Chapter 5

Exercises 5.1

1. **a.** $S' = \{5, 6, 7\}$

b. $S \cup T = \{1, 2, 3, 4, 5, 7\}$

c. $S \cap T = \{1, 3\}$

d. $S' \cap T = \{5, 7\}$

2. **a.** $S' = \{4, 5\}$

b. $S \cup T = \{1, 2, 3, 5\}$

c. $S \cap T = \varnothing$

d. $S' \cap T = \{5\}$

3. **a.** $R \cup S = \{a, b, c, \text{e, i, o, u}\}$

b. $R \cap S = \{\text{a}\}$

c. $S \cap T = \varnothing$

d. $S' \cap R = \{b, c\}$

4. **a.** $R \cup S = \{a, b\}$

b. $R \cap S = \{a\}$

c. $T' = \{a, c\}$

d. $T' \cup S = \{a, b, c\}$

5. $\varnothing$, {1}, {2}, {1, 2}

6. $\varnothing$, {1}, {2}, {3}, {4}, {1,2}, {1,3}, {1,4}, {2,3}, {2,4}, {3,4}, {1,2,3}, {1,3,4}, {1,2,4}, {2,3,4}, {1,2,3,4}

7. **a.** $F \cap B =$ {all freshman college students who like basketball}

b. $B' =$ {all college students who do not like basketball}

c. $F' \cap B' =$ {all college students who are neither freshman nor like basketball}

d. $F \cup B =$ {all college students who are either freshman or like basketball}

8. **a.** $S' =$ {all corporations with headquarters not in New York City}

b. $T' =$ {all publicly owned corporations}

c. $S \cap T =$ {all privately owned corporations with headquarters in New York City}

d. $S \cap T' =$ {all publicly owned corporations with headquarters in New York City}

9. **a.** $S =$ {1999, 2003, 2006, 2010, 2013}

b. $T =$ {1996, 1997, 1998, 1999, 2003, 2009, 2013}

c. $S \cap T =$ {1999, 2003, 2013}

d. $S \cup T =$ {1996, 1997, 1998, 1999, 2003, 2006, 2009, 2010, 2013}

e. $S' \cap T =$ {1996, 1997, 1998, 2009}

f. $S \cap T' =$ { 2006, 2010}

10. **a.** $A =$ {1998, 2000, 2001, 2005, 2007, 2008, 2014}

b. $B =$ {2000, 2001, 2002, 2008, 2015}

c. $A \cap B =$ { 2000, 2001, 2008}

d. $A' \cap B =$ {2002, 2015}

e. $A \cap B' =$ {1998, 2005, 2007, 2014}

11. From 1996 to 2015, there were only two years in which the Standard and Poor's Index increased by 2% or more during the first five days and not increase by 16% or more for the entire year.

12. From 1996 to 2015, there were only two years in which the Standard and Poor's Index did not decline during the first five days but declined for the entire year.

13. **a.** $R \cup S = \{a, b, c, d\}$
$(R \cup S)' = \{e, f\}$

b. $R \cup S \cup T = \{a, b, c, d, e, f\}$

c. $R \cap S = \{a, b\}$
$R \cap S \cap T = (R \cap S) \cap T = \varnothing$

d. $T' = \{a, b, c, d\}$
$R \cap S \cap T' = (R \cap S) \cap T' = \{a, b\}$

e. $R' = \{d, e, f\}$; $S \cap T = \varnothing$
$R' \cap S \cap T = R' \cap (S \cap T) = \varnothing$

f. $S \cup T = \{a, b, d, e, f\}$

g. $R \cup S = \{a, b, c, d\}$;
$R \cup T = \{a, b, c, e, f\}$
$(R \cup S) \cap (R \cup T) = \{a, b, c\}$

h. $R \cap S = \{a, b\}$; $R \cap T = \varnothing$
$(R \cap S) \cup (R \cap T) = \{a, b\}$

i. $R' = \{d, e, f\}; T' = \{a, b, c, d\}$
$R' \cap T' = \{d\}$

14. **a.** $R \cap S = \{3, 5\}$
$R \cap S \cap T = (R \cap S) \cap T = \varnothing$

b. $T' = \{1, 3, 5\}$
$R \cap S \cap T' = (R \cap S) \cap T' = \{3, 5\}$

c. $S' = \{1, 2\}; R \cap S' = \{1\}$
$R \cap S' \cap T = (R \cap S') \cap T = \varnothing$

d. $R' = T = \{2, 4\}$
$R' \cap T = T = \{2, 4\}$

e. $R \cup S = \{1, 3, 4, 5\}$

f. $R' \cup R = U = \{1, 2, 3, 4, 5\}$

g. $S \cap T = \{4\}$
$(S \cap T)' = \{1, 2, 3, 5\}$

h. $S' = \{1, 2\}; T' = \{1, 3, 5\}$
$S' \cup T' = \{1, 2, 3, 5\}$

15. $(S')' = S$

16. $S \cap S' = \varnothing$

17. $S \cup S' = U$

18. $S \cap \varnothing = \varnothing$

19. $T \cap S \cap T' = S \cap (T \cap T') = S \cap \varnothing = \varnothing$

20. $S \cup \varnothing = S$

21. {divisions that had increases in labor costs or total revenue} = $L \cup T$

22. {divisions that did not make a profit} = P'

23. {divisions that made a profit despite an increase in labor costs} = $L \cap P$

24. {divisions that had an increase in labor costs and were either unprofitable or did not increase their total revenue}
$= L \cap (P' \cup T')$

25. {profitable divisions with increases in labor costs and total revenue} = $P \cap L \cap T$

26. {divisions that were unprofitable or did not have increases in either labor costs or total revenue}
$= P' \cup (L \cup T)'$

27. {applicants who have not received speeding tickets} = S'

28. {applicants who have caused accidents and been arrested for drunk driving} = $A \cap D$

29. {applicants who have received speeding tickets, caused accidents, or were arrested for drunk driving}
$= S \cup A \cup D$

30. {applicants who have not been arrested for drunk driving but have received speeding tickets or have caused accidents}
$= D' \cap (S \cup A)$

31. {applicants who have not both caused accidents and received speeding tickets but who have been arrested for drunk driving}
$= (A \cap S)' \cap D$

32. {applicants who have not caused accidents or have not been arrested for drunk driving} = $A' \cup D'$

33. $A \cap D$ = {students at Mount College who are younger than 35}

34. $B \cap C$ = {teachers at Mount College who are older than 35}

35. $A \cap B$ = {people who are both student and teachers at Mount College}

36. $B \cup C$ = {people at Mount College who are teachers or older than 35}

37. $A \cup C' = A \cup D$ = {people at Mount College who are students or are at most 35}

38. $(A \cap D)'$ = {people at Mount College who are not students younger than 35}

39. $D' = C$ = {people at Mount College who are at least 35}

40. $D \cap U = D$ = {people at Mount College who are younger than 35}

41. {people who don't like vanilla ice cream} = V'

42. {people who like vanilla but not chocolate ice cream} $= V \cap C'$

43. {people who like vanilla but not chocolate or strawberry ice cream} = $V \cap (C \cup S)'$

44. {people who don't like any flavor of the three flavors of the ice cream} $= (S \cup V \cup C)'$

45. {people who like neither chocolate nor vanilla ice cream} = $(V \cup C)'$

46. {people who like only strawberry and chocolate ice cream} = $S \cap C \cap V'$

47. **a.** $R = \{B, C, D, E\}$

b. $S = \{C, D, E, F\}$

c. $T = \{A, D, E, F\}$

d. $R' = \{A, F\}$
$R' \cup S = \{A, C, D, E, F\}$

e. $R' \cap T = \{A, F\}$

f. $R \cap S = \{C, D, E\}$
$R \cap S \cap T = (R \cap S) \cap T = \{D, E\}$

48. **a.** $R = \{C, D, E\}$

b. $S = \{A, B, C, D\}$

c. $T = \{A, B, C, F\}$

d. $S' = \{E, F\}$
$R \cap S' = \{ E\}$

e. $R' = \{A, B, F\}$

$R' \cup T = \{A, B, C, F\}$

f. $R' \cap S' = \{F\}$
$R' \cap S' \cap T' = (R' \cap S') \cap T' = \varnothing$

49. There are eight different ways. They are no toppings; peppers; onions; mushrooms; peppers and onions; peppers and mushrooms; onions and mushrooms; all three toppings.

50. There are 16 different ways. They are no toppings; butter; cheese; chives; bacon; butter and cheese; butter and chives; butter and bacon; cheese and chives; cheese and bacon; chives and bacon; butter, cheese, and chives; butter, cheese, and bacon; butter, chives, and bacon; cheese, chives, and bacon; all four toppings.

51. Any subset of T with 2 as an element is an example. Possible answer: {2}

52. If *T* is a subset of *S*, then $S \cap T = T$.

53. If *S* is a subset of *T*, then $S \cup T = T$.

54. There are many possible subsets. Possible answer: $R = \{1\}$, $S = \{1, 2\}$, $T = \{2, 3\}$. Then $R \cup (S \cap T) = \{1, 2\}$ and $(R \cup S) \cap T = \{2\}$.

55. True; 5 is an element of the set {3, 5, 7}.

56. True; {1, 3} is a subset of the set {1, 2, 3}.

57. True; {b} is a subset of the set {b, c}.

58. False; 0 is not an element of the set {1, 2, 3}.

59. False; 0 is not an element of the empty set ∅.

60. True; the empty set ∅ is a subset of any set.

61. True; any set is a subset of itself.

62. False; 1 *is* an element of the set {1}.

Exercises 5.2

1. $n(S\cup T)=n(S)+n(T)-n(S\cap T)$
$=4+4-2=6$

2. $n(S\cup T)=n(S)+n(T)-n(S\cap T)$
$=17+13-0$
$=30$

3. $n(S\cup T)=n(S)+n(T)-n(S\cap T)$
$15 = 6 + 9 - n(S\cap T)$
$n(S\cap T)=6+9-15=0$

4. $n(S\cup T)=n(S)+n(T)-n(S\cap T)$
$15=4+12-n(S\cap T)$
$n(S\cap T)=4+12-15=1$

5. $n(S\cup T)=n(S)+n(T)-n(S\cap T)$
$10 = n(S) + 7 - 5$
$n(S) = 10 - 7 + 5 = 8$

6. $n(S\cup T)=n(S)+n(T)-n(S\cap T)$
$14 = 14 + n(T) - 6$
$n(T) = 14 - 14 + 6 = 6$

7. *S* is a subset of *T*.

8. *S* is a subset of *T*.

9. Let *P* = {adults in South America fluent in Portuguese} and
S = {adults in South America fluent in Spanish}.
Then $P\cup S$ = {adults in South America fluent in Portuguese or Spanish} and
$P\cap S$ = {adults in South America fluent in Portuguese and Spanish}.
$n(P) = 170$, $n(S) = 155$,
$n(P\cup S) = 314$ (numbers in millions)
$n(P\cup S)=n(P)+n(S)-n(P\cap S)$
$314 = 170 + 155 - n(P\cap S)$
$n(P\cap S) = 170 + 155 - 314 = 11$
11 million are fluent in both languages.

10. Let *M* = {fist-year students enrolled in a math course} and *E* = {first-year students enrolled in an English course}.
Then $M\cup E$ = {all first-year students} and $M\cap E$ = {first-year students enrolled in both math and English}.
$n(E)=600, n(M\cap E)=400, n(M\cup E)=1000$
$n(M\cup E)=n(M)+n(E)-n(M\cap E)$
$1000=n(M)+600-400$
$n(M) = 1000 - 600 + 400 = 800$
800 are taking a math course.

11. Let *U* = {all letters of the alphabet},
let *V* = {letters with vertical symmetry}, and
H = {letters with horizontal symmetry}.
Then $V\cup H=$ {letters with vertical or horizontal symmetry} and
$V\cap H=$ {letters with both vertical and horizontal symmetry}.
$n(V) = 11$, $n(H) = 9$, $n(V\cap H)=4$
$n(V\cup H)=n(V)+n(H)-n(V\cap H)$
$n(V\cup H)=11+9-4=16$
$n((V\cup H)')=n(U)-n(V\cup H)=26-16=10$
There are 10 letters with no symmetry.

12. Let *V* = {subscribers to streaming video service} and *M* = {subscribers to streaming music service}.
Then $V\cup M$ = {subscribers to streaming video or music} and
$V\cap M$ = {subscribers to both streaming video and music}.
$n(V)=250, n(M)=75, n(V\cap M)=25$
$n(V\cup M)=n(V)+n(M)-n(V\cap M)$
$=250+75-25$
$=300$
300 subscribe to at least one of these streaming services.

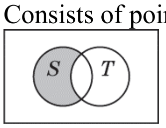
13. Let *A* = {cars with a navigation system} and *P* = {cars with push-button start}.
Then $A\cup P$ = {cars with a navigation system or push-button start} and
$A\cap P$ = {cars with both a navigation system and push-button start},
$n(A) = 325$, $n(P) = 216$, $n(A\cap P)=89$

$$n(A \cup P) = n(A) + n(P) - n(A \cap P)$$
$$= 325 + 216 - 89$$
$$= 452$$

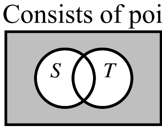
452 cars were manufactured with at least one of the two options.

14. Let S = {investors in stocks} and B = {investors in bonds}.
Then $S \cup B$ = {investors in either stocks or bonds} and $S \cap B$ = {investors in both stocks and bonds}.
$n(S) = 90, n(B) = 70, n(S \cup B) = 120$
$n(S \cup B) = n(S) + n(B) - n(S \cap B)$
$120 = 90 + 70 - n(S \cap B)$
$n(S \cap B) = 90 + 70 - 120 = 40$
40 investors owned both.

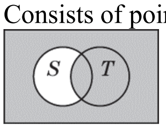
15. Consists of points not in T but in S.



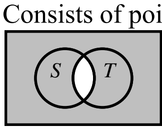
16. Consists of points not in S and not in T.



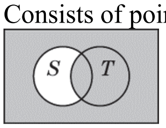
17. Consists of points in T or not in S.



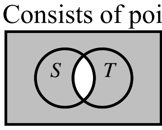
18. Consists of points not in S or not in T.



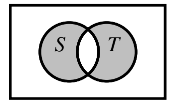
19. $(S \cap T')' = S' \cup T$
Consists of points in T or not in S.



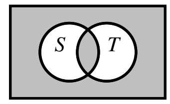
20. $(S \cap T)' = S' \cup T'$
Consists of points not in S or not in T.



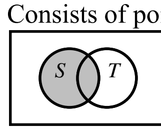
21. Consists of points in S but not in T or points in T but not in S.



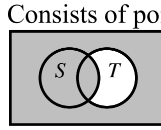
22. Consists of points in both S and T or points in neither S nor T.



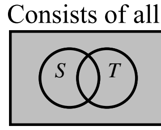
23. $S \cup (S \cap T) = S$
Consists of points in S.



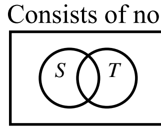
24. $S \cup (T' \cup S) = S \cup T'$
Consists of points in S or not in T.



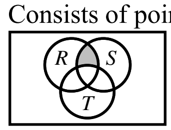
25. $S \cup S' = U$
Consists of all points.



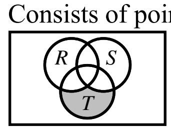
26. $S \cap S' = \varnothing$
Consists of no points.



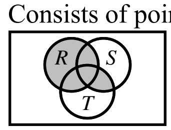
27. Consists of points in R and S but not in T.



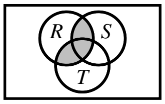
28. Consists of points in T but not in R and not in S.



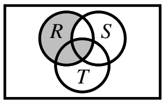
29. Consists of points in R or points in both S and T.



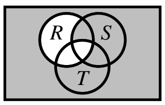
30. Consists of points in both R and S or points in both R and T.



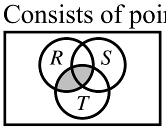
31. Consists of points in R but not in S or points in both R and T.



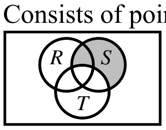
32. Consists of points not in R or points in S but not in T.



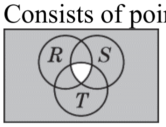
33. Consists of points in both R and T.



34. Consists of points in S but not in T.

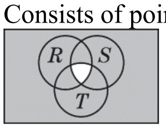


35. Consists of points not in R, S, and T.

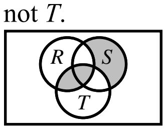


36. $(R \cap S \cap T)' = R' \cup S' \cup T'$

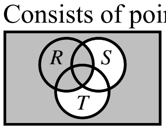
Consists of points not in R, S, and T.



37. Consists of points in R and T or points in S but not T.



38. Consists of points in R or points not in S or T.



39. $S' \cup (S \cap T)' = S' \cup S' \cup T' = S' \cup T'$

40. $S \cap (S \cup T)' = S \cap S' \cap T' = (S \cap S') \cap T' = \varnothing$

41. $(S' \cup T)' = S \cap T'$

42. $(S' \cap T')' = S \cup T$

43.
$$\begin{aligned} T \cup (S \cap T)' &= T \cup S' \cup T' \\ &= (T \cup T') \cup S' \\ &= U \end{aligned}$$

44. $(S' \cap T)' \cup S = S \cup T' \cup S = S \cup T'$

45. S'

46. $(S \cap T') \cup (S' \cap T)$

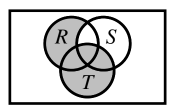
47. $R \cap T$

48. $R' \cap S \cap T'$

49. $R' \cap S \cap T$

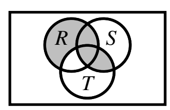
50. $R \cap (T \cup S')$

51.



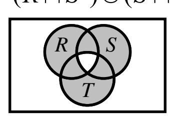
$T \cup (R \cap S')$

52.



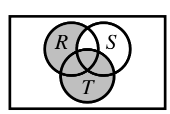
$(R \cap T') \cup (S \cap T)$

53. First draw a Venn diagram for $(R \cap S') \cup (S \cap T') \cup (T \cap R')$.



The set consists of the complement.
$(R \cap S \cap T) \cup (R' \cap S' \cap T')$

54.



$R \cup (S \cap T')$

55. Everyone who is not a citizen or is both over the age of 18 and employed

56. People between the ages of 5 and 18 who are citizens and employed

57. Everyone over the age of 18 who is unemployed

58. All citizens who are over the age of 18 or employed

59. Noncitizens who are 5 years of age or older

60. No one (everyone who is under the age of 5 and employed).

Exercises 5.3

1. $5 + 6 = 11$

2. $15 + 20 = 35$

3. $6 + 5 + 15 + 20 = 46$

4. $14 + 19 + 10 + 11 = 54$

5. 11

6. 19

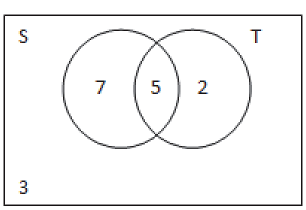
7. $19 + 10 + 5 + 6 + 15 + 20 = 75$

8. $10 + 5 + 6 = 21$

9. $10 + 5 + 15 = 30$

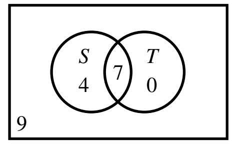
10. $19 + 10 + 5 + 6 + 11 + 15 + 20 = 86$

11. $n(S \cap T') = n(S) - n(S \cap T) = 12 - 5 = 7$
$n(S' \cap T) = n(T) - n(S \cap T) = 7 - 5 = 2$
$n(S \cup T) = n(S) + n(T) - n(S \cap T)$
$= 7 + 2 + 5 = 14$
$n(S' \cap T') = n(U) - n(S \cup T) = 17 - 14 = 3$

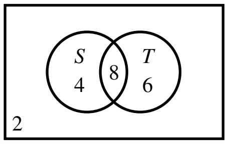


12. $n(S \cap T') = n(S) - n(S \cap T) = 11 - 7 = 4$
$n(S' \cap T) = n(T) - n(S \cap T) = 7 - 7 = 0$
$n(S \cup T) = n(S) + n(T) - n(S \cap T) = 11 + 7 - 7 = 11$

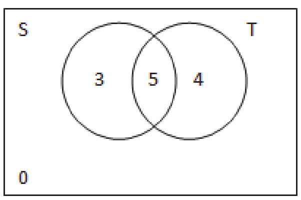
$n(S' \cap T') = n(U) - n(S \cup T) = 20 - 11 = 9$



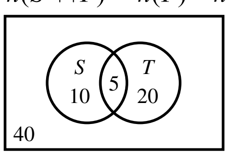
13. $n(S \cap T) = n(S) + n(T) - n(S \cup T)$
$= 12 + 14 - 18 = 8$
$n(S \cap T') = n(S) - n(S \cap T) = 12 - 8 = 4$
$n(S' \cap T) = n(T) - n(S \cap T) = 14 - 8 = 6$
$n(S' \cap T') = n(U) - n(S \cup T) = 20 - 18 = 2$



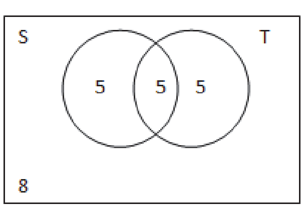
14. $n(S) = n(S \cap T) + n(S \cup T) - n(T) = 5 + 12 - 9 = 8$
$n(U) = n(S) + n(S') = 8 + 4 = 12$
$n(S \cap T') = n(S) - n(S \cap T) = 8 - 5 = 3$
$n(S' \cap T) = n(T) - n(S \cap T) = 9 - 5 = 4$
$n(S' \cap T') = n(U) - n(S \cup T) = 12 - 12 = 0$



15. $n(S \cup T) = n(U) - n(S' \cap T') = 75 - 40 = 35$
$n(S \cap T) = n(S) + n(T) - n(S \cup T) = 15 + 25 - 35 = 5$
$n(S \cap T') = n(S) - n(S \cap T) = 15 - 5 = 10$
$n(S' \cap T) = n(T) - n(S \cap T) = 25 - 5 = 20$



16. $n(U) = n(S) + n(S') = 10 + 13 = 23$
$n(S \cup T) = n(S) + n(T) - n(S \cap T)$
$= 10 + 10 - 5$
$= 15$
$n(S \cap T') = n(S) - n(S \cap T) = 10 - 5 = 5$
$n(S' \cap T) = n(T) - n(S \cap T) = 10 - 5 = 5$
$n(S' \cap T') = n(U) - n(S \cup T) = 23 - 15 = 8$



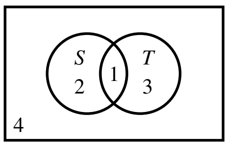
17. $n(S \cap T) = n(S) + n(T) - n(S \cup T) = 3 + 4 - 6 = 1$

$n(U) = n(S' \cup T') + n(S \cap T) = 9 + 1 = 10$

$n(S \cap T') = n(S) - n(S \cap T) = 3 - 1 = 2$

$n(S' \cap T) = n(T) - n(S \cap T) = 4 - 1 = 3$

$n(S' \cap T') = n(U) - n(S \cup T) = 10 - 6 = 4$

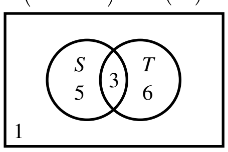


18. $n(S \cap T) = n(S) + n(T) - n(S \cup T) = 8 + 9 - 14 = 3$

$n(S \cap T') = n(S) - n(S \cap T) = 8 - 3 = 5$

$n(T \cap S') = n(T) - n(S \cap T) = 9 - 3 = 6$

$n(S' \cap T') = n(U) - n(S \cup T) = 15 - 14 = 1$



19. $n(R \cap S \cap T') = n(R \cap S) - n(R \cap S \cap T) = 5 - 2 = 3$

$n(R \cap S' \cap T) = n(R \cap T) - n(R \cap S \cap T) = 7 - 2 = 5$

$n(R' \cap S \cap T) = n(S \cap T) - n(R \cap S \cap T) = 3 - 2 = 1$

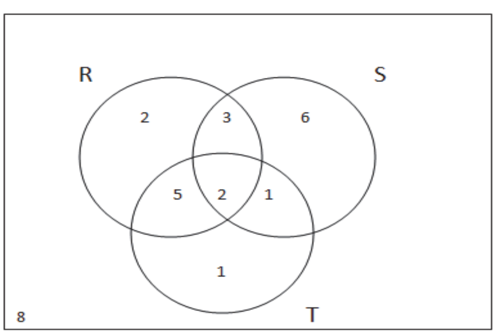
$n(R \cap S' \cap T') = n(R) - n(R \cap S \cap T') - n(R \cap S' \cap T) - n(R \cap S \cap T) = 12 - 3 - 5 - 2 = 2$

$n(R' \cap S \cap T') = n(S) - n(R \cap S \cap T') - n(R' \cap S \cap T) - n(R \cap S \cap T) = 12 - 3 - 1 - 2 = 6$

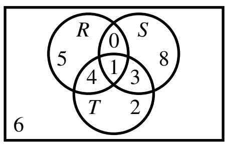
$n(R' \cap S' \cap T) = n(T) - n(R \cap S' \cap T) - n(R' \cap S \cap T) - n(R \cap S \cap T) = 9 - 5 - 1 - 2 = 1$

$n(R \cup S \cup T) = 3 + 5 + 1 + 2 + 6 + 1 + 2 = 20$

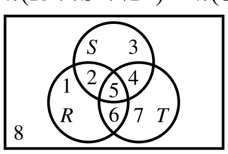
$n(R' \cap S' \cap T') = n(U) - n(R \cup S \cup T) = 28 - 20 = 8$



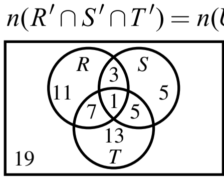
20. $n(R \cap S \cap T') = n(R \cap S) - n(R \cap S \cap T) = 1 - 1 = 0$
$n(R \cap S' \cap T) = n(R \cap T) - n(R \cap S \cap T) = 5 - 1 = 4$
$n(R' \cap S \cap T) = n(S \cap T) - n(R \cap S \cap T) = 4 - 1 = 3$
$n(R \cap S' \cap T') = n(R) - n(R \cap S \cap T') - n(R \cap S' \cap T) - n(R \cap S \cap T) = 10 - 0 - 4 - 1 = 5$
$n(R' \cap S \cap T') = n(S) - n(R \cap S \cap T') - n(R' \cap S \cap T) - n(R \cap S \cap T) = 12 - 0 - 3 - 1 = 8$
$n(R' \cap S' \cap T) = n(T) - n(R \cap S' \cap T) - n(R' \cap S \cap T) = n(R \cap S \cap T) = 10 - 4 - 3 - 1 = 2$
$n(R \cup S \cup T) = 5 + 8 + 2 + 0 + 4 + 3 + 1 = 23$
$n(R' \cap S' \cap T') = n(U) - n(R \cup S \cup T) = 29 - 23 = 6$



21. $n(R) = n(R \cup S) + n(R \cap S) - n(S) = 21 + 7 - 14 = 14$
$n(U) = n(R) + n(R') = 22 + 14 = 36$
$n(R \cap S \cap T') = n(R \cap S) - n(R \cap S \cap T) = 7 - 5 = 2$
$n(R \cap S' \cap T) = n(R \cap T) - n(R \cap S \cap T) = 11 - 5 = 6$
$n(R' \cap S \cap T) = n(S \cap T) - n(R \cap S \cap T) = 9 - 5 = 4$
$n(R \cap S' \cap T') = n(R) - n(R \cap S \cap T') - n(R \cap S' \cap T) - n(R \cap S \cap T) = 14 - 2 - 6 - 5 = 1$
$n(R' \cap S \cap T') = n(S) - n(R \cap S \cap T') - n(R' \cap S \cap T) - n(R \cap S \cap T) = 14 - 2 - 4 - 5 = 3$
$n(R' \cap S' \cap T) = n(T) - n(R \cap S' \cap T) - n(R' \cap S \cap T) - n(R \cap S \cap T) = 22 - 6 - 4 - 5 = 7$
$n(R \cup S \cup T) = 1 + 3 + 7 + 2 + 6 + 4 + 5 = 28$
$n(R' \cap S' \cap T') = n(U) - n(R \cup S \cup T) = 36 - 28 = 8$



22. $n(R \cup T) = n(R) + n(T) - n(R \cap T) = 22 + 26 - 8 = 40$
$n((R \cup T) \cap S) = n((R \cap S) \cup (T \cap S)) = n(R \cap S) + n(T \cap S) - n(R \cap S \cap T) = 4 + 6 - 1 = 9$
$n(S) = n(R \cup S \cup T) + n((R \cup T) \cap S) - n(R \cup T) = 45 + 9 - 40 = 14$
$n(R \cap S \cap T') = n(R \cap S) - n(R \cap S \cap T) = 4 - 1 = 3$
$n(R \cap S' \cap T) = n(R \cap T) - n(R \cap S \cap T) = 8 - 1 = 7$
$n(R' \cap S \cap T) = n(S \cap T) - n(R \cap S \cap T) = 6 - 1 = 5$
$n(R \cap S' \cap T') = n(R) - n(R \cap S \cap T') - n(R \cap S' \cap T) - n(R \cap S \cap T) = 22 - 3 - 7 - 1 = 11$
$n(R' \cap S \cap T') = n(S) - n(R \cap S \cap T') - n(R' \cap S \cap T) - n(R \cap S \cap T) = 14 - 3 - 5 - 1 = 5$
$n(R' \cap S' \cap T) = n(T) - n(R \cap S' \cap T) - n(R' \cap S \cap T) - n(R \cap S \cap T) = 26 - 7 - 5 - 1 = 13$
$n(R' \cap S' \cap T') = n(U) - n(R \cup S \cup T) = 64 - 45 = 19$

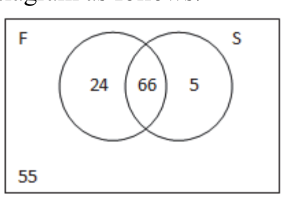


23. Let U = {high school students surveyed}, R = {students who like rock music}, and H = {students who like hip-hop music}.
$n(U) = 70$; $n(R) = 35$; $n(H) = 15$; $n(R \cap H) = 5$
$n(R \cup H) = n(R) + n(H) - n(R \cap H) = 35 + 15 - 5 = 45$
$n((R \cup H)') = n(U) - n(R \cup H) = 70 - 45 = 25$
25 students do not like either rock or hip-hop music.

24. Let U = {Nobel Prizes awarded}, L = {Nobel Prizes in literature}, and S = {Nobel Prizes awarded to Scandinavians}.
$n(U) = 900$; $n(L) = 112$; $n(S) = 57$; $n(L \cap S) = 14$
The number of Literature or Scandinavian winners = no. literature + no. Scandinavians – no. Literature and Scandinavians = 112 + 57 – 14 = 155. Since total number of Nobel prizes is 900, then the number of non-literature Nobel prizes awarded to non-Scandinavians is 900 – 155 = 745.

25. Let U = {lines}, V = {lines with verbs}, A = {lines with adjectives}
$n(U) = 14$; $n(V) = 11$; $n(A) = 9$;
$n(V \cap A) = 7$
$n(V \cap A') = n(V) - n(V \cap A) = 11 - 7 = 4$
Four lines have a verb with no adjective.
$n(V' \cap A) = n(A) - n(V \cap A) = 9 - 7 = 2$
Two lines have an adjective but no verb.
$n(V \cup A) = n(V) + n(A) - n(V \cap A) = 11 + 9 - 7 = 13$
$n((V \cup A)') = n(U) - n(V \cap A) = 14 - 13 = 1$
One line has neither an adjective nor a verb.

For Exercises 26–30, let U = {students who took the exam}, F = {students who correctly answered the first question}, S = {students who correctly answered the second question}. Then $n(U) = 150$, $n(F) = 90$, $n(S) = 71$, $n(F \cap S) = 66$. Draw and complete the Venn diagram as follows.



26. $n(F \cup S) = 24 + 66 + 5 = 95$

27. $n((F \cup S)') = 55$

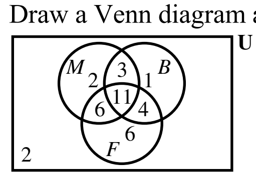
28. $n((F \cap S') \cup (F' \cap S)) = 24 + 5 = 29$

29. $n(S \cap F') = 5$

30. $n(S') = 55 + 24 = 79$

31. Let U = {students in finite math}, M = {male students}, B = {students who are business majors}, and F = {first-year students}.
$n(U) = 35$; $n(M) = 22$; $n(B) = 19$; $n(F) = 27$;
$n(M \cap B) = 14$; $n(M \cap F) = 17$;
$n(B \cap F) = 15$; $n(M \cap B \cap F) = 11$

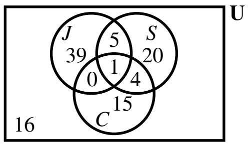
a. Draw a Venn diagram as shown.



b. $n(F' \cap M' \cap B') = 2$
There are two upperclass women nonbusiness majors.

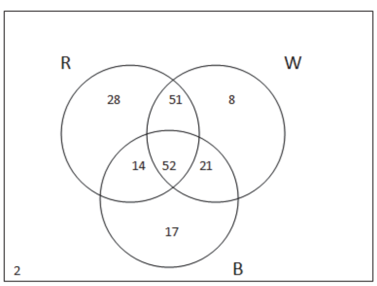
c. $n(M' \cap B) = 1 + 4 = 5$
There are five women business majors.

32. Let U = {college faculty}, J = {faculty who jog}, S = {faculty who swim}, and C = {faculty who cycle}.
$n(U) = 100$; $n(J) = 45$; $n(S) = 30$; $n(C) = 20$; $n(J \cap S) = 6$; $n(J \cap C) = 1$; $n(S \cap C) = 5$; $n(J \cap S \cap C) = 1$
Draw a Venn diagram as shown.



$n((J \cup S \cup C)') = 16$
16 members do not do any of these three activities.
$n(J \cap S' \cap C') = 39$
39 members just jog.

For Exercises 33–38, let U = {UN members}, R = {members with red in their flag}, W = {members with white in their flag}, B = {members with blue in their flag}.
Then $n(U) = 193$, $n(R) = 145$, $n(W) = 132$, $n(B) = 104$, $n(R \cap W) = 103$, $n(R \cap B) = 66$, $n(W \cap B) = 73$, $n(R \cap W \cap B) = 52$. Draw and complete the Venn diagram as follows.



33. $n(R \cap (W \cup B)') = 28$

34. $n((R \cap W' \cap B') \cup (R' \cap W \cap B') \cup (R' \cap W' \cap B))$
$= 28 + 8 + 17$
$= 53$

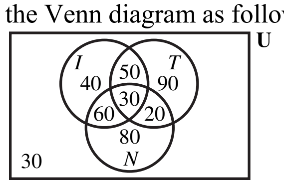
35. $n((R \cup W \cup B)') = 2$

36. $n((R \cap W \cap B') \cup (R' \cap W \cap B) \cup (R \cap W' \cap B))$
$= 51 + 21 + 14$
$= 86$

37. $n(R \cap W \cap B') = 51$

38. $n((R \cup W) \cap (R \cap W)') = 28 + 14 + 8 + 21$
$= 71$

For Exercises 39–44, let U = {people surveyed}, I = {people who learned from the Internet}, T = {people who learned from television}, N = {people who learned from newspapers}.
Then $n(U) = 400$, $n(I) = 180$, $n(T) = 190$, $n(N) = 190$, $n(I \cap T) = 80$, $n(I \cap N) = 90$, $n(T \cap N) = 50$, $n(I \cap T \cap N) = 30$. Draw and complete the Venn diagram as follows.



39. $n((I \cap N') \cup (I' \cap N)) = 40 + 50 + 80 + 20$
$= 190$

40. $n(I' \cap T' \cap N) = 80$

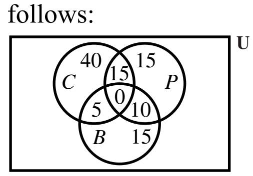
41. $n((I \cup T) \cap N') = 40 + 90 + 50 = 180$

42. $n((I \cap T) \cup (I \cap N) \cup (T \cap N))$
$= 50 + 60 + 20 + 30$
$= 160$

43. $n((I \cap T' \cap N') \cup (I' \cap T \cap N') \cup (I' \cap T' \cap N))$
$= 40 + 90 + 80$
$= 210$

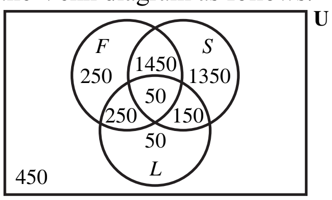
44. $n(I \cap T \cap N') = 50$

45. Draw and complete the Venn diagram as follows:



$40 + 15 + 15 + 15 + 5 + 10 + 0 = 100$

For Exercises 46–50, $n(U) = 4000$, $n(F) = 2000$, $n(S) = 3000$, $n(L) = 500$, $n(F \cap S) = 1500, n(F \cap L) = 300,$ $n(S \cap L) = 200, n(F \cap S \cap L) = 50.$ Draw and complete the Venn diagram as follows.



46. $L \cap (F \cup S) = 250 + 150 + 50 = 450$

47. $(L \cup F \cup S)' = 450$

48. $L' = 4000 - 500 = 3500$

49. $L \cup S \cup F' = 4000 - 250 = 3750$

50. $F \cap S' \cap L' = 250$

51. Let U = {college students surveyed}, F = {first-year students}, D = {voted Democratic}, $n(U) = 100$; $n(F) = 50$; $n(D) = 55$

$n(F' \cap D') = n((F \cup D)') = 25$

$n(F \cup D) = n(U) - n((F \cup D)') = 100 - 25 = 75$

$$\begin{aligned} n(F \cap D) &= n(F) + n(D) - n(F \cup D) \\ &= 50 + 55 - 75 \\ &= 30 \end{aligned}$$

30 freshmen voted Democratic.

52. Let U = {workers}, G = {college graduates}, M = {union members}.

$n(U) = 100$; $n(G') = 60; n(G \cap M') = 20;$ $n(M) = 30$

$n(G \cup M) = n(G \cap M') + n(M) = 20 + 30 = 50$

$$\begin{aligned} n(G' \cap M') &= n((G \cup M)') \\ &= n(U) - n((G \cup M)) \\ &= 100 - 50 \\ &= 50 \end{aligned}$$

50 workers were neither college graduates nor union members.

53. Let U = {students}, D = {students who passed the diagnostic test}, C = {students who passed the course}.

$n(U) = 30$; $n(D) = 21$; $n(C) = 23$; $n(D \cap C') = 2$

$n(D \cap C) = n(D) - n(D \cap C') = 21 - 2 = 19$

$n(D' \cap C) = n(C) - n(D \cap C) = 23 - 19 = 4$

Four students passed the course even though they failed the diagnostic test.

54. Let U = {applicants}, P = {pilots}, V = {veterans}.

$n(P) = 35$; $n(V) = 20$;

$n(P \cap V') = 30; n(P' \cap V') = 50$

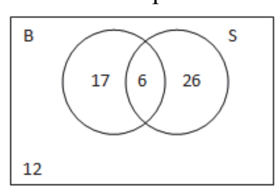
$n(P \cup V) = n(P \cap V') + n(V) = 30 + 20 = 50$

$$\begin{aligned} n(U) &= n(P \cup V) + n((P \cup V)') \\ &= n(P \cup V) + n(P' \cap V') \\ &= 50 + 50 \\ &= 100 \end{aligned}$$

There were 100 applicants.

For Exercises 55–60, let U = {students}, S = {seniors}, B = {biology majors}. Then $n(U) = 61$, $n(S \cap B) = 6$, $n(S' \cap B) = 17$, and $n(S' \cap B') = 12$. Therefore $n(S \cap B') = n(U) - n(B) =$ $n(U) - n(S \cap B) - n(S' \cap B) - n(S' \cap B') =$ $61 - 6 - 17 - 12 = 26$

Draw and complete the Venn diagram as follows.



55. 17 + 6 + 26 = 49

56. 6 + 26 = 32

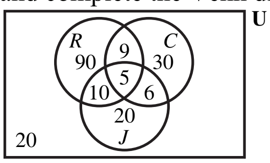
57. 17 +12 = 29

58. 17 + 6 = 23

59. 26

60. 26 + 12 = 38

For Exercises 61–68, let U = {surveyed students}, R = {students who like rock}, C = {students who like country}, J = {students who like rap}. Then $n(U) = 190$, $n(R) = 114$, $n(C) = 50$, $n(R \cap J) = 15$, $n(C \cap J) = 11$, $n(R' \cap C' \cap J) = 20$, $n(R \cap C' \cap J) = 10$, $n(R \cap C \cap J') = 9$ and $n((R \cup C \cup J)') = 20$. Draw and complete the Venn diagram as follows.



61. $n(R \cap C' \cap J') = 90$

62. $n(R' \cap C) = 30 + 6 = 36$

63. $n(R' \cap C \cap J) = 6$

64. $n((C \cup J) \cap R') = 30 + 20 + 6 = 56$

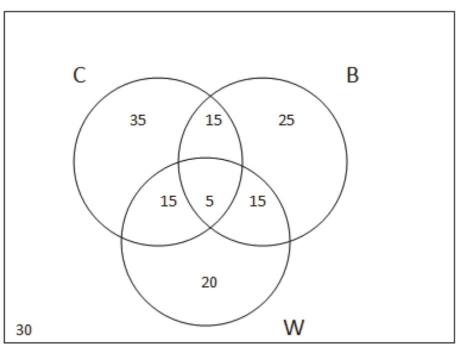
65. $n((R \cap C' \cap J') \cup (R' \cap C \cap J') \cup (R' \cap C' \cap J)) = 90 + 30 + 20 = 140$

66. $n(R \cap C \cap J) = 5$

67. $n((R \cap C) \cup (R \cap J) \cup (C \cap J)) = 9 + 10 + 6 + 5$
$= 30$

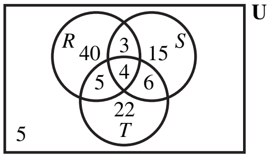
68. $n((R \cup C)') = 20 + 20 = 40$

69. Let U = {executives}, C = {executives who visited *CNN Money*}, B = {executives who visited *Bloomberg*}, W = {executives who visited *The Wall Street Journal*}.
$n(U) = 160$, $n(C) = 70$, $n(B) = 60$, $n(W) = 55$, $n(C \cap B) = 20$, $n(B \cap W) = 20$, $n(M \cap T \cap N) = 5$
Draw and complete the Venn diagram as follows.



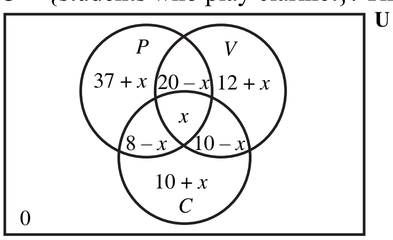
$n((C \cup B \cup W)') = 30$

70. Let U = {small businesses which had failed}, R = {undercapitalized}, S = {inexperienced management}, T = {poor location}.
$n(U) = 100$, $n(R \cup S \cup T) = 95$, $n(R \cap S \cap T) = 4$, $n(R \cap S' \cap T') = 40$, $n(R' \cap S \cap T') = 15$, $n(R \cap S) = 7$, $n(R \cap T) = 9$, $n(S \cap T) = 10$
Draw and complete the Venn diagram as follows.



$n(T) = 22 + 5 + 6 + 4 = 37$
37 businesses had poor location.
$n(R) = 40 + 3 + 5 + 4 = 52$
$n(S) = 15 + 3 + 6 + 4 = 28$
The most prevalent characteristic was being undercapitalized.

71. Let U = {students}, P = {students who play piano}, V = {students who play violin} and C = {students who play clarinet}. Then let $x = n(P \cap V \cap C)$ and complete the Venn diagram as follows.



$37 + x + 12 + x + 10 + x + (20 - x) + (8 - x) + (10 - x) + x = 97 + x = 100$, so $x = 3$.

72. Let M = {students taking mathematics}, H = {students taking history}. Then $n(M \cap H) = 0.1x$, $n((M \cap H)') = 0.2x$, $n(M \cap H') = 160$, $n(M' \cap H) = 120$, $n(U) = x$.

$x = 160 + 0.1x + 120 + 0.2x$ Therefore, $n(M) = 160 + 0.1(400)$
$x = 400$ $n(M) = 200$

Exercises 5.4

1. $4 \cdot 2 = 8$ routes

2. $3 \cdot 3 = 9$ routes

3. $3 \cdot 2 = 6$ routes

4. $4 \cdot 4 = 16$ routes

5. $44 \cdot 43 \cdot 42 = 79,464$ possibilities

6. $18 \cdot 17 \cdot 16 = 4896$ possibilities

7. $20 \cdot 19 \cdot 18 = 6840$ possibilities

8. $6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 720$ possibilities

9. 30 because $30 \cdot 29 = 870$

10. 25 because $25 \cdot 24 = 600$

11. **a.** $8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 40,320$ ways

b. $5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 3 \cdot 2 \cdot 1 = 720$ ways

12. **a.** $9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 362,880$ ways

b. $3 \cdot 2 \cdot 1 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 4320$ ways

13. $4 \cdot 3 \cdot 2 \cdot 1 = 24$ words

14. $26 \cdot 25 \cdot 25 = 16,250$ words

15. $2 \cdot 3 = 6$ outfits

16. $2 \cdot 4 \cdot 2 = 16$ outfits

17. $3 \cdot 12 \cdot 10 \cdot 10 \cdot 10 \cdot 10 = 360,000$ serial numbers

18. $9 \cdot 26 \cdot 26 \cdot 26 \cdot 9 \cdot 9 \cdot 9 = 115,316,136$ license plates

19. $10^9 - 1 = 999,999,999$ social security numbers

20. $1 \cdot 26 \cdot 26 + 1 \cdot 26 \cdot 26 \cdot 26 = 18,252$ call letters

21. $8 \cdot 2 \cdot 10 = 160$ area codes

22. $8 \cdot 10 \cdot 10 = 800$ area code

23. $9 \cdot 10 \cdot 10 \cdot 1 \cdot 1 = 900$ 5-digit palindromes

24. $9 \cdot 10 \cdot 10 \cdot 1 \cdot 1 \cdot 1 = 900$ 6-digit palindromes

25. $26 \cdot 26 \cdot 1 \cdot 1 = 676$ 4-letter palindromes

26. $26 \cdot 26 \cdot 1 = 676$ 3-letter palindromes

27. $15 \cdot 15 = 225$ matchups

28. $16 \cdot 16 = 256$ matchups

29. $3200 \cdot 2 \cdot 24 \cdot 52 = 7,987,200$ deals per year

30. $25 \cdot 25 = 625$ with repetition

$25 \cdot 24 = 600$ without repetition

31. $26 \cdot 26 \cdot 26 = 17,576$ sets of unique initials. Since there are 20,000 students, at least two students have the same set of initials

32. $26 \cdot 26 = 676$ sets of unique initials. Since there are 700 employees, at least two employees have the same set of initial.

33. $7 \cdot 5 = 35$ different possible halftime scores

34. If Gloria has 7 of each, she would have $7 \cdot 7 \cdot 7 = 343$ outfits (which is not enough). If Gloria would need 8 of each, so she would have $8 \cdot 8 \cdot 8 = 512$ outfits. Therefore, she has 8 of each.

35. $5 \cdot 4 = 20$ different mismatched sets

36. $11 \cdot 10 = 110$ different mismatched sets

37. $2^6 = 64$ possible sequences

38. $2^5 = 32$ possible sequences

39. $2^5 = 32$ possible ways

40. $3^5 = 243$ possible ways

41. $4^{10} = 1,048,576$ possible ways

42. $5^{10} = 9,765,625$ possible ways

43. $10^5 = 100,000$ possible zip codes

44. $10^4 = 10,000$ possible zip codes

45. $8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 40,320$ ways

$40320 \cdot 15 = 604,800$ seconds

$\frac{604800}{60} = 10,080$ minutes

$\frac{10080}{60} = 168$ hours

$\frac{168}{24} = 7$ days

46. $25 \cdot 25 \cdot 9 \cdot 9 \cdot 9 \cdot 25 \cdot 25 = 284,765,625$ ways

$\frac{284765625}{500000} = 569.53125$ weeks

$\frac{569.53125}{52} \approx 11$ years

47. $6 \cdot 7 \cdot 4 = 168$ days or 24 weeks

48. $7 \cdot 10 \cdot 4 = 280$ different meals

49. $5 \cdot 11 \cdot (7 \cdot 2 + 1) \cdot 10 = 8250$ different ways

50. $5 \cdot 5 \cdot 2 = 50$ possibilities

51. $2^4 = 16$ possible ways

52. $2^8 = 256$ possible ways

53. $2 \cdot 38 \cdot 38 = 2888$ different outcomes

54. $18 \cdot 17 \cdot 16 = 4896$ different outcomes

55. **a.** $9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 362,880$

b. $8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 1 = 40,320$

c. $1 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 1 \cdot 1 = 720$

56. $4 \cdot 3 \cdot 10 = 120$ different subjects are needed

57. $\frac{10 \cdot 9}{2} + 10 \cdot 10 = 145$ handshakes

58. $3 \cdot 2 \cdot 1 \cdot 3 \cdot 2 \cdot 1 = 36$ different ways

59. $4 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 = 972$ ways

60. $3 \cdot 20 = 60$ ways

61. $7 \cdot 4 \cdot 2^6 = 1792$ different ballots

$8 \cdot 5 \cdot 3^6 = 29,160$ different ballots

62. $3 \cdot 6 + 3 \cdot 7 = 39$ different segments.

63. $2^4 = 16$ possible ways

64. $20 \cdot 19 = 380$ different tickets

$\frac{380}{2} = 190$ different tickets

Exercises 5.5

1. $P(4, 2) = 4 \cdot 3 = 12$

2. $P(5, 1) = 5 = 5$

3. $P(6, 3) = 6 \cdot 5 \cdot 4 = 120$

4. $P(5, 4) = 5 \cdot 4 \cdot 3 \cdot 2 = 120$

5. $C(10, 3) = \frac{P(10, 3)}{3!} = \frac{10 \cdot 9 \cdot 8}{3 \cdot 2 \cdot 1} = 120$

6. $C(12, 2) = \frac{P(12, 2)}{2!} = \frac{12 \cdot 11}{2 \cdot 1} = 66$

7. $C(5, 4) = \frac{P(5, 4)}{4!} = \frac{5 \cdot 4 \cdot 3 \cdot 2}{4 \cdot 3 \cdot 2 \cdot 1} = 5$

8. $C(6, 3) = \frac{P(6, 3)}{3!} = \frac{6 \cdot 5 \cdot 4}{3 \cdot 2 \cdot 1} = 20$

9. $P(7, 1) = 7$

10. $P(5, 5) = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$

11. $P(n, 1) = n$

12. $P(n, 2) = n \cdot (n-1) = n^2 - n$

13. $C(4, 4) = \frac{P(4, 4)}{4!} = \frac{4 \cdot 3 \cdot 2 \cdot 1}{4 \cdot 3 \cdot 2 \cdot 1} = 1$

14. $C(n, 2) = \frac{P(n, 2)}{2!} = \frac{n \cdot (n-1)}{2 \cdot 1} = \frac{n(n-1)}{2}$

15.
$$\begin{aligned} C(n, n-2) &= \frac{P(n, n-2)}{(n-2)!} \\ &= \frac{n \cdot (n-1) \text{L } 4 \cdot 3}{(n-2) \cdot (n-3) \text{L } 2 \cdot 1} \\ &= \frac{n \cdot (n-1)}{2 \cdot 1} \\ &= \frac{n(n-1)}{2} \end{aligned}$$

16. $C(n, 1) = \frac{P(n, 1)}{1!} = \frac{n}{1} = n$

17. $6! = 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 720$

18.
$$\begin{aligned} \frac{10!}{4!} &= \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{4 \cdot 3 \cdot 2 \cdot 1} \\ &= 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \\ &= 151{,}200 \end{aligned}$$

19. $\frac{9!}{7!} = \frac{9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 9 \cdot 8 = 72$

20. $7! = 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 5040$

21. Permutation; order matters

22. Permutation; order matters

23. Combination; order does not matter

24. Permutation; order matters

25. Neither

26. Neither

27. $4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24$ ways

28. $6! = 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 720$ ways

29. $C(9,7) = \frac{P(9,7)}{7!} = \frac{9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3}{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 36$ selections

30. $C(5,3) = \frac{P(5,3)}{3!} = \frac{5 \cdot 4 \cdot 3}{3 \cdot 2 \cdot 1} = 10$ pizzas

31. $C(8,4) = \frac{P(8,4)}{4!} = \frac{8 \cdot 7 \cdot 6 \cdot 5}{4 \cdot 3 \cdot 2 \cdot 1} = 70$ ways

32.
$$\begin{aligned} C(100,5) &= \frac{P(100,5)}{5!} \\ &= \frac{100 \cdot 99 \cdot 98 \cdot 97 \cdot 96}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} \\ &= 75{,}287{,}520 \text{ ways} \end{aligned}$$

33.
$$\begin{aligned} P(65,5) &= 65 \cdot 64 \cdot 63 \cdot 62 \cdot 61 \\ &= 991{,}186{,}560 \text{ ways} \end{aligned}$$

34.
$$\begin{aligned} P(36,5) &= 36 \cdot 35 \cdot 34 \cdot 33 \cdot 32 \\ &= 45{,}239{,}040 \text{ ways} \end{aligned}$$

35. $C(10,5)=\dfrac{P(10,5)}{5!}=\dfrac{10\cdot 9\cdot 8\cdot 7\cdot 6}{5\cdot 4\cdot 3\cdot 2\cdot 1}=252$ ways

36. $C(8,4)=\dfrac{P(8,4)}{4!}=\dfrac{8\cdot 7\cdot 6\cdot 5}{4\cdot 3\cdot 2\cdot 1}=70$ ways

37. $C(100,3)=\dfrac{P(100,3)}{3!}=\dfrac{100\cdot 99\cdot 98}{3\cdot 2\cdot 1}=161,700$ possible samples

$C(7,3)=\dfrac{P(7,3)}{3!}=\dfrac{7\cdot 6\cdot 5}{3\cdot 2\cdot 1}=35$ defective samples

38. $C(17,10)=\dfrac{P(17,10)}{10!}=\dfrac{17\cdot 16\cdot 15\cdot 14\cdot 13\cdot 12\cdot 11\cdot 10\cdot 9\cdot 8}{10\cdot 9\cdot 8\cdot 7\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1}=19,448$ possibilities

39. $P(150,3)=150\cdot 149\cdot 148=3,307,800$ ways

40. $5!=5\cdot 4\cdot 3\cdot 2\cdot 1=120$ ways

41. $C(52,5)=\dfrac{P(52,5)}{5!}=\dfrac{52\cdot 51\cdot 50\cdot 49\cdot 48}{5\cdot 4\cdot 3\cdot 2\cdot 1}=2,598,960$ hands

42. $C(8,5)=\dfrac{P(8,5)}{5!}=\dfrac{8\cdot 7\cdot 6\cdot 5\cdot 4}{5\cdot 4\cdot 3\cdot 2\cdot 1}=56$ hands

43. $C(13,5)=\dfrac{P(13,5)}{5!}=\dfrac{13\cdot 12\cdot 11\cdot 10\cdot 9}{5\cdot 4\cdot 3\cdot 2\cdot 1}=1287$ hands

44. $C(26,5)=\dfrac{P(26,5)}{5!}=\dfrac{26\cdot 25\cdot 24\cdot 23\cdot 22}{5\cdot 4\cdot 3\cdot 2\cdot 1}=65,780$ hands

45. $5!=5\cdot 4\cdot 3\cdot 2\cdot 1=120$ ways

46. $P(6,3)=6\cdot 5\cdot 4=120$ signals

47. **a.** $C(10,4)=\dfrac{P(10,4)}{4!}=\dfrac{10\cdot 9\cdot 8\cdot 7}{4\cdot 3\cdot 2\cdot 1}=210$ ways

b. $C(10,6)=\dfrac{P(10,6)}{6!}=\dfrac{10\cdot 9\cdot 8\cdot 7\cdot 6\cdot 5}{6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1}=210$ ways

c. They are the same because taking four sweaters is the same as leaving 6 sweaters.

48. **a.** $C(12,3)\cdot C(9,4)=\dfrac{P(12,3)}{3!}\cdot\dfrac{P(9,4)}{4!}=\dfrac{12\cdot 11\cdot 10}{3\cdot 2\cdot 1}\cdot\dfrac{9\cdot 8\cdot 7\cdot 6}{4\cdot 3\cdot 2\cdot 1}=220\cdot 126=27,720$ ways

b. $C(12,4)\cdot C(8,3)=\dfrac{P(12,4)}{4!}\cdot\dfrac{P(8,3)}{3!}=\dfrac{12\cdot 11\cdot 10\cdot 9}{4\cdot 3\cdot 2\cdot 1}\cdot\dfrac{8\cdot 7\cdot 6}{3\cdot 2\cdot 1}=495\cdot 56=27,720$ ways

c. The order of giving the books does not matter.

49. $C(8, 2) = \frac{P(8, 2)}{2!} = \frac{8 \cdot 7}{2 \cdot 1} = 28$ games

50. $3C(6, 2) = \frac{3P(6, 2)}{2!} = \frac{3 \cdot 6 \cdot 5}{2 \cdot 1} = 45$ games

51. $26C(69, 5) = \frac{26P(69, 5)}{5!} = \frac{26 \cdot 69 \cdot 68 \cdot 67 \cdot 66 \cdot 65}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 292,201,338$ outcomes

52. $4! \cdot 5 \cdot 4! = 4 \cdot 3 \cdot 2 \cdot 1 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 2880$ batting orders

53. $\frac{C(59,6)}{C(49,6)} = \frac{45,057,474}{13,983,816} \approx 3.22$ Choice (b)

54. You buy (2)(110) = 220 tickets per week
$\frac{C(59,6)}{220} = \frac{45,057,474}{220} \approx 204,806.7$ weeks or 3938.6 years; choice (d)

55. Moe: $C(10, 3) = \frac{P(10, 3)}{3!} = \frac{10 \cdot 9 \cdot 8}{3 \cdot 2 \cdot 1} = 120$ choices

Joe: $C(7,4) = \frac{P(7,4)}{4!} = \frac{7 \cdot 6 \cdot 5 \cdot 4}{4 \cdot 3 \cdot 2 \cdot 1} = 35$ choices

Thus Joe is correct.

56. $4 \cdot 4 \cdot 4 \cdot C(9,3) = 4 \cdot 4 \cdot 4 \cdot \frac{P(9,3)}{3!} = 4 \cdot 4 \cdot 4 \cdot \frac{9 \cdot 8 \cdot 7}{3 \cdot 2 \cdot 1} = 64 \cdot 84 = 5376$ ways

57. $4! \cdot P(4,3) \cdot P(5,3) \cdot P(6,3) \cdot P(7,3)$
$24 \cdot 24 \cdot 60 \cdot 120 \cdot 210 = 870,912,000$ pictures

58. 5! 3! 3! 3! 3! 3!
$120 \cdot 6 \cdot 6 \cdot 6 \cdot 6 \cdot 6 = 933,120$ ways

59. 3! 3! 3! 3!
$6 \cdot 6 \cdot 6 \cdot 6 = 1296$ ways

60. 3! 4! 4! 4!
$6 \cdot 24 \cdot 24 \cdot 24 = 82,944$ ways

61. Through trial are error, you will find that 10 people were at the party.

62. Through trial are error, you will find that 11 teams are in the league.

63. $C(15,3) + (15 \cdot 14) + 15 = 455 + 210 + 15 = 680$ side dish options

64. $C(16,3) + (16 \cdot 15) + 16 = 560 + 240 + 16 = 816$ possibilities

65. $720 - 3! - 5!$
$720 - 6 - 120 = 594$

66. $5^5 - 5 \cdot 5 \cdot 3 \cdot 1 \cdot 1 = 3125 - 75 = 3050$

67. **a.** $C(45,5) = 1,221,759$ possible lottery tickets

b. $C(100,4) = 3,921,225$ possible lottery tickets

c. The first lottery has a better chance of winning

68. **a.** $\frac{C(48,9)}{C(52,13)} \approx 0.00264 = 0.264\%$

b. $\frac{C(44,9)}{C(52,13)} \approx 0.00112 = 0.112\%$

c. The hand with 4 aces is more likely.

69. Yes; the number of ways to shuffle a deck of cards is $52! \approx 8 \times 10^{67}$.

70. No; the number of ways to rearrange the letters of the alphabet is $26! \approx 4 \times 10^{26}$.

Exercises 5.6

1. a. $2^8 = 256$ outcomes

b. $C(8, 4) = \frac{8 \cdot 7 \cdot 6 \cdot 5}{4 \cdot 3 \cdot 2 \cdot 1} = 70$ outcomes

2. a. $2^9 = 512$ outcomes

b. $C(9,2) = \frac{9 \cdot 8}{2 \cdot 1} = 36$ outcomes

3. a. $C(7,5) + C(7,6) + C(7,7) =$
$21 + 7 + 1 = 29$ outcomes

b. $2^7 - 29 = 128 - 29 = 99$ outcomes

4. a. $C(6,0) + C(6,1) + C(6,2) + C(6,3)$
$1 + 6 + 15 + 20$
42 outcomes

b. $2^6 - 42 = 64 - 42 = 22$ outcomes

5. a. $2C(6,3) = \frac{2 \cdot 6 \cdot 5 \cdot 4}{3 \cdot 2 \cdot 1} = 40$ ways

b. $2C(5,3) = \frac{2 \cdot 5 \cdot 4 \cdot 3}{3 \cdot 2 \cdot 1} = 20$ ways

6. $2 + 2C(4,3) + 2C(5,3) + 2C(6,3) =$
$2 + 8 + 20 + 40 = 70$ ways

7. $C(11,5) \cdot C(6,5) \cdot C(1,1) =$
$462 \cdot 6 \cdot 1 = 2772$ ways

8. $C(15,10) \cdot C(5,2) \cdot C(3,3) =$
$3003 \cdot 10 \cdot 1 = 30,030$ ways

9. $C(8,5) = \frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 56$ ways

10. $C(6,3) = \frac{6 \cdot 5 \cdot 4}{3 \cdot 2 \cdot 1} = 20$ ways

11. $C(7,2) = \frac{7 \cdot 6}{2 \cdot 1} = 21$ ways

12. $C(9,4) = \frac{9 \cdot 8 \cdot 7 \cdot 6}{4 \cdot 3 \cdot 2 \cdot 1} = 126$ ways

13. $C(5,2) \cdot C(4,2) =$
$10 \cdot 6 = 60$ ways

14. $C(3,1) \cdot C(4,1) =$
$3 \cdot 4 = 12$ ways

15. $C(6,2) = \frac{6 \cdot 5}{2 \cdot 1} = 15$ ways

16. $C(6,4) = \frac{6 \cdot 5 \cdot 4 \cdot 3}{4 \cdot 3 \cdot 2 \cdot 1} = 15$ ways

17. c. The two points where the combinations stop are the two points that make up the intersection B. Therefore, the sum of these two points will be the single combination.

d. $C(8,3) + C(8,4) = C(9,4)$
$56 + 70 = 126$

18. Answer will vary

19. $C(8,3) = 56$ outcomes

20. $C(7,3) = 35$ arrangements

21. $C(10,6)-C(7,4)=$
$210-35=175$ ways

22. $C(7,3)=35$ ways

23. **a.** $C(12,5)=792$ samples

b. $C(7,5)=21$ samples

c. $C(7,2)\cdot C(5,3)=21\cdot 10=210$ samples

d. $C(7,4)\cdot C(5,1)+21=$
$35\cdot 5+21=196$ samples

24. **a.** $C(15,6)=5005$ ways

b. $C(9,6)=84$ ways

c. $C(6,2)\cdot C(9,4)=15\cdot 126=1890$ ways

d. $5005-(84+C(9,5)\cdot C(6,1))$
$5005-(84+126\cdot 6)$
$5005-(84+756)$
$5005-840$
4165 ways

25. **a.** $C(10,3)=120$ ways

b. $C(8,3)=56$ ways

c. $120-56=64$

26. **a.** $C(100,5)=75,287,520$ ways

b $C(10,2)\cdot C(90,3)=$
$45\cdot 117,480=5,286,600$ ways

c. $75,287,520-C(90,5)$
$75,287,520-43,949,268$
$31,338,252$ ways

27. $C(4,2)\cdot C(6,2)=6\cdot 15=90$ ways

28. $C(9,5)\cdot C(7,6)=126\cdot 7=882$ ways

29. $C(4,3)\cdot C(4,2)=4\cdot 6=24$ ways

30. $C(4,2)\cdot 12C(4,2)\cdot 44=6\cdot 12\cdot 6\cdot 44$
$=19,008$ ways

31. $13C(4,3)\cdot 12C(4,2)=13\cdot 4\cdot 12\cdot 6$
$=3744$ ways

32. $C(13,2)\cdot C(4,2)\cdot C(4,2)\cdot 44=78\cdot 6\cdot 6\cdot 44$
$=123,552$ ways

33. $C(7,5)\cdot 5!\ \cdot 21\cdot 20=21\cdot 120\cdot 21\cdot 20$
$=1,058,400$ ways

34. $C(9,5)\cdot P(21,4)=126\cdot 143,640$
$=18,098,640$ ways

35. $C(10,5)\cdot P(21,5)=252\cdot 2,441,880$
$=615,353,760$ ways

36. $C(8,5)\cdot 5!\ \cdot P(21,3)=56\cdot 120\cdot 7980$
$=53,625,600$ ways

37. $6!\ \cdot 7\cdot 3!=720\cdot 7\cdot 6$
$=30,240$ ways

38. $4!\ \cdot 5\cdot 2!=24\cdot 5\cdot 2$
$=240$ ways

39. $C(9,5)=126$ ways

40. $C(10,4)=210$ ways

41. $C(26,22)\cdot C(10,7)=14,950\cdot 120$
$=1,794,000$ ways

42. $C(26,24)\cdot C(10,6)=325\cdot 210$
$=68,250$ ways

43. $C(12,6)=924$ ways

44. $C(8,5)=56$ ways

45. $\dfrac{C(100,50)}{2^{100}}\approx 0.07959=7.96\%$

46. $\dfrac{C(200,100)}{2^{200}}\approx 0.05634=5.63\%$

47. $\frac{C(50,10)\cdot C(50,10)}{C(100,20)} \approx 0.1969 = 19.7\%$

48. $\frac{C(100,10)\cdot C(100,10)}{C(200,20)} \approx 0.1857 = 18.6\%$

Exercises 5.7

1. $\binom{18}{16} = C(18, 16) = C(18, 2) = \frac{18\cdot 17}{2\cdot 1} = 153$

2. $\binom{25}{24} = C(25, 24) = C(25, 1) = \frac{25}{1} = 25$

3. $\binom{6}{2} = C(6, 2) = \frac{6\cdot 5}{2\cdot 1} = 15$

4. $\binom{7}{3} = C(7, 3) = \frac{7\cdot 6\cdot 5}{3\cdot 2\cdot 1} = 35$

5. $\binom{8}{1} = C(8, 1) = \frac{8}{1} = 8$

6. $\binom{9}{9} = C(9, 9) = 1$

7. $\binom{7}{0} = C(7, 0) = 1$

8. $\binom{6}{1} = C(6, 1) = \frac{6}{1} = 6$

9. $\binom{8}{8} = C(8, 8) = 1$

10. $\binom{9}{0} = C(9, 0) = 1$

11. $\binom{n}{n-1} = C(n, n-1) = C(n, 1) = \frac{n}{1} = n$

12. $\binom{n}{n} = C(n, n) = 1$

13. $0! = 1$

14. $1! = 1$

15. $n \cdot (n - 1)! = n!$

16. $\frac{n!}{n} = \frac{n\cdot(n-1)!}{n} = (n-1)!$

17. $\binom{6}{0}+\binom{6}{1}+\binom{6}{2}+\binom{6}{3}+\binom{6}{4}+\binom{6}{5}+\binom{6}{6}$
$= 2^6$
$= 64$

18. $\binom{7}{0}+\binom{7}{1}+\binom{7}{2}+\binom{7}{3}+\binom{7}{4}+\binom{7}{5}+\binom{7}{6}+\binom{7}{7}$
$= 2^7$
$= 128$

19. The number of terms in a binomial expansion is the exponent plus 1, so there are 20 terms.

20. The number of terms in a binomial expansion is the exponent plus 1, so there are 26 terms.

21. $\binom{10}{0}x^{10} + \binom{10}{1}x^9 y + \binom{10}{2}x^8 y^2$
$= x^{10} + 10x^9 y + 45x^8 y^2$

22. $\binom{20}{0}x^{20} + \binom{20}{1}x^{19} y + \binom{20}{2}x^{18} y^2$
$= x^{20} + 20x^{19} y + 190x^{18} y^2$

23. $\binom{15}{13}x^2 y^{13} + \binom{15}{14}xy^{14} + \binom{15}{15}y^{15}$
$= 105x^2 y^{13} + 15xy^{14} + y^{15}$

24. $\binom{12}{10}x^2 y^{10} + \binom{12}{11}xy^{11} + \binom{12}{12}y^{12}$
$= 66x^2 y^{10} + 12xy^{11} + y^{12}$

25. $\binom{20}{10}x^{10}y^{10} = 184,756x^{10}y^{10}$

26. $\binom{10}{5}x^5y^5 = 252x^5y^5$

27. $\binom{4}{2} = C(4,2) = 6$

28. $\binom{6}{3}\cdot 2^3 = C(6,3)\cdot 8 = 20\cdot 8 = 160$

29. $\binom{11}{7} = 330$

30. $\binom{13}{4} = 715$

31. $\binom{9}{0}x^9 + \binom{9}{1}x^8(2y) + \binom{9}{2}x^7(2y)^2$
$= x^9 + 9x^8(2y) + 36x^7(4y^2)$
$= x^9 + 18x^8y + 144x^7y^2$

32. $\binom{8}{6}x^2(-y)^6 + \binom{8}{7}x(-y)^7 + \binom{8}{8}(-y)^8$
$= 28x^2y^6 + 8x(-y^7) + 1(y^8)$
$= 28x^2y^6 - 8xy^7 + y^8$

33. $\binom{12}{6}x^6(-3y)^6 = 924x^6(729y^6)$
$= 673,596x^6y^6$

34. $\binom{6}{2}x^4(3y)^2 = 15x^4(9y^2) = 135x^4y^2$

Therefore, the coefficient is 135.

35. $\binom{7}{3}x^4(-3y)^3 = 35x^4(-27y^3) = -945x^4y^3$

36. $\binom{8}{5}x^3(-3y)^5 = 56x^3(243y^5)$
$= -13,608x^3y^5$

37. $2^8 = 256$ subsets

38. $2^9 = 512$ subsets

39. $2^4 = 16$ tips

40. $2^4 = 16$ types

41. $2^5 = 32$ options (The number of subsets of any size taken from a set of five elements.)

42. For dressings, there are six choices, including no dressing.
There are $2^6 \cdot 6 = 384$ possible salads.

43. $2^8 - 1 = 255$ ways

44. $2^6 - 1 = 63$ ways

45. $2\cdot 3\cdot 2^{13} = 49,152$ types

46. There are 4 choices of ice cream and $2^5 - 1 = 31$ ways to choose a selection of toppings (since at least one topping must be selected), and hence 4 · 31 = 124 possible sundaes.

47. $2^7 - C(7,\,6) - C(7,\,7) = 128 - 7 - 1$
$= 120$ ways

48. $2^7 - C(7,\,0) - C(7,\,1) = 128 - 1 - 7$
$= 120$ ways

49. $2^8 - C(8,\,0) - C(8,\,1) = 256 - 1 - 8$
$= 247$ ways

50. $2^8 - C(8,7) - C(8,8) = 256 - 8 - 1$
$= 247$ ways

51. $\left(2^9 - C(9,1) - C(9,0)\right)\left(2^{10} - C(10,1) - C(10,0)\right)$
$= (512 - 9 - 1)(1024 - 10 - 1)$
$= (502)(1013)$
$= 508,526$ ways

52. $\left(2^5 - C(5,1) - C(5,0)\right)\left(2^6 - C(6,0)\right)$
$= (32 - 5 - 1)(64 - 1)$
$= (26)(63)$
$= 1638$ ways

53. No, the exponents on the variables add to 7, but the combination portion of the terms shows that they should add to 8.

54. Yes, the second term in the binomial could be the value 1.

55. $\binom{5}{0} + \binom{5}{2} + \binom{5}{4} = \binom{5}{1} + \binom{5}{3} + \binom{5}{5}$
$1 + 10 + 5 = 5 + 10 + 1$
$16 = 16$

56. **a.** $\binom{5}{0} + \binom{5}{2} + \binom{5}{4} = \binom{5}{5} + \binom{5}{3} + \binom{5}{1}$
by equation (4), so
$\binom{5}{0} - \binom{5}{1} + \binom{5}{2} - \binom{5}{3} + \binom{5}{4} - \binom{5}{5}$
$= 0.$

b. The same reasoning applies when *n* is odd:
$\binom{n}{0} + \binom{n}{2} + \binom{n}{4} + \text{L} = \binom{n}{1} + \binom{n}{3} + \binom{n}{5} + \text{L}$
so $\binom{n}{0} - \binom{n}{1} + \binom{n}{2} - \binom{n}{3} + \text{L} - \binom{n}{n} = 0.$

c. $0 = [1 + (-1)]^5$
$= \binom{5}{0} - \binom{5}{1} + \binom{5}{2} - \binom{5}{3} + \binom{5}{4} - \binom{5}{5}$

d. For any *n*:
$0 = [1 + (-1)]^n$
$= \binom{n}{0} - \binom{n}{1} + \binom{n}{2} - \binom{n}{3} + \text{L} + (-1)^n \binom{n}{n}$

57. $2^{10} - 2 = 1022$

58. $2^{10} - 2 = 1022$

Exercises 5.8

1. $\frac{5!}{3!1!1!} = 20$

2. $\frac{5!}{2!1!2!} = 30$

3. $\frac{6!}{2!1!2!1!} = 180$

4. $\frac{6!}{3!3!} = 20$

5. $\frac{7!}{3!2!2!} = 210$

6. $\frac{7!}{4!1!2!} = 105$

7. $\frac{12!}{4!4!4!} = 34,650$

8. $\frac{8!}{3!3!2!} = 560$

9. $\frac{12!}{5!3!2!2!} = 166,320$

10. $\frac{8!}{2!2!2!2!} = 2520$

11. $\frac{1}{5!}\cdot\frac{15!}{(3!)^5} = 1,401,400$

12. $\frac{1}{2!}\cdot\frac{10!}{(5!)^2} = 126$

13. $\frac{1}{3!}\cdot\frac{18!}{(6!)^3} = 2,858,856$

14. $\frac{1}{3!}\cdot\frac{12!}{(4!)^3} = 5775$

15. $\binom{20}{7,\ 5,\ 8} = \frac{20!}{7!5!8!} = 99,768,240$ reports

16. $\binom{15}{5,\ 5,\ 5} = \frac{15!}{5!5!5!} = 756,756$ ways

17. $\binom{8}{2,1,4,1} = \frac{8!}{2!1!4!1!} = 840$ words

18. $\binom{10}{1,3,4,1,1} = \frac{10!}{1!3!4!1!1!} = 25,200$ words

19. $\binom{9}{3,2,4} = \frac{9!}{3!2!4!} = 1260$ ways

20. $\binom{6}{3,2,1} = \frac{6!}{3!2!1!} = 60$ ways

21. $\frac{1}{5!}\cdot\frac{20!}{(4!)^5} = 2,546,168,625$ ways

22. $\frac{1}{4!}\cdot\frac{20!}{(5!)^4} = 488,864,376$ ways

23. $\binom{30}{10,\ 2,\ 18} = \frac{30!}{10!2!18!}$
$= 5,708,552,850$ ways

24. $\binom{9}{3,\ 3,\ 3} = \frac{9!}{3!3!3!} = 1680$ ways

25. $\binom{4}{1,\ 1,\ 2} = \frac{4!}{1!1!2!} = 12$ ways

26. $\binom{10}{4,\ 4,\ 2} = \frac{10!}{4!4!2!} = 3150$ ways

27. $\frac{1}{7!}\cdot\frac{14!}{(2!)^7} = 135,135$ ways

28. First partition the six days traveling and four days at the office. There are $\binom{10}{6,\ 4} = 210$ ways to do this without any restriction. Next partition the six travel days among the three cities. There are $\binom{6}{2,\ 2,\ 2} = \frac{6!}{2!2!2!} = 90$ ways to do this.
Thus there are 210 · 90 = 18,900 ways for her to schedule her travel.

29. The number of ways ten students are to be divided into two five member teams for a basketball game is $\frac{10!}{(2)!(5!)^2} = 126.$

30. $\binom{12}{6,\ 4,\ 2} = \frac{12!}{6!4!2!} = 13,860$ ways

31. $\binom{n}{1,1,\cdots,1} = \frac{n!}{1!1!\cdots 1!} = n!$

32. Select the number of elements of S_1:

$$p_1 = \binom{n}{n_1} = \frac{n!}{n_1!(n-n_1)!} \text{ ways}$$

Select the number of elements of S_2:

$$p_2 = \binom{n-n_1}{n_2} = \frac{(n-n_1)!}{n_2!(n-n_1-n_2)!} \text{ ways}$$

Select the number of elements of $S_k (1 < k < m)$:

$$p_k = \binom{n-n_1-n_2-\cdots-n_{k-1}}{n_k} = \frac{(n-n_1-n_2-\cdots-n_{k-1})!}{n_k!(n-n_1-n_2-\cdots-n_{k-1}-n_k)!} \text{ ways}$$

Select the number of elements of S_m:

$$p_m = \binom{n-n_1-n_2-\cdots-n_{m-1}}{n_m} = \binom{n_m}{n_m} = \frac{n_m!}{n_m!} \text{ way}$$

Observe that for $1 < k < m$, the numerator of P_k is $(n-n_1-\cdots-n_{k-1})!$ which also occurs in the denominator of p_{k-1}, which is $n_{k-1}!(n-n_1-\cdots-n_{k-1})!$. Using the generalized multiplication principle, the number of ordered partitions is $p_1 \cdot p_2 \cdot \cdots \cdot p_m$. All of the numerators cancel in this multiplication for $1 < k \le m$, leaving $n_1! \cdot n_2! \cdot \cdots \cdot n_m!$ in the denominator. Thus,

$$p_1 \cdot p_2 \cdot \cdots \cdot p_m = \frac{n!}{n_1! \cdot n_2! \cdot n_3! \cdot \cdots \cdot n_m!}.$$

33. $\binom{38}{10,\ 12,\ 10,\ 6}$
$= \frac{38!}{10!12!10!6!}$
= 115,166,175,166,136,334,240 ways

34. $\frac{65!}{(13!)^5} \approx 8.81 \times 10^{41}$ ways

35. $\binom{52}{13,13,13,13} = \frac{52!}{13!13!13!13!} \approx 5.4 \times 10^{28}$

Therefore, there are more than one octillion.

Chapter 5 Fundamental Concept Check

1. A collection of objects

2. Set *B* is a subset of set *A* if every element of *B* is also an element of *A*.

3. an object in the set

4. The set of all elements under consideration; usually denoted by the capital letter *U*.

5. The set containing no elements; denoted by the symbol ∅.

6. The set consisting of those elements of *U* that are not in *A*.

7. The set consisting of those elements that are in both *A* and *B*.

8. The set consisting of those elements that are in A or *B* or both.

9. If a task consists of t choices performed consecutively, and the first choice can be performed in m_1 ways, for each of these the second choice can be performed in m_2 ways, and so on, then the task can be performed in $m_1 \cdot m_2 \cdot \text{L} \cdot m_t$ ways.

10. an arrangement of r of the n objects in a specific order.

11. $P(n,r) = \underbrace{n(n-1)(n-2)\cdots(n-r+1)}_{r \text{ terms}}$

12. A combination does not take order into account.

13. $C(n,r) = \dfrac{\overbrace{n(n-1)(n-2)\cdots(n-r+1)}^{r \text{ terms}}}{r!}$

14. $n! = n(n-1)(n-2)\text{L } 1$

$$\binom{n}{r} = C(n,r) = \frac{\overbrace{n(n-1)(n-2)\cdots(n-r+1)}^{r \text{ terms}}}{r!}$$

$$P(n,r) = \underbrace{n(n-1)(n-2)\cdots(n-r+1)}_{r \text{ terms}}$$

15. $(x+y)^n = \binom{n}{0}x^n + \binom{n}{1}x^{n-1}y + \binom{n}{2}x^{n-2}y^2 + \text{L } + \binom{n}{n-1}xy^{n-1} + \binom{n}{n}y^n$

16. 2^n

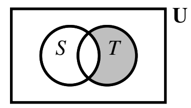
17. A decomposition of a set into an ordered sequence of disjoint subsets.

18. $\dfrac{n!}{n_1!n_2!\text{L } n_m!}$, where the partition is of the type $(n_1, n_2, \cdots, n_m)$ and $n = n_1 + n_2 + \cdots + n_m$.

Chapter 5 Review Exercises

1. $\varnothing, \{a\}, \{b\}, \{a, b\}$

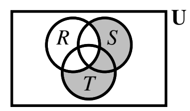
2. $(S \cup T')' = S' \cap T$



3. $C(16, 2) = \dfrac{16!}{2!14!} = 120$ possibilities

4. 2 · 5! = 240 ways

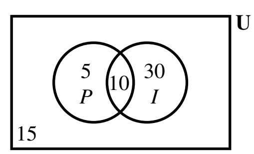
5.



6. $\binom{12}{0}x^{12} + \binom{12}{1}x^{11}(-2y) + \binom{12}{2}x^{10}(-2y)^2$
$= x^{12} - 24x^{11}y + 264x^{10}y^2$

7. $C(8, 3) \cdot C(6, 2) = \dfrac{8!}{3!5!} \cdot \dfrac{6!}{2!4!} = 56 \cdot 15 = 840$

8. Let U = {people given pills}, P = {people who received placebos}, I = {people who showed improvement}.
$n(U) = 60$; $n(P) = 15$; $n(I) = 40$; $n(P' \cap I) = 30$
Draw and complete a Venn diagram as shown.

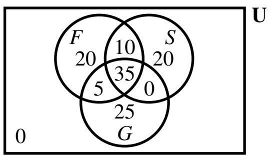


$n(P' \cap I') = 15$
Fifteen of the people who received the drug showed no improvement.

9. $7 \cdot 5 = 35$ combinations

10. $\binom{12}{2,\ 4,\ 6} = \frac{12!}{2!4!6!} = 13,860$

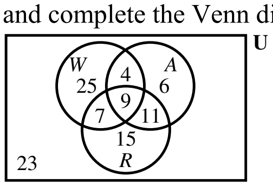
11. Let U = {applicants}, F = {applicants who speak French}, S = {applicants who speak Spanish}, and G = {applicants who speak German}.
$n(U) = 115$; $n(F) = 70$; $n(S) = 65$; $n(G) = 65$; $n(F \cap S) = 45$; $n(S \cap G) = 35$; $n(F \cap G) = 40$;
$n(F \cap S \cap G) = 35$. Draw and complete a Venn diagram.



$n((F \cup S \cup G)') = 0$
None of the people speak none of the three languages.

12. $\binom{17}{15} = \binom{17}{2} = \frac{17 \cdot 16}{2 \cdot 1} = 136$

For Exercises 13–20, let U = {members of the Earth Club}, W = {members who thought the priority is clean water}, A = {members who thought the priority is clean air}, R = {members who thought the priority is recycling}. Then $n(U) = 100$, $n(W) = 45$, $n(A) = 30$, $n(R) = 42$, $n(W \cap A) = 13$, $n(A \cap R) = 20$, $n(W \cap R) = 16$, $n(W \cap A \cap R) = 9$. Draw and complete the Venn diagram as follows.



13. $n(A \cap W' \cap R') = 6$

14. $n((W \cap A') \cup (W' \cap A)) = (25 + 7) + (6 + 11) = 32 + 17 = 49$

15. $n((W \cup R) \cap A') = 25 + 15 + 7 = 47$

16. $n(A \cap R \cap W') = 11$

17. $n((W \cap A' \cap R') \cup (W' \cap A \cap R') \cup (W' \cap A' \cap R)) = 25 + 6 + 15 = 46$

18. $n(R') = 23 + 25 + 6 + 4 = 58$

19. $n(R \cap A') = 15 + 7 = 22$

20. $n((W \cup A \cup R)') = 23$

21. $C(9, 4) = C(9, 5) = 126$

22. $2^{20} = 1,048,576$ ways

23. Let S = {students who ski}, H = {students who play ice hockey}. Then

$$n(S \cup H) = n(S) + n(H) - n(S \cap H) = 400 + 300 - 150 = 550.$$

24. 6 · 10 · 8 = 480 meals

25. $\binom{5}{1,\ 3,\ 1} = \frac{5!}{1!3!1!} = 20$ ways

26. The first digit can be anything but 0, the hundreds digit must be 3, and the last digit must be even.

$9 \cdot 1 \cdot 5 \cdot 10^4 = 450{,}000$

27. $9^2 \cdot 10^8 = 8{,}100{,}000{,}000$

28. $P(7,3) = 7 \cdot 6 \cdot 5 = 210$ ways

29. Strings of length 8 formed from the symbols *a*, *b*, *c*, *d*, *e*:

$5^8 = 390{,}625$ strings

Strings of length 8 formed from the symbols *a*, *b*, *c*, *d*:

$4^8 = 65{,}536$ strings

Strings with at least one *e*:

390,625 – 65,536 = 325,089 strings

30. $C(12, 5) = \frac{12!}{5!7!} = 792$

31. $C(30, 14) = \frac{30!}{14!16!} = 145{,}422{,}675$ groups

32. U = {households}

F = {households that get *Fancy Diet Magazine*}

C = {households that get *Clean Living Journal*}

$$n(F \cup C) = n(F) + n(C) - n(F \cap C) = 4000 + 10{,}000 - 1500 = 12{,}500$$

$$n((F \cup C)') = n(U) - n(F \cup C) = 40{,}000 - 12{,}500 = 27{,}500$$

27,500 households get neither.

33. $3^{10} = 59{,}049$ paths

34. $C(60, 10) = \frac{60!}{10!50!} = 75{,}394{,}027{,}566$ ways

35. $5^{10} = 9{,}765{,}625$ tests

36. $21^6 = 85{,}766{,}121$ strings

37. $C(10, 4) = \frac{10!}{4!6!} = 210$ ways

38. $\frac{1}{3!} \cdot \frac{21!}{(7!)^3} = 66{,}512{,}160$ ways

39. 14! = 87,178,291,200 ways

40. $\frac{1}{4!} \cdot \frac{20!}{(5!)^4} = 488{,}864{,}376$ ways

41. $3 \cdot 5 \cdot 4 = 60$ ways

42. 3 · C(10, 2) = 135

43. A diagonal corresponds to a pair of vertices, except that adjacent pairs must be excluded. Hence there are

$C(n,\ 2) - n = \frac{n(n-1)}{2} - n = \frac{n(n-3)}{2}$ diagonals.

44. 8 · 6 = 48

45. 5! · 4! · 3! · 2! · 1! = 120 · 24 · 6 · 2 · 1 = 34,560

46. P(12, 5) = 12 · 11 · 10 · 9 · 8 = 95,040

47. There are four suits. Once a suit is chosen, select 5 of the 13 cards.

Poker hands with cards of the same suit:

4 · C(13, 5) = 4 · 1287 = 5148 hands

48. C(4, 3) · C(48, 2) = 4 · 1128 = 4512 hands

49. Exactly the first two digits alike: 9 · 1 · 9 = 81

First and last digit alike: 9 · 9 · 1 = 81

Last two digits alike: 9 · 9 · 1 = 81

Numbers with exactly two digits alike:

81 + 81 + 81 = 243

50. 9 · 9 · 8 = 648 numbers

51. 3 · 3 = 9 pairings

52. 6 · 3 · 2 · 2 · 1 · 1 = 72 ways

53. $24 \cdot 23 + 24 \cdot 23 \cdot 22 = 552 + 12{,}144$
$= 12{,}696$ names

54. $3 \cdot 4! \cdot 2 = 3 \cdot 24 \cdot 2 = 144$ ways

55. Any two lines will intersect, so the number of intersections is $C(10, 2) = 45$.

56. First teacher: $\frac{1}{4!} \cdot \frac{24!}{(6!)^4} = 96{,}197{,}645{,}544$ ways

Second teacher:

$\frac{1}{6!} \cdot \frac{24!}{(4!)^6} = 4{,}509{,}264{,}634{,}875$ ways

The second teacher has more options.

57. $\binom{10}{3,\ 4,\ 3} = \frac{10!}{3!4!3!} = 4200$

58. Let n be the number of books.
$n! = 120$, so $n = 5$. There are five books.

59. $7 \cdot 6 \cdot 5 \cdot 1 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 1 = 5040$ orders

60. **a.** $C(6, 2) \cdot C(6, 3) = 15 \cdot 20 = 300$

b. Select five out of twelve states. Then determine the junior or senior senator from each state.
$C(12, 5) \cdot 2^5 = 25{,}344$

61. $C(7, 2) + C(7, 1) + C(7, 0) = 29$ ways

62. There are two choices for every row, so the number of paths from the *A* at the top to the final *A* is $2^6 = 64$ ways.

63. There are $2 \cdot 26^2$ 3-letter call letters and $2 \cdot 26^3$ 4-letter call letters, so $2 \cdot 26^2 + 2 \cdot 26^3 = 36{,}504$ call letters are possible.

64. Find n such that $n! = 479{,}001{,}600$.
Testing numbers on computer, $n = 12$.

65. $\binom{25}{10,\ 9,\ 6}$ =16,360,143,800 ways

66. First choose the three letters. There are $C(26, 3)$ ways to do this. Then choose the three numbers. There are $C(10, 3)$ ways to do this. Then order the six. There are 6! ways to do this.
The number of license plates is
$C(26, 3) \cdot C(10, 3) \cdot 6! = 224{,}640{,}000$.

Conceptual Exercises

67. If $A \cap B = \varnothing$ then *A* and *B* have no elements in common.

68. If $n(A \cup B) = n(A) + n(B)$, then *A* and *B* are disjoint sets and have no elements in common.

69. The intersection of sets *S* and *T* will be the same as set *T* when all the elements of *T* are also in set *S*, in symbols, when $T \subseteq S$.

70. The union of sets *S* and *T* will be the same as set *T* when all the elements of *S* are also in set *T*, in symbols, when $S \subseteq T$.

71. True;
$$n(S \cup T) = n(S) + n(T) - n(S \cap T)$$
$$n(S \cup T) + n(S \cap T) = n(S) + n(T)$$

72. True; the empty set is a subset of every set.

73. $n(n-1)! = n(n-1)(n-2)(n-3)...1 = n!$
For example,
$5 \cdot 4! = 5(4 \cdot 3 \cdot 2 \cdot 1) = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 5!$.

74. Let $n = 1$. We have $1(1-1)! = 1(0)! = 1! = 1$ So, since 1 times any number is the number, we have $0! = 1! = 1$.

75. A permutation is an arrangement of items in which order is important. A combination is a subset of objects taken from a larger set in which the order of the objects does not make a difference.

76. The number of committees of size 6 is the same as the number of committees of size 4. For each committee of size 6, there is a committee consisting of the four people who were not chosen. For example, use letters A, B,C, D, E, F, G, H, I, J to represent the people. For the committee
{A, B, C, D, E, F} the complementary committee is {G, H, I, J}.

77. $C(10, 3) = C(10, 7)$. The number of subsets of size 3 taken from a 10 member set is the same as the number of subsets of size 7. For example, consider the set $N = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$. For the subset $A = \{1, 2, 3\}$, there is the corresponding subset $B = \{4, 5, 6, 7, 8, 9, 10\}$.

78. A committee of size 5 either includes or excludes John Doe. If the committee includes him, there are $C(10, 4)$ ways to choose the other 4 members. If the committee excludes him, there are $C(10, 5)$ ways to choose the 5 members. Hence $C(10, 4) + C(10, 5) = C(11, 5)$.

Chapter 5 Project

1. $n = 5$, $r = 2$, and $r = 3$

2.
$$\begin{aligned}
\binom{n-1}{r-1}+\binom{n-1}{r} &= \frac{(n-1)!}{(r-1)![(n-1)-(r-1)]!}+\frac{(n-1)!}{r![(n-1)-r]} \\
&= \frac{(n-1)!}{(r-1)!(n-r)!}+\frac{(n-1)!}{r!(n-r-1)!} \\
&= \frac{r(n-1)!}{r!(n-r)!}+\frac{(n-r)(n-1)!}{r!(n-r)!} \\
&= \frac{r(n-1)!+(n-r)(n-1)!}{r!(n-r)!} \\
&= \frac{(n-1)![r+n-r]}{r!(n-r)!} \\
&= \frac{n!}{r!(n-r)!} \\
&= C(n, r) \\
&= \binom{n}{r}
\end{aligned}$$

3. Using the hint, there are $C(n-1, r-1)$ ways to select r objects including x because this is equivalent to selecting $r-1$ from the remaining $n-1$ objects. The number of ways to select a subset of r objects not containing x is $C(n-1, r)$ because r objects must be selected from the $n-1$ objects that are not x. The total number of ways to do this is $C(n-1, r-1)+C(n-1, r)=\binom{n-1}{r-1}+\binom{n-1}{r}$.

4.

$$
\begin{array}{ccccccccccccccccccccccccc}
 & & & & & & 1 & & 7 & & 21 & & 35 & & 35 & & 21 & & 7 & & 1 & & & & \\
 & & & & & 1 & & 8 & & 28 & & 56 & & 70 & & 56 & & 28 & & 8 & & 1 & & & \\
 & & & & 1 & & 9 & & 36 & & 84 & & 126 & & 126 & & 84 & & 36 & & 9 & & 1 & & \\
 & & & 1 & & 10 & & 45 & & 120 & & 210 & & 252 & & 210 & & 120 & & 45 & & 10 & & 1 & \\
 & & 1 & & 11 & & 55 & & 165 & & 330 & & 462 & & 462 & & 330 & & 165 & & 55 & & 11 & & 1 \\
 & 1 & & 12 & & 66 & & 220 & & 495 & & 792 & & 924 & & 792 & & 495 & & 220 & & 66 & & 12 & & 1
\end{array}
$$

$\binom{12}{5} = 792, \binom{12}{6} = 924$

5. a. $2^n = (1+1)^n = \binom{n}{0}1^n + \binom{n}{1}1^{n-1}\cdot 1 + \binom{n}{2}1^{n-2}\cdot 1^2 + \ldots + \binom{n}{n-1}1\cdot 1^{n-1} + \binom{n}{n}1^n$

$= \binom{n}{0} + \binom{n}{1} + \binom{n}{2} + \binom{n}{3} + \ldots + \binom{n}{n}$

b. $0 = 0^n = [1+(-1)]^n = \binom{n}{0}1^n + \binom{n}{1}1^{n-1}(-1) + \binom{n}{2}1^{n-2}(-1)^2 + \ldots + \binom{n}{n-1}1\cdot(-1)^{n-1} + \binom{n}{n}(-1)^n$

$= \binom{n}{0} - \binom{n}{1} + \binom{n}{2} - \binom{n}{3} + \ldots \pm \binom{n}{n}$

c. $\binom{7}{0} + \binom{7}{2} + \binom{7}{4} + \binom{7}{6} = 1 + 21 + 35 + 7 = 64$

$\binom{7}{1} + \binom{7}{3} + \binom{7}{5} + \binom{7}{7} = 7 + 35 + 21 + 1 = 64$

d. Part (b) shows that the sum of the even-numbered elements equals the sum of the odd-numbered elements.
If n is even, then

$\binom{n}{0} - \binom{n}{1} + \binom{n}{2} - \binom{n}{3} + \ldots - \binom{n}{n-1} + \binom{n}{n} = 0$

so $\binom{n}{0} + \binom{n}{2} + \ldots + \binom{n}{n} = \binom{n}{1} + \binom{n}{3} + \ldots + \binom{n}{n-1}$.

If n is odd, then

$\binom{n}{0} - \binom{n}{1} + \binom{n}{2} - \binom{n}{3} + \ldots + \binom{n}{n-1} - \binom{n}{n} = 0$

so $\binom{n}{0} + \binom{n}{2} + \ldots + \binom{n}{n-1} = \binom{n}{1} + \binom{n}{3} + \ldots + \binom{n}{n}$.

The common sum is $\frac{1}{2}(2^n)$, or 2^{n-1}.

e. Since there are an equal number of even and odd-numbered subsets, a set *S* with *n* elements has 2^n subsets so that half of them, or $\frac{2^n}{2} = 2^{n-1}$, have an even number of elements.

6. a. Let

$$\begin{array}{rcl} S &=& 1 + 2 + 4 + 8 + \cdots + 2^n \\ 2S &=& \phantom{1 +{}} 2 + 4 + 8 + 16 + \cdots + 2^{n+1} \\ -(S &=& 1 + 2 + 4 + 8 + \cdots + 2^n) \\ \hline S &=& -1 + 2^{n+1} \end{array}$$

Therefore $1+2+4+8+\ldots+2^n = 2^{n+1} - 1$.

b. From 5(a) we know that the sum of the n^{th} row is 2^n. Rearranging the equation in 6(a), we get

$$\underbrace{\left(1+2+4+8+\ldots+2^n\right)}_{\text{The sum of all previous rows}} + 1 = \underset{\substack{\uparrow \\ \text{The sum of} \\ \text{the } (n+1)^{\text{st}} \text{ row}}}{2^{n+1}}$$

7. a. $n = 1, 2, 3, 5, 7, 11$

b. $\binom{p}{r} = \frac{p(p-1)(p-2)\ldots(p-r+1)}{1 \cdot 2 \cdot 3 \cdot \ldots \cdot r}$

Since the result is always an integer,
$p(p-1)(p-2) \cdot \ldots \cdot (p-r+1)$ must be divisible by $1 \cdot 2 \cdot 3 \cdots r = r!$. But *p* is prime and $p > r$, so $(p-1)(p-2) \cdot \ldots \cdot (p-r+1)$ must be divisible by $r!$. Call the result of this division m_r. Then $\binom{p}{r} = m_r p$, and this is divisible by *p*.

c. The sum of all of the numbers in the p^{th} row is 2^p (by 5(a)). There are 2 numbers in the row that are not interior numbers, and they are both 1. Therefore the sum of the interior number is $2^p - 2$.

d. $2^7 - 2 = 126$

$\frac{126}{7} = 18$

e. By 7(c), for any prime number *p*, the sum of the interior numbers of the p^{th} row is $2^p - 2$. By 7(b), each of these numbers is divisible by *p*, so their sum must also be divisible by *p*. Thus, for any prime number *p*, $2^p - 2$ is a multiple of *p*.

8.

row	number of odd numbers
0	$1 = 2^0$
1	$2 = 2^1$
2	$2 = 2^1$
3	$4 = 2^2$
4	$2 = 2^1$
5	$4 = 2^2$
6	$4 = 2^2$
7	$8 = 2^3$
8	$2 = 2^1$
9	$4 = 2^2$
10	$4 = 2^2$
11	$8 = 2^3$
12	$4 = 2^2$

9. Answers may vary. *Sample answer*:
Each row of the small triangle has a number of dots that is equal to a power of 2. These small triangles then appear in multiples of 2 in the larger triangle. Since the original power of 2 is always multiplied by a power of 2, the result is always a power of 2.