mutually exclusive because $E \cap E' = \varnothing.$

13. All combinations of members of S: $\varnothing, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, S$

14. For each outcome, there are 2 possibilities; in the event or not in it. That leads to 2^n possible events.

15. Yes; $(E \cup F) \cap (E' \cap F') = \{1, 2, 3\} \cap \{4\} = \varnothing$

16. Yes; $E' \cap F' = (E \cup F)'$, so $(E \cup F) \cap (E' \cup F') = (E \cup F) \cap (E \cup F)' = \varnothing.$

17. a. $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

b. More than half heads: $\{6, 7, 8, 9, 10\}$

18. a. $\{178, 187, 718, 781, 817, 871\}$

b. $\{718, 781, 817, 871\}$

19. a. No; there are blue-eyed people at least 18 years old.

b. Yes; a brown-eyed person younger than 18 doesn't have blue eyes.

c. Yes; a brown-eyed person younger than 18 is not at least 18 years old.

20. a. $E \cup F =$ "blue eyes or at least 18 years old"

b. $E \cap G = \varnothing$

c. $E' =$ "not blue eyes"

d. $F' =$ "younger than 18 years old"

e. $(G \cup F) \cap E = (G \cap E) \cup (F \cap E)$
$= \varnothing \cup (F \cap E)$
$= F \cap E$
= "blue eyes and at least 18"

f. $G' \cap E = E =$ "blue eyes"

21. $\{0, 1, 2, 3, 4, 5, 6, 7, 8\}$

22. $\{2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$

23. A possible outcome is (7, 4). There are $9 \cdot 9 = 81$ outcomes in the sample space.

24. A possible outcome is (7, 4). There are $9 \cdot 8 = 72$ outcomes in the sample space.

25. A possible outcome is $\{2, 6, 9, 10\}$. There are $C(14, 4) - 1 = 1000$ possible combinations, so $\frac{250}{1000} = 0.25 = 25\%$ were assigned to the Minnesota Timberwolves.

26. $2 \cdot 6 = 12$

27. A sample space for the choice of murderers would be {Colonel Mustard, Miss Scarlet, Professor Plum, Mrs. White, Mr. Green, Mrs. Peacock}

A sample space for the entire solution would be formed by placing each suspect, with each murder weapon, in each room.

a. 6 suspects $\times$ 6 weapons $\times$ 9 rooms = 324 outcomes

b. $E \cap F =$ "The murder occurred in the library with a gun."

c. $E \cup F =$ "Either the murder occurred in the library, or it was done with a gun."

Exercises 6.2

1. Judgemental; the probability is an opinion.

2. Empirical; the probability is based on past data.

3. Logical; the probability is based on theory.

4. Judgemental; the probability is an opinion.

5. The probability distribution is as follows:

Number of Heads	Probability
0	$\frac{1}{4}$
1	$\frac{2}{4}=\frac{1}{2}$
2	$\frac{1}{4}$

6. The probability distribution is as follows:

Letter	Probability
A	$\frac{1}{7}$
B	$\frac{1}{7}$
C	$\frac{3}{7}$
D	$\frac{2}{7}$

7. $\frac{1}{38}+\frac{1}{38}=\frac{2}{38}=\frac{1}{19}$

8. $\frac{6}{50}=\frac{3}{25}$

9. a. $\frac{191}{4487}\approx 0.04257$

b. $\frac{191+81}{4487}=\frac{272}{4487}\approx 0.06062$

c. $\frac{4487-272}{4487}=\frac{4215}{4487}\approx 0.9394$

10. a. $\frac{54}{193}\approx 0.2798$

b. $\frac{54+23}{193}=\frac{77}{193}\approx 0.3990$

c. $\frac{193-77}{193}=\frac{116}{193}\approx 0.6010$

11. a. $\frac{6}{26}=\frac{3}{13}\approx 0.2308$

b. $\frac{5}{26}\approx 0.1923$

c. $\frac{6+5-2}{26}=\frac{9}{26}\approx 0.3462$

12. a. $\frac{3}{9}=\frac{1}{3}\approx 0.3333$

b. $\frac{5}{9}\approx 0.5556$

c. The probability of a 1, 2, 3, 5, 7, or 9 is $\frac{6}{9}=\frac{2}{3}\approx 0.6667.$

13. a. E = "the numbers add up to 9" = {(3, 6), (4, 5), (5, 4), (6, 3)}

Pr(E) = $\frac{4}{36}=\frac{1}{9}\approx 0.1111$

b. Pr(sum is 2) = Pr((1, 1)) = $\frac{1}{36}$;

Pr(sum is 3) = Pr((1, 2)) + Pr((2, 1)) = $\frac{2}{36}$

Pr(sum is 4) = Pr((1, 3)) + Pr((2, 2)) + Pr((3, 1)) = $\frac{3}{36}$

The probability that the sum is less than 5 is $\frac{1}{36}+\frac{2}{36}+\frac{3}{36}=\frac{1}{6}\approx 0.1667.$

14. a. $\Pr(\text{at least two boys}) = \frac{4}{8} = \frac{1}{2} = 0.5$

b. $\Pr(\text{Oldest is a girl}) = \frac{1}{2} = 0.5$

15.

Kind of High School	Probability
Public	$\frac{115620}{141000} = 0.820$
Private	$\frac{24252}{141000} = 0.172$
Home School	$\frac{1128}{141000} = 0.008$

16.

Highest Academic Degree Planned	Probabilities
Master's	$\frac{59361}{141000} = 0.421$
Bachelor's	$\frac{29751}{141000} = 0.211$
Ph.D or Ed. D	$\frac{26931}{141000} = 0.191$
M.D, D.O., D.D.S., D.V.M.	$\frac{15792}{141000} = 0.112$
Other	$\frac{9165}{141000} = 0.065$

17. $\Pr(B, C, \text{ or } D) = 0.34 + 0.21 + 0.09 = 0.64$

18. $0.13 + 0.13 + 0.20 = 0.46$

19. a. $\Pr(E) = 0.1 + 0.5 = 0.6$;
$\Pr(F) = 0.5 + 0.2 = 0.7$

b. $\Pr(E') = 1 - 0.6 = 0.4$

c. $\Pr(E \cap F) = 0.5$

d. $\Pr(E \cup F) = 0.6 + 0.7 - 0.5 = 0.8$

20. a. $\Pr(E) = 0.05 + 0.25 = 0.3$;
$\Pr(F) = 0.05 + 0.63 + 0.01 = 0.69$

b. $\Pr(E') = 1 - 0.3 = 0.7$

c. $\Pr(E \cap F) = 0$

d. $\Pr(E \cup F) = 0.3 + 0.69 = 0.99$

21. a.

Number of Colleges Applied to	Probability
1	0.10
2	$0.17 - 0.10 = 0.07$
3	$0.27 - 0.17 = 0.10$
4	$0.40 - 0.27 = 0.13$
≥ 5	$1 - 0.40 = 0.60$

b. $0.10 + 0.13 + 0.60 = 0.83$

22. **a.**

Age (in Years)	Probability
20 – 34	.15
35 – 49	$0.70 - 0.15 = 0.55$
50 – 64	$0.90 - 0.70 = 0.20$
65 - 79	$1 - 0.90 = 0.10$

b. $0.20 + 0.10 = 0.30$

23. **a.** No; The probabilities add to more than 1.

b. No; One of the probabilities is a negative value.

c. No; The probabilities do not add to 1.

24. **a.** No; The probabilities do not add 1.

b. No; One of the probabilities is a negative value.

c. Yes; The probabilities are positive and they add to 1.

25. $1-\left(\frac{2}{3}+\frac{1}{4}\right)=\frac{1}{12}$

26. **a.** $1-0.17-0.47-0.20=0.16$

b. $0.17+0.47=0.64$

c. $1-0.20=0.80$

27. Pr(Liberal) $=.28$; Pr(middle) $=2x$; Pr(Conseravtive) $=x$

$$\begin{aligned}0.28+2x+x&=1\\3x&=0.72\\x&=0.24\end{aligned}$$

28. Pr(Alice wins) $=2x$; Pr(Ben wins) $=x$

$$\begin{aligned}2x+x&=1\\3x&=1\\x&=\tfrac{1}{3}\end{aligned}$$

Pr(Alice wins) $=\frac{2}{3}$; Pr(Ben wins) $=\frac{1}{3}$

29. The probability is 1 because the sum of a pair of dice must be odd or even.

30. The probability is 1 because the number of heads must be odd or even.

31. $$\begin{aligned}\Pr(E\cup F)&=\Pr(E)+\Pr(F)\\&=0.4+0.5\\&=0.9\end{aligned}$$

32. $$\begin{aligned}\Pr(F)&=\Pr(E\cup F)-\Pr(E)\\&=0.7-0.3\\&=0.4\end{aligned}$$

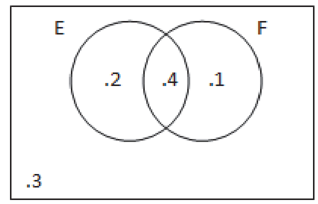
33. $\Pr(S)=0.05+0.40=0.45$

34. Pr(Exactly 1 event) $=0.05+0.25=0.30$

35. $\Pr(T \text{ only})=0.25$

36. $\Pr(S')=1-0.45=0.55$

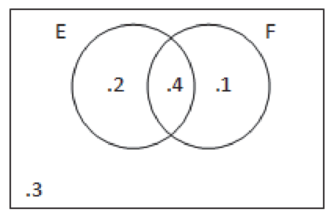
37.



a. $\Pr(E\cup F)=0.2+0.4+0.1=0.7$

b. $\Pr(E\cap F')=0.2$

38.



a. $\Pr(E\cap F)=0.4$

b. $\Pr(E\cup F')=0.2+0.4+0.3=0.9$

39. $$\begin{aligned}\Pr(H\cap P)&=\Pr(H)+\Pr(P)-\Pr(H\cup P)\\&=0.7+0.8-0.9\\&=0.6\end{aligned}$$

40. $$\begin{aligned}\Pr(M\cap C)&=\Pr(M)+\Pr(C)-\Pr(M\cup C)\\&=\tfrac{1}{2}+\tfrac{3}{8}-\tfrac{3}{4}\\&=\tfrac{1}{8}\end{aligned}$$

41. $10 \text{ to } 1=\frac{10}{10+1}=\frac{10}{11}$

42. $4 \text{ to } 5=\frac{4}{4+5}=\frac{4}{9}$

43. $.2=\frac{1}{5}\Rightarrow 1 \text{ to } (5-1)=1 \text{ to } 4$

44. $\frac{3}{7} \Rightarrow 3 \text{ to } (7-3) = 3 \text{ to } 4$

45. $.3125 = \frac{5}{16} \Rightarrow 5 \text{ to } (16-5) = 5 \text{ to } 11$

46. $.05 = \frac{1}{20} \Rightarrow 1 \text{ to } (20-1) = 1 \text{ to } 19$

47. $2 \text{ to } 9 = \frac{2}{2+9} = \frac{2}{11}$

48. $2 \text{ to } 5 = \frac{2}{2+5} = \frac{2}{7}$

49. **a.** Sparks: $5 \text{ to } 3 = \frac{3}{5+3} = \frac{3}{8}$

Meteors: $3 \text{ to } 1 = \frac{1}{3+1} = \frac{1}{4}$

Asteroids: $3 \text{ to } 2 = \frac{2}{3+2} = \frac{2}{5}$

Suns: $4 \text{ to } 1 = \frac{1}{4+1} = \frac{1}{5}$

b. $\frac{3}{8} + \frac{1}{4} + \frac{2}{5} + \frac{1}{5} = \frac{15}{40} + \frac{10}{40} + \frac{16}{40} + \frac{8}{40}$
$= \frac{49}{40}$

c. Bookies have to make a living. The payoffs are a little lower than they should be; thus allowing the bookie to make a profit.

50. $.63 = \frac{63}{100} \Rightarrow 63 \text{ to } (100-63) = 63 \text{ to } 37$

51. There are more members (13) than Zodiac signs (12) so two or more members will always have the same Zodiac sign; thus the probability is 1.

52. This event never occurs; if 5 of the people receive the correct coat then so must the remaining person. Thus the probability is 0.

Exercises 6.3

1. **a.** $\frac{9}{17} \approx 0.5294$

b. $\frac{8}{17} \approx 0.4706$

c. $\Pr(\{3, 6, 9, 12, 15\}) = \frac{5}{17} \approx 0.2941$

d. $\Pr(\{1,3,5,6,7,9,11,12,13,15,17\}) = \frac{11}{17}$
≈ 0.6471

2. a. $\frac{10}{100} = \frac{1}{10} = 0.1$

b. $\frac{50}{100} = \frac{1}{2} = 0.5$

c. $0.1 + 0.5 = 0.6$

3. **a.** $\frac{C(5, 2)}{C(11, 2)} = \frac{10}{55} = \frac{2}{11} \approx 0.1818$

b. $1 - \frac{C(5, 2)}{C(11, 2)} = 1 - \frac{10}{55} = \frac{9}{11} \approx 0.8182$

4. **a.** $\frac{C(7,3)}{C(12,3)} = \frac{35}{220} = \frac{7}{44} \approx 0.1591$

b. $1 - \frac{C(7,3)}{C(12,3)} = 1 - \frac{35}{220} = \frac{37}{44} \approx 0.8409$

5. **a.** $\frac{C(6,4) + C(7,4)}{C(13,4)} = \frac{15+35}{715}$
$= \frac{50}{715} = \frac{10}{143} \approx 0.0699$

b. $\frac{C(6,4) + C(6,3)C(7,1)}{C(13,4)} = \frac{15 + 20 \cdot 7}{715}$
$= \frac{15+140}{715}$
$= \frac{155}{715}$
$= \frac{31}{143} \approx 0.2168$

6. a. $\frac{C(8,3)+C(6,3)}{C(14,3)}=\frac{56+20}{364}$

$=\frac{76}{364}=\frac{19}{91}\approx 0.2088$

b. $\frac{C(6,3)+C(6,2)C(8,1)}{C(14,3)}=\frac{20+15\cdot 8}{364}$

$=\frac{20+120}{364}$

$=\frac{140}{364}$

$=\frac{5}{13}\approx 0.3846$

7. $1-\frac{C(5,\ 3)}{C(7,\ 3)}=1-\frac{10}{35}$

$=\frac{5}{7}\approx 0.7143$

8. $\frac{C(10,\ 4)\times C(5,\ 2)}{C(15,\ 6)}=\frac{60}{143}\approx 0.4196$

9. $1-\frac{C(9,\ 3)}{C(13,\ 3)}=1-\frac{84}{286}=\frac{101}{143}\approx 0.7063$

10. The total number of ways to select 5 senators is

$C(100,\ 5)=\frac{100!}{95!5!}=75{,}287{,}520.$

The total number of ways to select senators from different states is

$\frac{100\cdot 98\cdot 96\cdot 94\cdot 92}{5!}=67{,}800{,}320.$

Pr(no two members from same state)

$=\frac{67{,}800{,}320}{75{,}287{,}520}$

≈ 0.9006

11. $1-\frac{C(4,\ 3)}{C(10,\ 3)}=1-\frac{4}{120}=\frac{29}{30}\approx 0.9667$

12. $1-\frac{C(8,3)}{C(9,3)}=1-\frac{2}{3}=\frac{1}{3}\approx 0.3333$

13. $\frac{C(10,\ 7)}{C(22,\ 7)}=\frac{5}{7106}\approx 0.0007$

14. $\frac{12\cdot 11\cdot 10}{22\cdot 21\cdot 20}=\frac{1}{7}\approx 0.1429$

15. Ways for no girls to be chosen: $C(12,\ 7)$

Ways for exactly 1 girl to be chosen:

$C(12,\ 6)\times C(10,\ 1)$

$1-\frac{C(12,\ 7)+C(12,\ 6)\times C(10,\ 1)}{C(22,\ 7)}$

$=1-\frac{792+924\times 10}{170{,}544}=\frac{16}{17}\approx 0.9412$

16. Ways for no boy to be chosen: $C(10,7)=120$

Ways for exactly 1 boy to be chosen:

$C(10,6)C(12,1)=(210)(12)=2520$

Ways for exactly 2 boys to be chosen:

$C(10,5)C(12,2)=(252)(66)=16{,}632$

$1-\frac{120+2520+16632}{C(22,\ 7)}$

$=1-\frac{19272}{170{,}544}$

$=\frac{151{,}272}{170{,}544}$

$=\frac{573}{646}\approx 0.8870$

17. $1-\frac{7\cdot 6\cdot 5}{7\cdot 7\cdot 7}=1-\frac{210}{343}=\frac{133}{343}\approx 0.3878$

18. $1-\frac{12\cdot 11\cdot 10\cdot 9}{12\cdot 12\cdot 12\cdot 12}=1-\frac{11880}{20736}$

$=\frac{8856}{20736}$

$=\frac{41}{96}\approx 0.4271$

19. $1-\frac{30\times 29\times 28\times 27}{30^4}=\frac{47}{250}=0.188$

20. $1-\frac{16\times 15\times 14\times 13\times 12}{16^5}=\frac{4097}{8192}\approx 0.5001$

21. $1-\frac{P(20,8)}{20^8}\approx 0.8016$

22. $1-\frac{P(100,10)}{100^{10}}\approx 0.3718$

23. Pr(at least one birthday on June 13)

$$=1-\left(\frac{364}{365}\right)^{25} \approx 0.06629$$

Because in Table 1 no particular date is being matched. Any two (or more) identical birthdays count as a success.

24. Pr(at least one birthday on October 23)

$$=1-\left(\frac{364}{365}\right)^{100} \approx 0.2399$$

Johnny Carson's reasoning was wrong because he was looking for a particular date, the theory is true when looking for two people with the same date.

25. $\frac{6\cdot 5}{6^2}=\frac{30}{36}=\frac{5}{6}\approx 0.8333$

26. $\frac{6\cdot 5\cdot 4}{6\cdot 6\cdot 6}=\frac{120}{216}=\frac{5}{9}\approx 0.5556$

27. $\frac{3^4}{6^4}=\frac{81}{1296}=\frac{1}{16}\approx 0.0625$

28. $\frac{5^3}{6^3}=\frac{125}{216}\approx 0.5787$

29. $\frac{C(10, 4)}{2^{10}}=\frac{210}{1024}=\frac{105}{512}\approx 0.2051$

30. $\frac{C(7, 5)}{2^7}=\frac{21}{128}\approx 0.1641$

31. $1-\frac{7\times 6\times 5\times 4}{7^4}=\frac{223}{343}\approx 0.6501$

32. $\frac{5!}{5^5}=\frac{120}{3125}=\frac{24}{625}\approx .0384$

33. The tourist must travel 8 blocks of which 3 are south. Thus he has $C(8, 3) = 56$ ways to get to B from A.

a. To get from A to B through C there are $C(3, 1)\cdot C(5, 2) = 30$ ways.
The probability is $\frac{30}{56}=\frac{15}{28}\approx 0.5357$.

b. To get from A to B through D there are $C(5, 1)\cdot C(3, 2) = 15$ ways.
The probability is $\frac{15}{56}\approx 0.2679$.

c. To get from A to B through C and D there are $C(3, 1)\cdot C(2, 0)\cdot C(3, 2) = 9$ ways.
The probability is $\frac{9}{56}\approx 0.1607$.

d. The number of ways to get from A to B through C or D is $30 + 15 - 9 = 36$.
The probability is $\frac{36}{56}=\frac{9}{14}\approx 0.6429$.

34. The tourist must travel 10 blocks of which 4 are south. Thus he has $C(10, 4) = 210$ ways to get to B from A.

a. To get from A to B through C there are $C(5, 2)\cdot C(5, 2) = 100$ ways.
The probability is $\frac{100}{210}=\frac{10}{21}\approx 0.4762$.

b. To get from A to B through D there are $C(7, 3)\cdot C(3, 1) = 105$ ways.
The probability is $\frac{105}{210}=\frac{1}{2}=0.5$.

c. To get from A to B through C and D there are $C(5, 2)\cdot 2\cdot C(3, 1) = 60$ ways.
The probability is $\frac{60}{210}=\frac{2}{7}\approx 0.2857$.

d. The number of ways to get from A to B through C or D is $100 + 105 - 60 = 145$.
The probability is $\frac{145}{210}=\frac{29}{42}\approx 0.6905$.

35.
$$1-\frac{4\cdot 4\cdot 4}{5\cdot 5\cdot 5}=1-\frac{64}{125}=\frac{61}{125}\approx 0.488$$

36.
$$1-\frac{3\cdot 3\cdot 3}{4\cdot 4\cdot 4}=1-\frac{27}{64}=\frac{37}{64}\approx 0.5781$$

37.
$$1-\frac{12\cdot 11\cdot 10}{15\cdot 14\cdot 13}=1-\frac{1320}{2730}=\frac{1410}{2730}\approx 0.5165$$

His chances are increased.

38. $1-\frac{9\cdot 8\cdot 7}{12\cdot 11\cdot 10}=1-\frac{504}{1320}$
$=\frac{816}{1320}\approx 0.6182$

His chances are increased.

39. $\frac{5\cdot 4\cdot 3}{5^3}=\frac{60}{125}=\frac{12}{25}=0.48$

40. $\frac{1\cdot 1}{7\cdot 6}=\frac{1}{42}\approx 0.0238$

41. There are 5! = 120 ways to arrange the family. First determine where the parents will stand. There are two ways with the man at one end. If the man doesn't stand at one end, there are 3 · 2 = 6 ways for the couple to stand together. Next determine the order of the children. There are 3! = 6 ways to order the children. Thus there are (2 + 6) · 6 = 48 ways to stand with the parents together.

The probability is $\frac{48}{120}=\frac{2}{5}=0.4.$

42. There are 5! or 120 ways to arrange 5 letters. In 36 of these, the 3 *E*'s will be adjacent.

Pr(all *E*'s adjacent) = $\frac{36}{120}=\frac{3}{10}=0.3$

43. There are 13 · 12 = 156 ways to choose the two denominations, *C*(4, 3) = 4 ways to choose the suits for the three of a kind and *C*(4, 2) = 6 ways to choose the suits for the pair; thus there are 156 · 4 · 6 = 3744 possible full house hands, so the probability is $\frac{3744}{C(52,\ 5)}\approx 0.0014.$

44. There are 13 choices for the denomination and *C*(4, 3) = 4 choices for the suits for the three-of -a-kind; there are *C*(12, 2) = 66 choices for the denominations and 4 · 4 = 16 choices for the suits of the remaining two cards, for a total of 13 · 4 · 66 · 16 = 54,912 possible three-of-a-kind poker hands; so the probability is

$\frac{54{,}912}{C(52,\ 5)}\approx 0.0211.$

45. There are *C*(13, 2) = 78 choices for the denominations of the two pairs, and *C*(4, 2) = 6 choices each for their suits; the remaining card has 52 – 8 = 44 choices, for a total of $78\cdot 6^2\cdot 44=123{,}552$ possible two pair hands, so the probability is $\frac{123{,}552}{C(52,\ 5)}\approx 0.0475.$

46. There are 13 · *C*(4, 2) = 78 pairs (13 choices for denomination, *C*(4, 2) = 6 choices for suits); *C*(12, 3) = 220 choices for the denominations of the remaining cards, and 4 choices each for their suits, which gives a total of $78\cdot 220\cdot 4^3=1{,}098{,}240$ one-pair hands, and so the probability is $\frac{1{,}098{,}240}{C(52,\ 5)}\approx 0.4226.$

47. **a.** There are 4 ways to select the suit of the 4 card group, *C*(13, 4) ways to select their denominations, and $C(13,\ 3)^3$ ways to select 3 cards each from the remaining 3 suits. The probability of a 4-3-3-3 bridge hand is

$\frac{4\cdot C(13,\ 4)\cdot C(13,\ 3)^3}{C(52,\ 13)}\approx 0.1054.$

b. There are *C*(4, 2) = 6 ways to choose the suits of the 4-card groups and 2 ways to choose the suit of the 3-card group (then the suit of the 2-card group is uniquely determined). The denominations can then be chosen in $C(13,\ 4)^2\cdot C(13,\ 3)\cdot C(13,\ 2)$ ways, so the probability of a 4-4-3-2 bridge hand is

$\frac{6\cdot 2\cdot C(13,\ 4)^2\cdot C(13,\ 3)\cdot C(13,\ 2)}{C(52,\ 13)}\approx 0.2155.$

48. **a.** $\frac{1}{C(69,5)\cdot 26}=\frac{1}{292{,}201{,}338}$

b. 1 to $292{,}201{,}337$

49. $\frac{2}{C(40,\ 6)}=\frac{1}{1{,}919{,}190}=0.0000005211$

50. Many people think multiples of 7 are lucky, and some might also pick 13 just to prove they're not superstitious! To avoid sharing, avoid "lucky" numbers.

51. $\frac{C(5,3)\cdot C(34,2)}{C(39,\ 5)}=\frac{10\cdot 561}{575{,}757}$
$=\frac{5610}{575{,}757}$
≈ 0.0097

52. $\frac{C(47,6)}{C(53,\ 6)}=\frac{10{,}737{,}573}{22{,}957{,}480}$
≈ 0.4677

53. $\frac{25-20}{80}=\frac{5}{80}=\frac{1}{16}=0.0625=6.25\%$

$\frac{25-20}{25}=\frac{5}{25}=\frac{1}{5}=0.2=20\%$

16 people will be needed because it helps 1 out of every 16 people.

54. $\frac{14-8}{60}=\frac{6}{60}=\frac{1}{10}=0.1=10\%$

$\frac{14-8}{14}=\frac{6}{14}=\frac{3}{7}=0.4286=42.86\%$

10 people will be needed because it helps 1 out of every 10 people.

55. $\frac{20-15}{100}=\frac{5}{100}=\frac{1}{20}=0.05=5\%$

$\frac{20-15}{20}=\frac{5}{20}=\frac{1}{4}=0.25=25\%$

20 people will be needed because it helps 1 out of every 20 people.

56. $\frac{50-40}{200}=\frac{10}{200}=\frac{1}{20}=0.05=5\%$

$\frac{50-40}{50}=\frac{10}{50}=\frac{1}{5}=0.2=20\%$

20 people will be needed because it helps 1 out of every 20 people.

57. Let x represent the number of people in the control group that developed the condition.

$$\frac{x-12}{x}=\frac{0.25}{1}$$
$$0.25x=x-12$$
$$-0.75x=-12$$
$$x=16$$

58. Since 80 people were required for one person to be helped: $\frac{1}{80}=0.0125=1.25\%$

59. $1-\frac{P(100,15)}{100^{15}}\approx 0.6687$

60. $1-\frac{C(11,4)}{C(14,4)}\approx 0.6703$

61. The probability that no two people choose the same card is $\frac{P(52,\ n)}{52^n}$, so the probability that at least two people pick the same card is

$P_n=1-\frac{P(52,\ n)}{52^n}.$

For $n = 5$, $P_5\approx 0.1797.$

$P_8\approx 0.4324$ and $P_9\approx 0.5197.$

P_n increases as n increases, so $n = 9$ is the smallest value of n for which $P_n>0.5.$

62. $1-\frac{N\times(N-1)\times(N-2)\times \text{L} \times(N-20+1)}{N^{20}}$ dips below .5 starting with $N=281$.

63. The probability that one or more people in a group of size n were born on a given specific day is $P_n=1-\left(\frac{364}{365}\right)^n.$

For $n = 100$, $P_n\approx 0.240.$

$P_{252}\approx 0.4991$ and $P_{253}\approx 0.5005,$ so $n = 253$ is the smallest value of n for which $P_n>0.5.$

64. $1-\frac{P(100,N)}{100^N}$ rises above 50% starting with $N=13.$

65. When $k=6,\ 1-\frac{C(n-k+1,k)}{C(n,k)}$ drops below 50% starting when $n=48.$

66. $1-\left(\frac{35}{36}\right)^N$ rises above 50% starting with $N=25.$

Exercises 6.4

1. **a.** $\Pr(E)=0.3+0.2=0.5$

b. $\Pr(F)=0.2+0.4=0.6$

c. $\Pr(E|F)=\frac{0.2}{0.6}=0.3333$

d. $\Pr(F|E)=\frac{0.2}{0.5}=0.4$

2. **a.** $\Pr(E)=\frac{1}{3}+\frac{1}{6}=\frac{1}{2}$

b. $\Pr(F)=\frac{1}{6}+\frac{1}{2}=\frac{2}{3}$

c. $\Pr\left(E|F\right)=\frac{\frac{1}{6}}{\frac{2}{3}}=\frac{1}{4}$

d. $\Pr\left(F|E\right)=\frac{\frac{1}{6}}{\frac{1}{2}}=\frac{1}{3}$

3. **a.** $\Pr\left(E|F\right)=\frac{0.1}{0.4}=\frac{1}{4}$

b. $\Pr\left(F|E\right)=\frac{0.1}{0.5}=\frac{1}{5}$

c. $\Pr\left(E|F'\right)=\frac{0.4}{0.6}=\frac{2}{3}$

d. $\Pr\left(E'|F'\right)=\frac{0.2}{0.6}=\frac{1}{3}$

4. **a.** $\Pr\left(E|F\right)=\frac{0.2}{0.3}=\frac{2}{3}$

b. $\Pr\left(F|E\right)=\frac{0.2}{0.6}=\frac{1}{3}$

c. $\Pr\left(E|F'\right)=\frac{0.4}{0.7}=\frac{4}{7}$

d. $\Pr\left(E'|F'\right)=\frac{0.3}{0.7}=\frac{3}{7}$

5. **a.** $\Pr(E\cap F)=\frac{1}{3}+\frac{5}{12}-\frac{2}{3}=\frac{1}{12}$

b. $\Pr\left(E|F\right)=\frac{\frac{1}{12}}{\frac{5}{12}}=\frac{1}{5}$

c. $\Pr\left(F|E\right)=\frac{\frac{1}{12}}{\frac{1}{3}}=\frac{1}{4}$

6. **a.** $\Pr(E\cap F)=\frac{1}{2}+\frac{1}{3}-\frac{7}{12}=\frac{3}{12}=\frac{1}{4}$

b. $\Pr\left(E|F\right)=\frac{\frac{1}{4}}{\frac{1}{3}}=\frac{3}{4}$

c. $\Pr\left(F|E\right)=\frac{\frac{1}{4}}{\frac{1}{2}}=\frac{1}{2}$

7. **a.** $\Pr(F\mid E)=\frac{\Pr(E\cap F)}{\Pr(E)}$

$0.25=\frac{\Pr(E\cap F)}{0.4}$

$\Pr(E\cap F)=0.1$

b. $\Pr(E\cup F)=0.4+0.3-0.1=0.6$

c. $\Pr\left(E|F\right)=\frac{0.1}{0.3}=\frac{1}{3}$

d. $\Pr(E'\cap F)=0.3-0.1=0.2$

8. **a.** $\Pr(F\mid E)=\frac{\Pr(E\cap F)}{\Pr(E)}$

$0.4=\frac{\Pr(E\cap F)}{0.5}$

$\Pr(E\cap F)=0.2$

b. $\Pr(E\cup F)=0.5+0.3-0.2=0.6$

c. $\Pr\left(E|F\right)=\frac{0.2}{0.3}=\frac{2}{3}$

d. $\Pr(E\cap F')=0.5-0.2=0.3$

9. $\Pr(8\mid \text{not } 7)=\frac{\Pr(8\cap \text{not } 7)}{\Pr(\text{not } 7)}$

$\Pr(8\mid \text{not } 7)=\frac{\frac{5}{36}}{\frac{30}{36}}$

$\Pr(8\mid \text{not } 7)=\frac{5}{30}=\frac{1}{6}$

10. $\Pr(8\mid \text{one } 3)=\frac{2}{5+5}=\frac{2}{10}=\frac{1}{5}=0.2$

11. 0; because exactly one coin shows heads therefore there are two tails.

12. 0; because exactly one coin shows tails

13. $\frac{[\text{number of outcomes that four are white}]}{[\text{number of outcomes that at least 1 is white}]}$

$$\frac{C(7,4)}{C(12,4)-C(5,4)} = \frac{35}{495-5} = \frac{35}{490} = \frac{1}{14} \approx 0.0714$$

14. $\frac{\text{[number of outcomes that two are white]}}{\text{[number of outcomes that at least 1 is white]}}$

$$= \frac{C(2,\ 2)}{C(5,\ 2)-C(3,2)} = \frac{1}{10-3} = \frac{1}{7} \approx 0.1429$$

15. $\Pr(\text{both girls}|\text{first girl}) = \frac{1}{2}$

16. $\Pr(\text{both girls}|\text{at least one girl}) = \frac{1}{3}$

17. $\Pr(\text{grad}|\text{ more \$45,000}) = \frac{\Pr(\text{grad and} >45000)}{\Pr(>45000)} = \frac{0.10}{0.25} = \frac{2}{5} = 0.4$

18. $\Pr(\text{masters}|\text{ female}) = \frac{\Pr(\text{masters and female})}{\Pr(\text{female})} = \frac{.40}{.60} = \frac{2}{3}$

19. **a.** $\Pr(\text{Masters}) = \frac{851}{2898} \approx 0.2937$

b. $\Pr(\text{Male}) = \frac{1201}{2898} \approx 0.4144$

c. $\Pr(\text{Female}|\text{Masters}) = \frac{522}{851} \approx 0.6134$

d. $\Pr(\text{Doctors}|\text{Female}) = \frac{93}{1697} \approx 0.0548$

20. **a.** $\Pr(\text{Voted}) = \frac{92.2}{185.3} \approx 0.4976$

b. $\Pr(\text{Male}) = \frac{88.5}{185.3} \approx 0.4776$

c. $\Pr(\text{Female}|\text{Voted}) = \frac{49.2}{92.2} \approx 0.5336$

d. $\Pr(\text{Voted}|\text{Male}) = \frac{43.0}{88.5} \approx 0.4859$

21. **a.** $\Pr(\text{Officer}) = \frac{228.6}{1291.8} \approx 0.1770$

b. $\Pr(\text{Marine}) = \frac{183.2}{1291.8} \approx 0.1418$

c. $\Pr(\text{Officer and Marine}) = \frac{20.7}{1291.8} \approx 0.0160$

d. $\Pr(\text{Officer}|\text{Marine}) = \frac{20.7}{183.2} \approx 0.1130$

e. $\Pr(\text{Marine}|\text{Officer}) = \frac{20.7}{228.6} \approx 0.0906$

22. **a.** $\Pr(\text{Business}) = \frac{362}{2500} = 0.1448$

b. $\Pr(\text{Female}) = \frac{1000}{2500} = 0.4$

c. $\Pr(\text{Female and Business}) = \frac{102}{2500} = 0.0408$

d. $\Pr(\text{Male}|\text{Social Studies}) = \frac{122}{252} \approx 0.4841$

e. $\Pr(\text{Social Studies}|\text{Female}) = \frac{130}{1000} = 0.13$

23. $\Pr(\$5|\$5) = \frac{\frac{1}{3}}{\frac{1}{2}} = \frac{2}{3}$

24. $\Pr(\text{Gold}|\text{Gold}) = \frac{\frac{1}{3}}{\frac{1}{2}} = \frac{2}{3}$

25. $\frac{4}{52}\cdot\frac{3}{51}=\frac{12}{2652}=\frac{1}{221}\approx 0.004525$

26. $\frac{13}{52}\cdot\frac{12}{51}=\frac{156}{2652}=\frac{1}{17}\approx 0.05882$

27. $\frac{1}{2}$; because the flip of the coin the fifth time is independent of the first four times.

28. $\frac{1}{2}$; because the flip of the coin the first time is independent of the second time.

29.
$$\begin{aligned}\Pr(\text{Kasich}|\text{Fem.}) &= \frac{\Pr(\text{Kasich and Fem.})}{\Pr(\text{Fem.})}\\ 0.09 &= \frac{\Pr(\text{Kasich and Fem.})}{0.48}\\ \Pr(\text{Kasich and Fem.}) &= 0.0432\end{aligned}$$

30. $\Pr(\text{Shanghai}) = 0.016\cdot 0.2 = 0.0032$

31. $\Pr(\text{Win with 2 point shot}) = 0.48\cdot 0.5 = 0.24$

$\Pr(\text{Win with 3 point shot}) = 0.29$

Therefore, there is a better chance of winning if you take the three – point shot.

32. $\frac{2}{10}=\frac{1}{5}$

33. $\Pr(E\cap F) = 0.4+0.5-0.7 = 0.2$

$\Pr(E\mid F) = \frac{0.2}{0.5} = 0.4 = \Pr(E)$

$\Pr(F\mid E) = \frac{0.2}{0.4} = 0.5 = \Pr(F)$

Therefore, the two events are independent.

34. $\Pr(E\cap F) = 0.2+0.5-0.6 = 0.1$

$\Pr(E\mid F) = \frac{0.1}{0.5} = 0.2 = \Pr(E)$

$\Pr(F\mid E) = \frac{0.1}{0.2} = 0.5 = \Pr(F)$

Therefore, the two events are independent.

35. $\Pr(E\cap F) = (0.5)(0.6) = 0.3$

$\Pr(E\cup F) = 0.5+0.6-0.3 = 0.8$

36. $\Pr(E\cap F) = (0.25)(0.4) = 0.1$

$\Pr(E\cup F) = 0.25+0.4-0.1 = 0.55$

37. Since the events are independent, $\Pr(F\mid E) = \Pr(F) = 0.6$

38. Since the events are independent,
$$\begin{aligned}\Pr(F) &= 1-\Pr(F'\mid E')\\ &= 1-\Pr(F')\\ &= 1-0.3\\ &= 0.7\end{aligned}$$

39. Since the events are independent,
$$\Pr(F) = \frac{\Pr(E\cap F)}{1-\Pr(E')} = \frac{0.1}{1-0.6} = \frac{0.1}{0.4} = 0.25$$

40. Since the events are independent,
$$\Pr(F) = \frac{\Pr(E\cap F)}{\Pr(E)} = \frac{0.4}{0.8} = 0.5$$

41. Since the events are independent,
$$\begin{aligned}\Pr[(A\cap B\cap C)'] &= 1-(0.4)(0.1)(0.2)\\ &= 1-0.008\\ &= 0.992\end{aligned}$$

42. Since the events are independent,
$$\Pr(B) = \frac{\Pr(A\cap B)}{\Pr(A)} = \frac{0.12}{0.2} = 0.6$$
$$\Pr(C) = \frac{\Pr(A\cap C)}{\Pr(A)} = \frac{0.06}{0.2} = 0.3$$
$$\begin{aligned}\Pr(B\cap C) &= \Pr(B)\Pr(C)\\ &= (0.6)(0.3)\\ &= 0.18\end{aligned}$$

43. No; because the selection of the first ball affects the selection of the second ball.

44. No; because the selection of the first ball affects the selection of the second ball.

45. Yes; because $\Pr(E \cap F) = \Pr(E) \cdot \Pr(F)$.

46. No; because $\Pr(E \cap F) \neq \Pr(E) \cdot \Pr(F)$.

47. No; because $\Pr(E \cap F) \neq \Pr(E) \cdot \Pr(F)$.

48. No; because $\Pr(E \cap F) \neq \Pr(E) \cdot \Pr(F)$.

49. No; because $\Pr(E \cap F) \neq \Pr(E) \cdot \Pr(F)$.

50. **a.** $\dfrac{\Pr(\text{Both corr.})}{\Pr(\text{first corr.})} = \dfrac{0.7+0.8-0.9}{0.7} \approx 0.8571$

b. They are not independent because the value that the second test is correct given the first is correct is 0.8751 from part (a) but the probability that the second test is correct is 0.8.

51. a. $\Pr(\text{pass all}) = (0.80)(0.75)(0.60) = 0.36$

b. $\Pr(\text{pass first 2}) = (0.80)(0.75)(0.40) = 0.24$

$\Pr(\text{pass 1st and 3rd}) = (0.80)(0.25)(0.60) = 0.12$

$\Pr(\text{pass last 2}) = (0.2)(0.75)(0.60) = 0.09$

$\Pr(\text{pass} \geq 2) = 0.36 + 0.24 + 0.12 + 0.09 = 0.81$

52. $\Pr(\text{all correct}) = \left(\dfrac{1}{2}\right)^{10} = \dfrac{1}{1024} \approx 0.0009766$

53. $(0.99)^5(0.98)^5(0.975)^3 \approx 0.7967$

54. $(1-0.003)^{72} = (0.997)^{72} \approx 0.8055$

55. $(1-0.7)^4 = (0.3)^4 = 0.0081$

56. $(0.15)^3 = 0.003375$

$1-(0.85)^3 = 0.3860$

57. **a.** $1-(0.7)^4 = 1-0.2401 = 0.7599$

b. $(0.7599)^{10} \approx 0.06420$

c. $1-(0.9358)^{20} \approx 0.7347$

58. **a.** 0.64

b. $1-\left(\dfrac{37}{38}\right)^{38} \approx 0.6370$

59. Scoring 0: $1-0.6 = 0.4$

Scoring 1: $(0.6)(1-0.6) = 0.24$

Scoring 2: $(0.6)(0.6) = 0.36$

60. Scoring 0: $1-0.7 = 0.3$

Scoring 1: $(0.7)(1-0.7) = 0.21$

Scoring 2: $(0.7)(0.7) = 0.49$

61. Answer will vary.

62. Let p = the probability of success.

$$\begin{aligned} p(1-p) &= p \cdot p \\ p - p^2 &= p^2 \\ 0 &= 2p^2 - p \\ 0 &= p(2p-1) \\ p &= 0 \\ p &= 0.5 \end{aligned}$$

Since the probability cannot be 0, the probability must be 0.5.

63. $(0.6)(0.4) = 0.24$

$(0.4)(0.6) = 0.24$

Answers will vary

64. No; All flips of a fair coin are independent of each other.

65. Answers will vary.

66. Answers will vary.

67. Answers will vary.

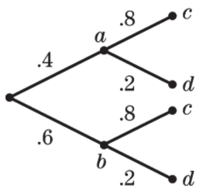
68. Answers will vary.

69. 26; $1-\left(\dfrac{37}{38}\right)^{26} \approx 0.5001$

70. 8; $1-\left(\frac{10}{19}\right)^8 \approx 0.994112$

Exercises 6.5

1.



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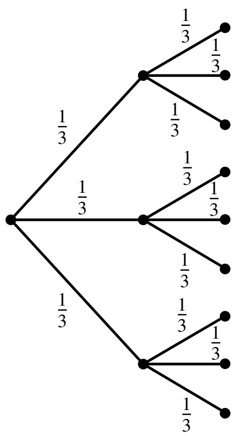

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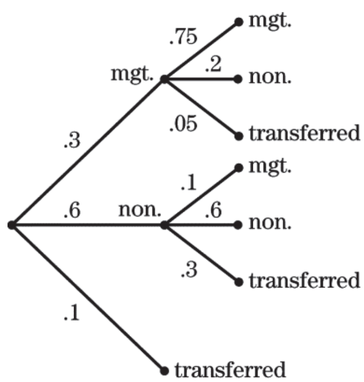

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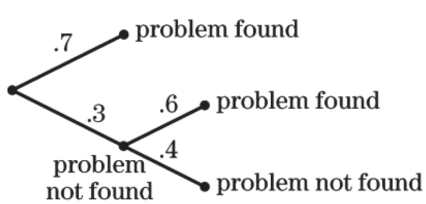


5. $0.30 \times 0.75 + 0.60 \times 0.10 = 0.285$

6. $0.2 \times 0.4 = 0.08$

7. $0.10 + 0.30 \times 0.05 + 0.60 \times 0.30 = 0.295$

8. $0.3 \times (0.2 + 0.05) = 0.075$

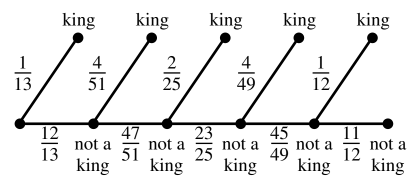
9. Pr(white, then red) + Pr(red, then red)

$$= \frac{2}{3}\times\frac{1}{2}+\frac{1}{3}\times\frac{3}{4}=\frac{7}{12}$$

10. $\text{Pr(6 on die)} = \frac{40}{52}\times\frac{1}{6}=\frac{5}{39}$;

$$\text{Pr(head on coin)}=\frac{12}{52}\times\frac{1}{2}=\frac{3}{26}$$

11.



Pr(king on 1st draw) + Pr(king on 2nd draw)
+ Pr(king on 3rd draw)
= 1 – Pr(not a king on 3rd draw)

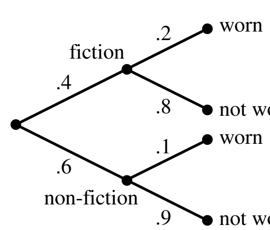
$$=1-\frac{12}{13}\times\frac{47}{51}\times\frac{23}{25}=1-\frac{4324}{5525}=\frac{1201}{5525}\approx 0.22$$

12. a. $\text{Pr(white)} = \frac{6}{8}=\frac{3}{4}=0.75$

b. $\text{Pr(red and white)} = \frac{2}{8}\cdot\frac{6}{7}=\frac{3}{14}$

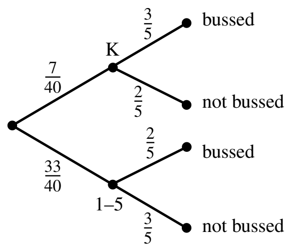
c. $\text{Pr(red and red and white)} = \frac{2}{8}\cdot\frac{1}{7}\cdot\frac{6}{6} = \frac{1}{28}$

13.



$0.40 \times 0.20 + 0.60 \times 0.10 = 0.14$

14.



Pr(elev. lead levels) =
$0.77 \cdot 0.06 + 0.23 \cdot 0.11 = 0.0715$

15. Pr(male|color-blind) $= \dfrac{\frac{1}{2} \times 0.08}{\frac{1}{2} \times 0.08 + \frac{1}{2} \times 0.005} = \dfrac{16}{17} \approx 0.9412$

16. **a.** $0.60 \times 0.03 + 0.40 \times 0.02 = 0.026$

b. Pr(machine I|defective) $= \dfrac{0.60 \times 0.03}{0.026} = \dfrac{9}{13}$

17. $0.5 \times 0.9 + 0.5 \times 0.7 = 0.8$

18. $(0.5)(0.9)(0.9) + (0.5)(0.1)(0.7) + (0.5)(0.7)(0.9) + (0.5)(0.3)(0.7) = 0.86$

19. $\Pr\left(\text{fake} \middle| \text{HH}\right) = \dfrac{\frac{1}{4} \times 1}{\frac{3}{4} \times \frac{1}{4} + \frac{1}{4} \times 1} = \dfrac{4}{7}$

20. Note that if the bag originally contains a white ball, then a white ball is always selected.

Pr(bag originally contains a white ball) $= \dfrac{1}{2}$

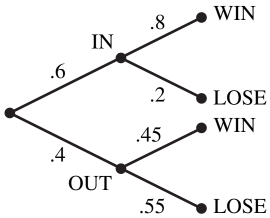
Pr(white ball is selected) $= \dfrac{1}{2} \times 1 + \dfrac{1}{2} \times \dfrac{1}{2} = \dfrac{3}{4}$

$\Pr\left(\text{bag originally contains a white ball} \middle| \text{white ball is selected}\right) = \dfrac{\frac{1}{2}}{\frac{3}{4}} = \dfrac{2}{3}$

21. **a.** Pr(wins the point) = $0.60 \times 0.75 + 0.40 \times 0.75 \times 0.50 = 0.60$

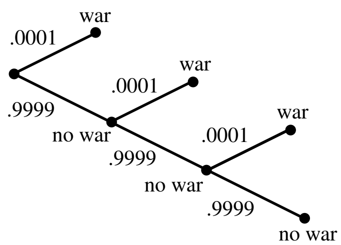
b. Pr(first serve good|wins service point) $= \dfrac{\Pr\left(\text{first serve good and wins service point}\right)}{\Pr\left(\text{wins service point}\right)} = \dfrac{0.60 \times 0.75}{0.60}$
$= 0.75$

22.



Pr(WIN) = $0.6 \times 0.8 + 0.4 \times 0.45 = 0.66$

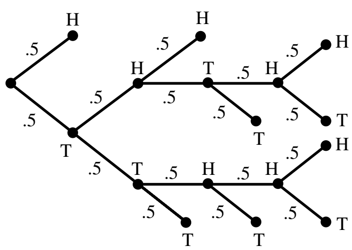
23.



$0.0001 + 0.9999 \times 0.0001 + 0.9999^2 \times 0.0001$ or $1 - (0.9999)^3 \approx 0.00029997$

24. $1 - (0.9999)^n$

25.



$.5 + (.5)^3 + 2(.5)^5 = \frac{11}{16}$

26. Same shape

$\text{Pr(winning)} = \frac{3}{6} + \frac{3}{6} \times \frac{3}{5} \times \frac{2}{4} + \frac{3}{6} \times \frac{3}{5} \times \frac{2}{4} \times \frac{2}{3} \times \frac{1}{2} + \frac{3}{6} \times \frac{2}{5} \times \frac{3}{4} \times \frac{2}{3} \times \frac{1}{2} = \frac{3}{4}$

Probability of winning card game is greater.

27. **a.** $\text{Pr(white)} = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

$\text{Pr(red)} = 1 - \frac{1}{4} = \frac{3}{4}$

b. Pr(red) = $0.6 \times 0.5 + 0.4 \times 1 = 0.7$

28. **a.** $\text{Pr(white)} = \frac{1}{3} \times 0 + \frac{1}{3} \times \frac{1}{2} + \frac{1}{3} \times 1 = \frac{1}{2}$

b. $\text{Pr(Cc | red)} = \frac{1}{6} \div \frac{1}{2} = \frac{1}{6} \times \frac{2}{1} = \frac{1}{3}$

29. $\Pr(\text{night}|\text{part-timer}) = \dfrac{0.60\times 2}{0.40\times 5+ .60\times 2} = \dfrac{3}{8} = 0.375$

30. We want

$$\begin{aligned}
&\Pr(\text{light is actually defective}|\text{found defective})\\
&= \frac{\Pr(\text{actually def and found def})}{\Pr(\text{found def})}\\
&= \frac{\Pr(\text{actually def and found def})}{\Pr(\text{def} \cap \text{found def}) + \Pr(\text{not def} \cap \text{found def})}\\
&= \frac{0.0005\cdot 0.99}{0.0005\cdot 0.99 + 0.9995\cdot 0.01}\\
&\approx 0.04719
\end{aligned}$$

31. $pr(W_2) = pr(R_1 \cap W_2) + pr(W_1 \cap W_2)$

$$= pr(R_1)pr\left(W_2 \middle| R_1\right) + pr(W_1)p\left(W_2 \middle| W_1\right)$$

$$= \left(\frac{5}{10}\right)\cdot\left(\frac{12}{13}\right) + \left(\frac{5}{10}\right)\cdot 1$$

$$= \frac{25}{26}$$

32. Pr(Green on second)

$$= pr(R_1)pr\left(G_2 \middle| R_1\right) + pr(G_1)\cdot pr\left(G_2 \middle| G_1\right)$$

$$= \frac{5}{8}\cdot\frac{4}{8} + \frac{3}{8}\cdot\frac{2}{8} = \frac{13}{32}$$

33. Pr(get same number of heads)
= Pr(2 heads) + Pr(1 head) + Pr(0 heads)

$$= \frac{1}{4}\cdot\frac{1}{4} + \frac{1}{2}\cdot\frac{1}{2} + \frac{1}{4}\cdot\frac{1}{4} = \frac{3}{8}$$

34. a. Pr(both red) $= \frac{4}{7}\cdot\frac{3}{6} = \frac{2}{7}$

Pr(both green) $= \frac{3}{7}\cdot\frac{2}{6} = \frac{1}{7}$

b. Pr(exactly 1 red) $= \frac{4}{7}\cdot\frac{3}{6} + \frac{3}{7}\cdot\frac{4}{6} = \frac{4}{7}$

35. a. Since printer B produces 201 models 99 out of 100 weeks, Pr (printer B produces more models than printer A) = 0.99.

b. For printer A to produce more than printer B, printer B would have to break down. Since this only occurs 1% of the time, the probability that printer B produces more models than printer A is $0.99^{200} \approx 0.1340$.

36. a. Lou's average score is 0.70(3) + 0.30(6) = 3.9, while Bud's average score is always a 4. Based upon long run averages, Lou will do better on a single par 3 course.

b. Summarize the outcomes of Lou in the table. The table represents Lou's possible scores on two consecutive par three holes:

3	3
3	6
6	3
6	6

Since Bud always scores a 4, his outcomes are

4	4
4	4
4	4
4	4

Lou only wins in the first case when he scores a 3 and a 3 on two consecutive holes. The probability he will do this is $(0.7)^2 = 0.49$, so Lou has a 0.49 chance of winning and Bud has a 0.51 chance of winning on two consecutive holes.

37. a. Pr(red die > blue die) $= \frac{1}{2} + \frac{1}{2}\cdot\frac{5}{6}$

$$= \frac{1}{2} + \frac{1}{12} = \frac{7}{12}$$

b. Pr(blue die > green die) $= \frac{1}{6} + \frac{5}{6}\cdot\frac{1}{2}$

$$= \frac{1}{6} + \frac{5}{12} = \frac{7}{12}$$

c. Pr(green die > red die) = $0 + \frac{5}{6}\cdot\frac{5}{6}$

$$= 0 + \frac{25}{36} = \frac{25}{36}$$

d. Since the red die beats the blue die more than half the time and the blue die beats the green die more than half the time, the red die appears to be the strongest of the three dice and the green appears to be the weakest. However, the green die beats the red die more than half the time.

38. a. $0.60 \times 0.28 = 0.168$

b. $0.60 \times 0.39 + 0.40 \times 0.33 = 0.336$

39. True; Sensitivity gives the percent of people that have a condition given the fact that they tested positive.

40. True; Specificity gives the percent of people that don't have a condition given the fact that they tested negative.

41. True; Based on the definition of specificity.

42. True; Based on the definition of sensitivity.

43. True; Sensitivity gives the percentage of people that have a given condition.

44. True; Specificity gives the percentage of people that don't have a given condition.

45. $$\begin{aligned}\Pr(\text{Hep}|\text{Pos.}) &= \frac{(0.0005)(0.95)}{(0.0005)(0.95)+(0.9995)(0.1)}\\ &= \frac{0.000475}{0.000475+0.09995}\\ &= \frac{0.000475}{0.100425} \approx 0.00473\end{aligned}$$

46. $$\begin{aligned}\Pr(\text{TB}|\text{Positive}) &= \frac{(0.02)(0.98)}{(0.02)(0.98)+(0.98)(0.01)}\\ &= \frac{0.0196}{0.0196+0.0098}\\ &= \frac{0.0196}{0.0294} = \frac{2}{3}\end{aligned}$$

47. $$\Pr(\text{Cond.}|\text{Positive}) = \frac{9}{19} \approx 0.474$$

48. $$\Pr(\text{Cond.}|\text{Positive}) = \frac{13}{20} = 0.65 = 65\%$$

49. $$\begin{aligned}\Pr(\text{Used}|\text{Positive}) &= \frac{(0.05)(1)}{(0.05)(1)+(0.95)(.01)}\\ &= \frac{0.05}{0.05+.0095}\\ &= \frac{0.05}{0.0595} \approx 0.84 = 84\%\end{aligned}$$

50. $$\begin{aligned}\Pr(\text{Guilty}|\text{Pos.}) &= \frac{(0.10)(0.88)}{(0.10)(0.88)+(0.9)(0.14)}\\ &= \frac{0.088}{0.088+0.126}\\ &= \frac{0.088}{0.214} \approx 0.4112 = 41.12\%\end{aligned}$$

Exercises 6.6

1. $$\begin{aligned}\Pr\left(\text{over 60}|\text{acc.}\right) &= \frac{0.10\times 0.04}{(0.05\times 0.06)+(0.10\times 0.04)+(0.25\times 0.02)+(0.20\times 0.015)+(0.30\times 0.025)+(0.10\times 0.04)}\\ &= \frac{0.004}{0.0265}\\ &= \frac{8}{53}\end{aligned}$$

2. $$\begin{aligned}\Pr(\text{type 1}|\text{fail.}) &= \frac{0.30\times 0.0002}{(0.3\times 0.0002)+(0.25\times 0.0004)+(0.2\times 0.0005)+(0.1\times 0.001)+(0.05\times 0.002)+(0.1\times 0.004)}\\ &= \frac{3}{43}\end{aligned}$$

3. $$\Pr\left(\text{sophomore}|\text{A}\right) = \frac{0.30\times 0.4}{(0.10\times 0.2)+(0.30\times 0.4)+(0.40\times 0.3)+(0.20\times 0.1)} = \frac{0.12}{0.28} = \frac{3}{7}$$

4. $$\Pr(\text{precinct 3}|\text{larceny}) = \frac{0.40\times 0.05}{(0.20\times 0.01)+(0.10\times 0.02)+(0.40\times 0.05)+(0.30\times 0.04)} = \frac{5}{9}$$

5. $$\Pr\left(\geq \$75{,}000 \middle| 2 \text{ or more cars}\right) = \frac{0.05\times 0.9}{(0.10\times 0.2)+(0.20\times 0.5)+(0.35\times 0.6)+(0.30\times 0.75)+(0.05\times 0.9)} = \frac{0.045}{0.6} = \frac{3}{40} = 0.075$$

6. $$\Pr(\text{reg. Indep.}|\text{turned out}) = \frac{0.30\times 0.7}{(0.50\times 0.4)+(0.20\times 0.5)+(0.30\times 0.7)} = \frac{7}{17}$$

7. $$\Pr\left(\text{passed exam}\middle|\text{A}\right) = \frac{0.80\times 0.40}{0.80\times 0.40+0.20\times 0.20} = \frac{0.32}{0.36} = \frac{8}{9}$$

8. **a.** $(0.07 \times 0.51) + (0.20 \times 0.51) + (0.33 \times 0.50) + (0.27 \times 0.48) + (0.13 \times 0.44) = 0.4895$

b. $$\Pr(5 \text{ to } 19|\text{male}) = \frac{0.20\times 0.51}{0.4895} \approx 0.2084$$

9. **a.** $(0.20 \times 0.20) + (0.15 \times 0.15) + (0.25 \times 0.12) + (0.30 \times 0.10) + (0.10 \times 0.10) = 0.1325$

b. $$\Pr\left(\text{division C}\middle|\text{bilingual}\right) = \frac{0.25\times 0.12}{0.1325} = \frac{0.03}{0.1325} = \frac{12}{53} \approx 0.23$$

10. Pr(four 2-spot die|one 2-spot in six)

$$= \frac{\Pr(\text{four 2-spot die})\times\Pr\left(\text{one 2-spot in six}\middle|\text{four 2-spot die}\right)}{\Pr(\text{four 2-spot die})\times\Pr\left(\text{one 2-spot in six}\middle|\text{four 2-spot die}\right)+\Pr(\text{three 2-spot die})\times\Pr\left(\text{one 2-spot in six}\middle|\text{three 2-spot die}\right)}$$

$$= \frac{\frac{1}{2}\times\left[6\times\frac{4}{6}\times\left(\frac{2}{6}\right)^5\right]}{\frac{1}{2}\times\left[6\times\frac{4}{6}\times\left(\frac{2}{6}\right)^5\right]+\frac{1}{2}\times\left[6\times\frac{3}{6}\times\left(\frac{3}{6}\right)^5\right]}$$

$$= \frac{\frac{2}{3}\times\left(\frac{1}{3}\right)^5}{\frac{2}{3}\times\left(\frac{1}{3}\right)^5+\left(\frac{1}{2}\right)^6}$$

$$\approx 0.1494$$

11. $$\Pr\left(\text{cancer}\middle|\text{positive}\right) = \frac{\Pr(\text{cancer})\times\Pr\left(\text{positive}\middle|\text{cancer}\right)}{\Pr(\text{cancer})\times\Pr\left(\text{positive}\middle|\text{cancer}\right)+\Pr(\text{no cancer})\times\Pr\left(\text{positive}\middle|\text{no cancer}\right)}$$

$$= \frac{0.02\times 0.75}{(0.02\times 0.75)+(0.98\times 0.30)} = \frac{5}{103} \approx 0.049$$

12. a. $$\Pr(\text{user}|\text{positive}) = \frac{\Pr(\text{user})\times\Pr\left(\text{positive}|\text{user}\right)}{\Pr(\text{user})\times\Pr\left(\text{positive}|\text{user}\right)+\Pr(\text{nonuser})\times\Pr\left(\text{positive}|\text{nonuser}\right)}$$
$$= \frac{0.10\times(1-0.02)}{[0.10\times(1-0.02)]+(0.90\times0.05)}$$
$$= \frac{98}{143}$$
$$\approx 0.685$$

b. $(0.05)^2 = 0.0025$

c. $$\Pr(\text{nonuser}|\text{twice positive}) = \frac{\Pr(\text{nonuser})\times\Pr\left(\text{twice positive}|\text{nonuser}\right)}{\Pr(\text{nonuser})\times\Pr\left(\text{twice positive}|\text{nonuser}\right)+\Pr(\text{user})\times\Pr\left(\text{twice positive}|\text{user}\right)}$$
$$= \frac{0.90\times 0.0025}{(0.90\times 0.0025)+[0.10\times(0.98)^2]}$$
$$\approx 0.0229$$

13. a. 1 – 0.99 = 0.01

b. $\Pr\left(\text{pregnant}|\text{positive}\right)$
$$= \frac{\Pr(\text{pregnant})\times\Pr\left(\text{positive}|\text{pregnant}\right)}{\Pr(\text{pregnant})\times\Pr\left(\text{positive}|\text{pregnant}\right)+\Pr(\text{not pregnant})\times\Pr\left(\text{positive}|\text{not pregnant}\right)}$$
$$= \frac{0.40\times 0.99}{(0.40\times 0.99)+(0.60\times0.02)} = \frac{33}{34} \approx 0.971$$

14. With 65% incidence, $$\Pr(\text{condition}|\text{pos}) = \frac{\Pr(\text{condition})\times\Pr\left(\text{pos}|\text{condition}\right)}{\Pr(\text{condition})\times\Pr\left(\text{pos}|\text{condition}\right)+\Pr(\text{no cond.})\times\Pr\left(\text{pos}|\text{no cond.}\right)}$$
$$= \frac{0.65\times0.90}{(0.65\times 0.90)+(0.35\times0.20)}$$
$$\approx 0.893$$

With 30% incidence, $$\Pr(\text{condition}|\text{pos}) = \frac{0.30\times0.90}{(0.30\times 0.90)+(0.70\times0.20)} \approx 0.659$$

15. $$\Pr\left(\text{steroids}|\text{positive}\right) = \frac{\Pr(\text{steroids})\times\Pr\left(\text{positive}|\text{steroids}\right)}{\Pr(\text{steroids})\times\Pr\left(\text{positive}|\text{steroids}\right)+\Pr(\text{no steroids})\times\Pr\left(\text{positive}|\text{no steroids}\right)}$$
$$= \frac{0.10\times0.93}{(0.10\times 0.93)+(0.90\times0.02)} = \frac{31}{37} \approx 0.838$$

16. $$\Pr\left(\text{condition}|\text{positive}\right) = \frac{\Pr(\text{condition})\times\Pr\left(\text{positive}|\text{condition}\right)}{\Pr(\text{condition})\times\Pr\left(\text{positive}|\text{condition}\right)+\Pr(\text{no condition})\times\Pr\left(\text{positive}|\text{no condition}\right)}$$
$$= \frac{0.05\times0.9}{(0.05\times 0.9)+(0.95\times\frac{11}{190})} = \frac{0.045}{0.045+0.055} = 0.45 = 45\%$$

17. $\Pr(\text{Male}|\text{Clinton}) = \dfrac{\Pr(\text{Male})\times\Pr(\text{Clinton}|\text{Male})}{\Pr(\text{Male})\times\Pr(\text{Clinton}|\text{Male})+\Pr(\text{Female})\times\Pr(\text{Clinton}|\text{Female})}$

$$= \frac{0.43\times 0.57}{(0.43\times 0.57)+(0.57\times 0.70)} = \frac{0.2451}{0.2451+0.399} = \frac{43}{113} \approx 0.3805$$

18. $\Pr(\geq \$50{,}000|\text{Repub.}) = \dfrac{\Pr(\geq \$50,000)\times\Pr(\text{Repub.}|\geq \$50,000)}{\Pr(\geq \$50,000)\times\Pr(\text{Repub.}|\geq \$50,000)+\Pr(< \$50,000)\times\Pr(\text{Repub.}|< \$50,000)}$

$$= \frac{0.64\times 0.55}{(0.64\times 0.55)+(0.36\times 0.43)} = \frac{0.352}{0.352+0.1548} \approx 0.6946$$

19. a. $\Pr(\text{one is}) = \dfrac{13}{52} = \dfrac{1}{4}$

b. $\Pr(\text{none is}|\text{random one isn't})$

$$= \frac{\Pr(\text{none is})\times(\text{random one isn't}|\text{none is})}{\Pr(\text{none is})\times\Pr(\text{random one isn't}|\text{none is})+\Pr(\text{one is})\times\Pr(\text{random one isn't}|\text{one is})}$$

$$= \frac{\frac{3}{4}\times 1}{\left(\frac{3}{4}\times 1\right)+\left(\frac{1}{4}\times\frac{12}{13}\right)} = \frac{13}{17} \approx 0.765$$

c. Pr(one is|10 randoms aren't)

$$= \frac{\Pr(\text{one is})\times\Pr(\text{10 randoms aren't}|\text{one is})}{\Pr(\text{one is})\times\Pr(\text{10 randoms aren't}|\text{one is})+\Pr(\text{none is})\times\Pr(\text{10 randoms aren't}|\text{none is})}$$

$$= \frac{\frac{1}{4}\times\left(\frac{12}{13}\right)^{10}}{\left[\frac{1}{4}\times\left(\frac{12}{13}\right)^{10}\right]+\left(\frac{3}{4}\times 1\right)}$$

$\approx .130$

20. $\Pr(\text{section I}|\text{math major}) = \dfrac{\#\text{ of section I math majors}}{\#\text{ of math majors}}$

$$= \frac{5}{16}$$

21. a. $\Pr(\text{Lakeside}|\text{winner})$

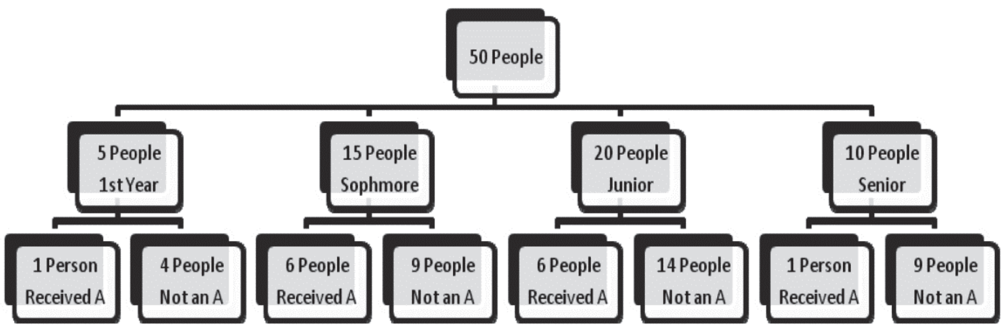
$$= \frac{\Pr(\text{Lakeside})\times\Pr(\text{winner}|\text{Lakeside})}{\Pr(\text{Lakeside})\times\Pr(\text{winner}|\text{Lakeside})+\Pr(\text{Pylesville})\times\Pr(\text{winner}|\text{Pylesville})+\Pr(\text{Millerville})\times\Pr(\text{winner}|\text{Millerville})}$$

$$= \frac{0.40\times 0.05}{(0.40\times 0.05)+(0.20\times 0.02)+(0.40\times 0.03)} = \frac{5}{9}$$

b. $\dfrac{0.20\times0.02}{(0.40\times 0.05)\times(0.20\times0.02)+(0.40\times0.03)}=\dfrac{1}{9}\approx 11\%$

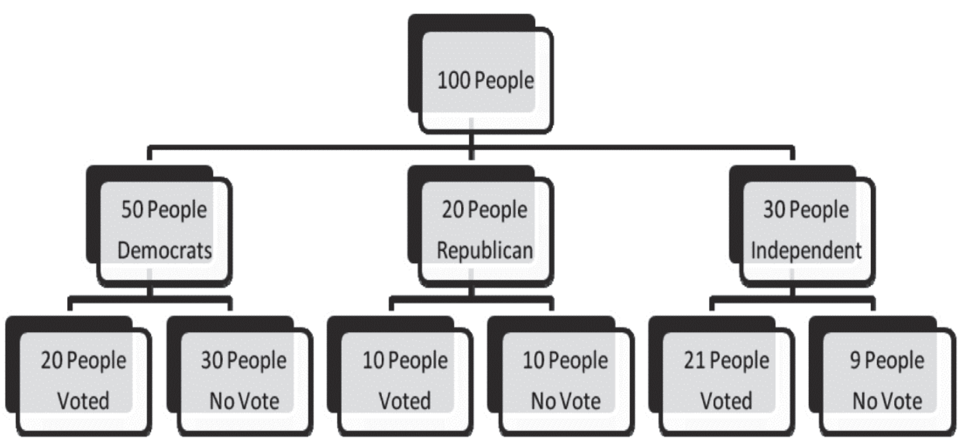
22. $$\text{Pr(Apex|def.)} = \frac{\Pr(\text{Apex})\times\Pr\left(\text{def.}\middle|\text{Apex}\right)}{\Pr(\text{Apex})\times\Pr\left(\text{def.}\middle|\text{Apex}\right)+\Pr(\text{B-ink})\times\Pr\left(\text{def.}\middle|\text{B-ink}\right)}$$
$$=\frac{0.70\times 0.10}{(0.70\times 0.10)+(0.30\times0.05)}$$
$$=\frac{14}{17}$$
$$\approx 0.824$$

23.



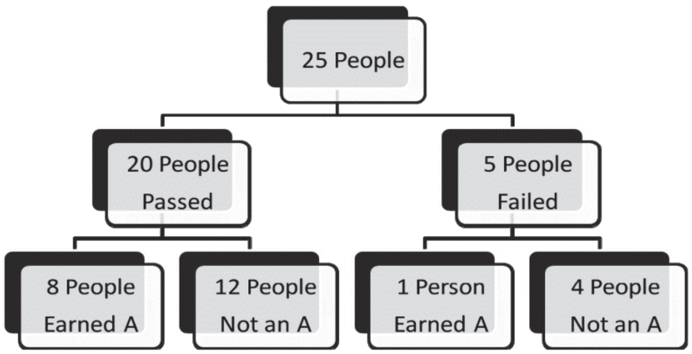
$$\text{Pr(Sophmore | A)}=\frac{6}{1+6+6+1}=\frac{6}{14}=\frac{3}{7}$$

24.



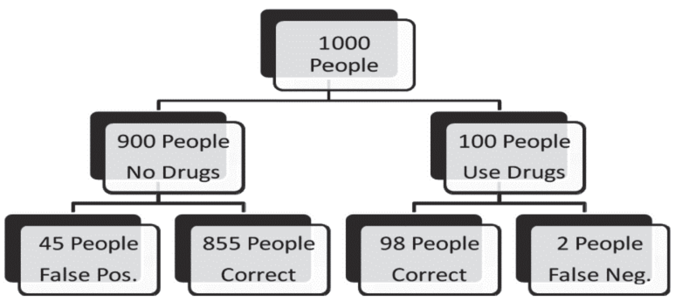
$$\text{Pr(Independent | Voted)}=\frac{21}{21+10+21}=\frac{21}{51}=\frac{7}{17}$$

25.



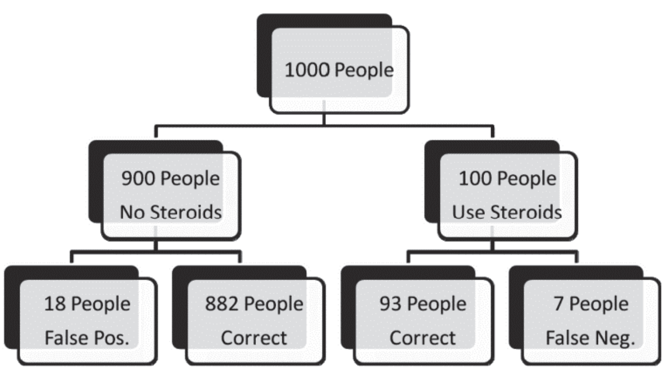
$$\Pr(\text{Passed} \mid \text{A}) = \frac{8}{8+1} = \frac{8}{9}$$

26.



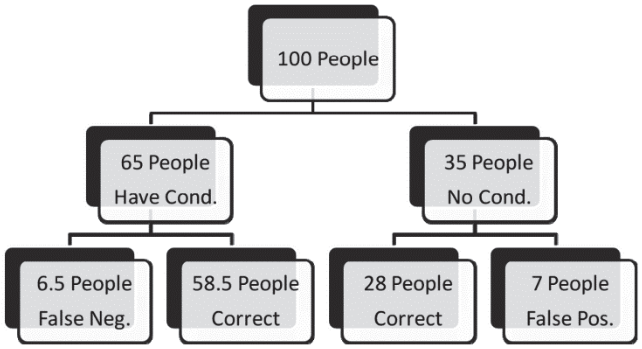
$$\Pr(\text{Used} \mid \text{Positive}) = \frac{98}{98+45} = \frac{98}{143} \approx 0.685$$

27.

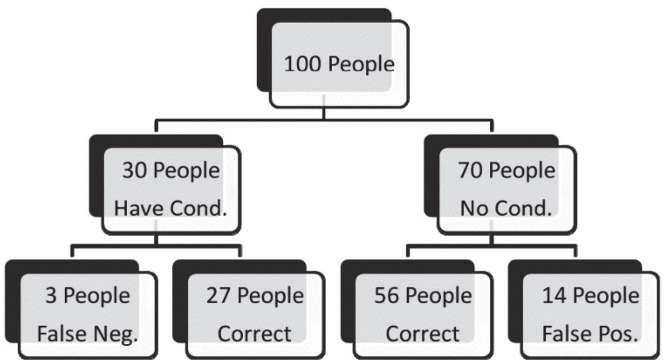


$$\Pr(\text{Used} \mid \text{Positive}) = \frac{93}{93+18} = \frac{93}{111} = \frac{31}{37} \approx 0.838$$

28.

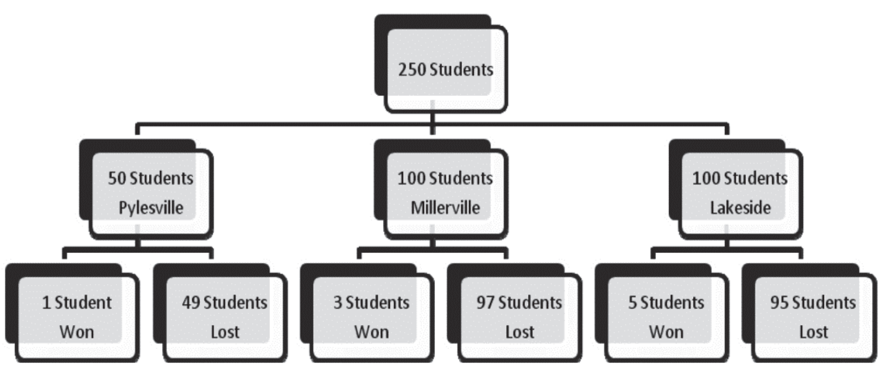


$$\Pr(\text{Condition} \mid \text{Positive}) = \frac{58.5}{58.5+7} = \frac{58.5}{65.5} \approx 0.893$$



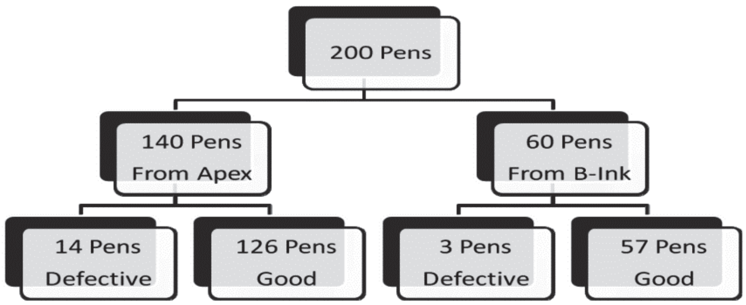
$$\Pr(\text{Condition} \mid \text{Positive}) = \frac{27}{27+14} = \frac{27}{41} \approx 0.659$$

29.



$$\Pr(\text{Lakeside} \mid \text{Winner}) = \frac{5}{1+3+5} = \frac{5}{9}$$

30.



$\Pr(\text{Apex} \mid \text{Defective}) = \dfrac{14}{14+3} = \dfrac{14}{17} \approx 0.824$

Exercises 6.7

1. Use seq(randInt(1,6),X,1,36,1)→ L_1.

   Theoretical probabilities: $\dfrac{1}{6}$ for each fall.

2. To simulate the sum of the faces of a pair of dice, use seq(randInt(1,6)+randInt(1,6),X,1,96,1)→L_1.

   Theoretical probabilities: $\Pr(2) = \dfrac{1}{36},\ \Pr(3) = \dfrac{2}{36} = \dfrac{1}{18},\ \Pr(4) = \dfrac{3}{36} = \dfrac{1}{12},\ \Pr(5) = \dfrac{4}{36} = \dfrac{1}{9},\ \Pr(6) = \dfrac{5}{36},$

   $\Pr(7) = \dfrac{6}{36} = \dfrac{1}{6},\ \Pr(8) = \dfrac{5}{36},\ \Pr(9) = \dfrac{4}{36} = \dfrac{1}{9},\ \Pr(10) = \dfrac{3}{36} = \dfrac{1}{12},\ \Pr(11) = \dfrac{2}{36} = \dfrac{1}{18},\ \Pr(12) = \dfrac{1}{36}$

   Repeat the sequence an additional 5 times. The theoretical probabilities will not change. Compare your answers to the above shown probabilities.

3. Use seq(randInt(1,1000),X,1,10,1)→L_1 where a number from 1–840 represents a successful freethrow and 841–1000 represents a miss.

4. Use seq(randInt(1,1000),X,1,10,1)→L_1 where 1–331 is hit and 332–1000 is not.

5. Use seq(randInt(1,4),X,1,10,1)→L_1, where a = 1, b = 2, c = 3, and d = 4.

6. Use randInt(1,30) and hit $\langle \text{ENTER} \rangle$ repeatedly, where red = 1 – 20 and green = 21 – 30.

   $\Pr(\text{no reds}) = \dfrac{10}{30}\cdot\dfrac{9}{29}\cdot\dfrac{8}{28}\cdot\dfrac{7}{27} = \dfrac{2}{261},\ \Pr(1\text{ red}) = 4\cdot\dfrac{20}{30}\cdot\dfrac{10}{29}\cdot\dfrac{9}{28}\cdot\dfrac{8}{27} = \dfrac{160}{1827},$

   $\Pr(2\text{ reds}) = 6\cdot\dfrac{20}{30}\cdot\dfrac{19}{29}\cdot\dfrac{10}{28}\cdot\dfrac{9}{27} = \dfrac{190}{609},\ \Pr(3\text{ reds}) = 4\cdot\dfrac{20}{30}\cdot\dfrac{19}{29}\cdot\dfrac{18}{28}\cdot\dfrac{10}{27} = \dfrac{760}{1827},$

   $\Pr(4\text{ reds}) = \dfrac{20}{30}\cdot\dfrac{19}{29}\cdot\dfrac{18}{28}\cdot\dfrac{17}{27} = \dfrac{323}{1827}$

7. Answers will vary (see Example 4).

8. Answers will vary.

9. Answers will vary.

10. Use seq(randInt(1,6)+randInt(1,6)+randInt(1,6),X,1,108,1)→L_1.

Chapter 6 Fundamental Concept Check

1. The set of all possible outcomes

2. $A \cup B$; $A \cap B$; A'

3. 0

4. The entire sample space.

5. $\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$

6. Two events are independent if the occurrence of one has no effect on the occurrence of the other. The two events might have outcomes in common. Two events are mutually exclusive if they have no outcomes in common.

7. The probability of an event is the sum of the probabilities of the outcomes in the event.

8. k to $n-k$

9. $\dfrac{a}{a+b}$

10. $\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$

11. $\dfrac{\Pr(E \cap F)}{\Pr(F)}$

12. Bayes' Theorem provides a way to calculate certain conditional probabilities. See page 285.

13. A tree diagram is a graphical device used to show all the possible outcomes of an experiment, and their probabilities, in a clear and uncomplicated manner. Each branch of the tree contains a probability.

Chapter 6 Review Exercises

1. **a.** The set of all possible pairs: {PN, PD, PQ, PH, ND, NQ, NH, DQ, DH, QH}

b. The set of pairs containing an even number of cents: {PN, PQ, NQ, DH}

2. a. A male junior is elected

b. A female junior is not elected

c. A male or a junior is elected

3. $\Pr(E \cap F) = 0.4 + 0.3 - 0.5 = 0.2$

4. $\Pr(E \cup F) = 0.5 + 0.3 = 0.8$

5. $\dfrac{120-(\text{speak Chinese or Spanish})-(\text{speak French only})}{120}=\dfrac{120-(30+50-12)-(75-30-15+7)}{120}=\dfrac{1}{8}$

6. There are 15 – 5 = 10 who like only bicycling and 20 – 5 = 15 who like only jogging, so the probability is $\dfrac{10+15}{50}=0.5.$

7. $\text{Pr(within 20 minutes)}=\dfrac{13}{13+12}=\dfrac{13}{25};\ \text{Pr(more than 20 minutes)}=1-\dfrac{13}{25}=\dfrac{12}{25}$

8. $\dfrac{1}{1+3708}=\dfrac{1}{3709}$

9. $26\%=\dfrac{13}{50};$ a = 13 and a + b = 50, so b = 37. The odds the person selected is under 18 are 13 to 37; the odds the person selected is 18 or older are 37 to 13.

10. $25\%=\dfrac{1}{4};$ a = 1 and a + b = 4, so b = 3. The odds are 1 to 3.

11. $\dfrac{5}{10}\times\dfrac{4}{9}\times\dfrac{3}{8}=\dfrac{1}{12}$

12. There are C(4, 2) = 6 ways to choose a "pair" of socks from the drawer; of these, 2 have the same color, so the probability is $\dfrac{2}{6}=\dfrac{1}{3}.$

13.
$$\begin{aligned}1-\frac{C(5,1)\cdot C(95,3)+C(95,4)}{C(100,4)}&=1-\frac{5\cdot 138,415+3,183,545}{3,921,225}\\&=1-\frac{3,875,620}{3,921,225}\\&\approx 0.01163\end{aligned}$$

14. $\dfrac{C(5,1)\cdot C(4,2)}{C(9,3)}=\dfrac{5\cdot 6}{84}=\dfrac{30}{84}=\dfrac{5}{14}\approx 0.3571$

15. a. $\text{Pr(prepared every question)}=\dfrac{C(8,6)}{C(10,6)}=\dfrac{28}{210}=\dfrac{2}{15}$

b. $\text{Pr(not prepared on test)}=\dfrac{C(8,4)}{C(10,6)}=\dfrac{70}{210}=\dfrac{1}{3}$

16. a.
$$\begin{aligned}\text{Pr(winning)}&=\text{Pr}(7)+\text{Pr}(11)\\&=\frac{6}{36}+\frac{2}{36}\\&=\frac{8}{36}=\frac{2}{9}\end{aligned}$$

$$\begin{aligned}\text{Pr(losing)} &= \text{Pr}(2) + \text{Pr}(3) + \text{Pr}(12)\\ &= \frac{1}{36} + \frac{2}{36} + \frac{1}{36}\\ &= \frac{4}{36} = \frac{1}{9}\end{aligned}$$

b. $\text{Pr(winning)} = \text{Pr}(6) = \frac{5}{36}$

$\text{Pr(losing)} = \text{Pr}(7) = \frac{6}{36} = \frac{1}{6}$

17. $1 - \left(\frac{1}{2}\right)^5 = \frac{31}{32}$

18. Pr(no tails) + Pr(1 tail each) + Pr(2 tails each) + Pr(3 tails each) $= \left(\frac{1}{8}\right)^2 + \left(\frac{3}{8}\right)^2 + \left(\frac{3}{8}\right)^2 + \left(\frac{1}{8}\right)^2 = \frac{5}{16}$

19. $\frac{2}{7} \times \frac{1}{6} = \frac{1}{21}$

20. $\text{Pr(select 4 winning teams)} = \left(\frac{1}{17}\right)^4 = \frac{1}{83{,}521}$

Odds against $= 1 - \frac{1}{83{,}521} : \frac{1}{83{,}521} = 83{,}520 : 1$

21. **a.** $\left(\frac{1}{36}\right)^3$

b. $\left(\frac{1}{10}\right)^4$

c. $\left(\frac{26}{36}\right)^3 \left(\frac{1}{2}\right)^4 = \left(\frac{17{,}576}{46{,}656}\right)\left(\frac{1}{16}\right) = \frac{2197}{93{,}312}$

22. **a.** Pr(all three cards are aces)

$= pr(A_1 \cap A_2 \cap A_3) = \frac{4}{52} \cdot \frac{4}{52} \cdot \frac{4}{52} = \frac{1}{2197}$

b. Pr (at least one ace) = 1 – Pr(no aces)

$= 1 - \left(\frac{48}{52}\right)^3 = 1 - \left(\frac{12}{13}\right)^3 = \frac{469}{2197}$

23. Record the six consecutive outcomes; there are 6^6 possibilities, of which 6! contain each number exactly once. Hence the probability is $\frac{6!}{6^6}=\frac{5}{324}\approx 0.0154.$

24. $\text{Pr(4 different numbers)}=\frac{6\cdot 5\cdot 4\cdot 3}{6\cdot 6\cdot 6\cdot 6}=\frac{5}{18},$ so the odds in favor of getting four different numbers are

$\frac{5}{18}:1-\frac{5}{18}=5$ to 13.

25. $1-\frac{365\cdot 364\cdot 363\cdot 362\cdot 361}{365^5}\approx 0.0271.$

26. $1-\frac{6\cdot 5\cdot 4}{7\cdot 7\cdot 7}=1-\frac{120}{343}=\frac{223}{343}\approx 0.6501$

27. $\Pr(E\mid F)=\frac{.4+.3-.5}{.3}=\frac{.2}{.3}=\frac{2}{3}$

28. $\Pr(F)=\frac{\frac{1}{10}}{\frac{1}{7}}=\frac{7}{10}$

29. Pr(at least one tail appears in three coins given that at least one head appeared)
1) Look at the sample space: HHH, HHT, HTH, HTT, THH, THT, TTH, TTT.
2) The first seven outcomes have at least one head.
3) Pr (one or more tails in the first seven outcomes) = 6/7

30. $\Pr\left(\text{one } 3\middle|\text{no doubles}\right)=\frac{5+5}{30}=\frac{1}{3}$

31. **a.** $\text{Pr(employed)=}\frac{147.48}{154.81}\approx 0.9527$

b. $\text{Pr(Male)=}\frac{80.74}{154.81}\approx 0.5215$

c. $\text{Pr(Female|employed)=}\frac{70.70}{147.48}\approx 0.4794$

d. $\text{Pr(employed|Female)=}\frac{70.70}{74.07}\approx 0.9545$

32. **a.** $\text{Pr(eng)=}\frac{15}{50}=\frac{3}{10}$

b. $\text{Pr(Eng|public)=}\frac{10}{25}=\frac{2}{5}$

c. $\text{Pr(Private|eng)=}\frac{5}{15}=\frac{1}{3}$

d. $\Pr(\text{Public}|\text{eng}) = \frac{10}{15} = \frac{2}{3}$

33. $(0.08)(0.5) = 0.04$

34. $\Pr(R_1 \cap G_2 \cap G_3 \cap R_4) = \frac{10}{30} \cdot \frac{20}{29} \cdot \frac{19}{28} \cdot \frac{9}{27}$

$$= \frac{95}{1827} \approx 0.0520$$

35. No; $\Pr\left(F \middle| E\right) = \frac{1}{6} > \Pr(F) = \frac{5}{36}$

36. No; $\Pr(F|E) = \frac{3}{4} \neq \Pr(F) = \frac{1}{2}$

37. Yes

38. Yes

39. a. $\frac{1}{4} \times \frac{1}{3} = \frac{1}{12}$

b. $\frac{1}{4} + \frac{1}{3} - \frac{1}{12} = \frac{1}{2}$

40. a. $(0.4)(0.75) = 0.3$

b. $0.4 + 0.75 - 0.3 = 0.85$

41. $\Pr(A \cup B) = \frac{1}{2}$; $\Pr(A' \cap B) = \frac{1}{3}$

$\Pr(A) + \Pr(A' \cap B) = \Pr(A \cup B)$

$\Pr(A) = \frac{1}{2} - \frac{1}{3} = \frac{1}{6}$

42. $\Pr(A \text{ and } B) = (0.4)(0.3) = 0.12$

$\Pr(A \text{ only}) = 0.3 - 0.12 = 0.18$

$\Pr(B \text{ only}) = 0.4 - 0.12 = 0.28$

$\Pr(\text{exactly } 1) = 0.18 + 0.28 = 0.46$

43. $\left(\frac{1}{3} \times \frac{2}{3}\right) + \left(\frac{2}{3} \times \frac{1}{3}\right) = \frac{4}{9}$

44. $\Pr(3 \text{ is drawn on 1st or 2nd draw}) = \frac{1}{3} + \left(\frac{1}{3}\right)\left(\frac{1}{2}\right) + \left(\frac{1}{3}\right)\left(\frac{1}{2}\right) = \frac{2}{3}$

45. You should switch. If you stay with your original choice your probability of winning remains $\frac{1}{3}$, whereas if you switch you lose only if your original choice was correct, so your probability of winning is $\frac{2}{3}$.

46. $(0.60 \times 0.90) + (0.40 \times 0.05) = 0.56 = 56\%$

47.
$$\begin{aligned}&\Pr\left(\text{both parents left-handed} \middle| \text{child left-handed}\right)\\ &= \frac{\Pr(\text{all three left-handed})}{\Pr(\text{child left-handed})}\\ &= \frac{0.4 \times 0.25 \times 0.25}{(0.4 \times 0.25 \times 0.25) + (0.2 \times 0.25 \times 0.75) + (0.2 \times 0.75 \times 0.25) + (0.1 \times 0.75 \times 0.75)}\\ &= \frac{4}{25}\end{aligned}$$

48.
$$\begin{aligned}\Pr\left(\text{correct} \middle| \text{rejected}\right) &= \frac{\Pr(\text{correct}) \times \Pr\left(\text{rejected} \middle| \text{correct}\right)}{\Pr(\text{correct}) \times \Pr\left(\text{rejected} \middle| \text{correct}\right) + \Pr(\text{incorrect}) \times \Pr\left(\text{rejected} \middle| \text{incorrect}\right)}\\ &= \frac{0.80 \times 0.05}{(0.80 \times 0.05) + (0.20 \times 0.90)} = \frac{2}{11}\end{aligned}$$

49.
$$\begin{aligned}\Pr\left(C \middle| \text{wrong}\right) &= \frac{\Pr(C) \times \Pr\left(\text{wrong} \middle| C\right)}{\Pr(C) \times \Pr\left(\text{wrong} \middle| C\right) + \Pr(A) \times \Pr\left(\text{wrong} \middle| A\right) + \Pr(B) \times \Pr\left(\text{wrong} \middle| B\right)}\\ &= \frac{0.20 \times 0.05}{(0.20 \times 0.05) + (0.40 \times 0.02) + (0.40 \times 0.03)}\\ &= \frac{0.01}{0.03}\\ &= \frac{1}{3}\end{aligned}$$

50. If n is the number of dragons, then $\frac{\text{\# of heads on 1-headed dragons}}{\text{\# of heads}} = \frac{\frac{n}{3}}{\frac{n}{3} + 2 \cdot \frac{n}{3} + 3 \cdot \frac{n}{3}} = \frac{1}{6}$

51. If E and F are independent events, then the outcome of F does not affect the outcome of E and vice versa. If the outcome of F does not affect the outcome of E, then neither would the outcome of F'.
Example: Let E = event you get a six on a die and F = event you get a H on a coin toss. Now, E and F are independent. F' is the event you get a T on a coin toss. The events E and F' are independent, so whether you get a H or T on the coin toss does not affect the probability of the six on a die.

52. If you know $\Pr(E \cup F)$, then you can compute
$\Pr(E \cap F) = \Pr(E) + \Pr(F) - \Pr(E \cup F)$. Alternatively, if you know that E and F are independent events, then you can compute
$\Pr(E \cap F) = \Pr(E) \cdot \Pr(F)$.

53. If two events are independent, then $\Pr(E \cap F) = \Pr(E) \cdot \Pr(F)$. Since the condition also states that they have nonzero probabilities, the right side of the equation has a nonzero value. Therefore, the probability of the intersection is nonzero, and events *E* and *F* must have at least one outcome in come, thus they are not mutually exclusive.

54. Suppose *A* and *B* are two mutually exclusive events with nonzero probabilities. Then $\Pr(A \cap B) = 0$ and $\Pr(A)\Pr(B) \neq 0$, so *A* and *B* are not independent.

55. True; The formula for a conditional probability is $\Pr(E \mid F) = \dfrac{\Pr(E \cap F)}{\Pr(F)}$.

56. No; because Pr(E) + Pr (F) > 1.

Chapter 6 Project

1. Answers may vary.

2. Pr(reward if CBC)
$= 0.5 \times 0.4 + 0.4 \times 0.5 - 0.5 \times 0.4 \times 0.5 = 0.3$

3. Pr(reward if BCB)
$= 0.4 \times 0.5 + 0.5 \times 0.4 - 0.4 \times 0.5 \times 0.4 = 0.32$

4. BCB

5. Answers may vary. *Sample answer*:
Alice has to win the second game, and she is better off playing Carol in that game than playing Betty.

6. $\Pr(\text{male}) = \dfrac{560+40}{1000} = 0.6$

$\Pr(\text{female}) = \dfrac{340+160}{1000} = 0.5$

7. Males

8. $\Pr(\text{male accepted to law school}) = \dfrac{560}{700} = 0.8$

$\Pr(\text{female accepted to law school}) = \dfrac{340}{400} = 0.85$

9. Pr(male accepted to medical school)
$= \dfrac{40}{300} = \dfrac{2}{15} \approx 0.13$
Pr(female accepted to medical school)
$= \dfrac{160}{600} = \dfrac{4}{15} \approx 0.27$

10. Females

11. Answers may vary. *Sample answer*:
The reason for apparent contradiction is that more women applied to medical school, the program with the tougher admissions requirements, than to law school. For the men, the opposite was true.

