Chapter 2

Exercises 2.1

1. $\begin{cases} \frac{1}{2}x - 3y = 2 \\ 5x + 4y = 1 \end{cases}$

$\xrightarrow{2R_1} \begin{cases} x - 6y = 4 \\ 5x + 4y = 1 \end{cases}$

2. $\begin{cases} x + 4y = 6 \\ -y = 2 \end{cases} \xrightarrow{(-1)R_2} \begin{cases} x + 4y = 6 \\ y = -2 \end{cases}$

3. $\begin{array}{rr} 5(\text{first}) & 5x + 10y = 15 \\ +\ (\text{second}) & \underline{-5x + 4y = 1} \\ & 14y = 16 \end{array}$

$\begin{cases} x + 2y = 3 \\ -5x + 4y = 1 \end{cases}$

$\xrightarrow{R_2 + 5R_1} \begin{cases} x + 2y = 3 \\ 14y = 16 \end{cases}$

4. $\begin{array}{rr} -\frac{1}{2}(\text{first}) & -\frac{1}{2}x + 3y = -2 \\ +\ (\text{second}) & \underline{\frac{1}{2}x + 2y = 1} \\ & 5y = -1 \end{array}$

$\begin{cases} x - 6y = 4 \\ \frac{1}{2}x + 2y = 1 \end{cases} \xrightarrow{R_2 + \left(-\frac{1}{2}\right)R_1} \begin{cases} x - 6y = 4 \\ 5y = -1 \end{cases}$

5. $\begin{array}{rr} -4(\text{first}) & -4x + 8y - 4z = 0 \\ +\ (\text{third}) & \underline{4x + y + 3z = 5} \\ & 9y - z = 5 \end{array}$

$\begin{cases} x - 2y + z = 0 \\ y - 2z = 4 \\ 4x + y + 3z = 5 \end{cases}$

$\xrightarrow{R_3 + (-4)R_1} \begin{cases} x - 2y + z = 0 \\ y - 2z = 4 \\ 9y - z = 5 \end{cases}$

6. $\begin{array}{rr} 3(\text{second}) & 3y + 9z = 3 \\ +(\text{third}) & \underline{-3y + 7z = 2} \\ & 16z = 5 \end{array}$

$\begin{cases} x + 6y - 4z = 1 \\ y + 3z = 1 \\ -3y + 7z = 2 \end{cases} \xrightarrow{R_3 + 3R_2} \begin{cases} x + 6y - 4z = 1 \\ y + 3z = 1 \\ 16z = 5 \end{cases}$

7. $\left[\begin{array}{rr|r} 1 & -\frac{1}{2} & 3 \\ 0 & 1 & 4 \end{array}\right] \xrightarrow{R_1 + \frac{1}{2}R_2} \left[\begin{array}{rr|r} 1 & 0 & 5 \\ 0 & 1 & 4 \end{array}\right]$

8. $\left[\begin{array}{rrr|r} 1 & 0 & 7 & 9 \\ 0 & 1 & -2 & 3 \\ 0 & 4 & 8 & 5 \end{array}\right]$

$\xrightarrow{R_3 + (-4)R_2} \left[\begin{array}{rrr|r} 1 & 0 & 7 & 9 \\ 0 & 1 & -2 & 3 \\ 0 & 0 & 16 & -7 \end{array}\right]$

9. $\left[\begin{array}{rr|r} -3 & 4 & -2 \\ 1 & -7 & 8 \end{array}\right]$

10. $\left[\begin{array}{rr|r} \frac{2}{3} & -3 & 4 \\ 0 & 1 & -5 \end{array}\right]$

11. $\left[\begin{array}{rrr|r} 1 & 13 & -2 & 0 \\ 2 & 0 & -1 & 3 \\ 0 & 1 & 0 & 5 \end{array}\right]$

12. $\left[\begin{array}{rrr|r} 0 & 1 & -1 & 22 \\ 2 & 0 & 0 & 17 \\ 1 & -3 & 0 & 12 \end{array}\right]$

13. $\begin{cases} -2y = 3 \\ x + 7y = -4 \end{cases}$

14. $\begin{cases} -5x + \frac{2}{3}y = 3 \\ x + 7y = -\frac{5}{8} \end{cases}$

15. $\begin{cases} 3x + 2y = -3 \\ y - 6z = 4 \\ -5x - y + 7z = 0 \end{cases}$

16. $\begin{cases} \frac{6}{5}x - y + 12z = -\frac{2}{3} \\ -x = 5 \\ 2y - z = 6 \end{cases}$

17. Multiply the second row of the matrix by $\frac{1}{3}$.

18. Change the second row of the matrix by adding to it -4 times the first row.

19. Change the first row of the matrix by adding to it 3 times the second row.

20. Multiply the first row of the matrix by -1.

21. Interchange rows 2 and 3.

22. Interchange rows 1 and 2.

23. $\left[\begin{array}{cc|c} 1 & 2 & 0 \\ 0 & 10 & 5 \end{array}\right]$

24. $\left[\begin{array}{cc|c} 1 & -4 & -\frac{3}{2} \\ -3 & 4 & 9 \end{array}\right]$

25. $\left[\begin{array}{cc|c} 1 & 2 & 3 \\ 3 & -2 & 0 \end{array}\right]$

26. $\left[\begin{array}{cc|c} 1 & 3 & -2 \\ 0 & -8 & 13 \end{array}\right]$

27. $\left[\begin{array}{cc|c} 1 & 3 & -5 \\ 0 & 1 & 7 \end{array}\right]$

28. $\left[\begin{array}{cc|c} 1 & 7 & 6 \\ -3 & 2 & 0 \end{array}\right]$

29. Use $R_2 + 2R_1$ to change the –2 to a 0.

30 Use $\frac{1}{2}R_2$ to change the 2 to a 1.

31. Use $R_1 + (-2)\,R_2$ to change the 2 to a 0.

32 Use $R_3 + (-4)\,R_1$ to change the 4 to a 0.

33. Interchange rows 1 and 2 or rows 1 and 3 to make the first entry in row 1 nonzero.

34. Use $\left(-\frac{1}{3}\right)R_2$ to change the –3 to a 1.

35. Use $R_1 + (-3)\,R_3$ to change the 3 to a 0 or $R_2 + (-2)\,R_3$ to change the 2 to a 0.

36. Interchange rows 2 and 3 to make the second entry in row 2 nonzero.

37. $\left[\begin{array}{ccc|c} 1 & 1 & -1 & 6 \\ -3 & 7 & 5 & 0 \\ 2 & -4 & 3 & -1 \end{array}\right]$

$$\xrightarrow[R_3+(-2)R_1]{R_2+3R_1} \left[\begin{array}{ccc|c} 1 & 1 & -1 & 6 \\ 0 & 10 & \boxed{2} & \boxed{18} \\ 0 & -6 & \boxed{5} & \boxed{-13} \end{array}\right]$$

38. $\left[\begin{array}{ccc|c} 1 & 2 & 7 & -3 \\ 1 & -5 & -4 & 2 \\ -4 & 6 & 9 & 3 \end{array}\right]$

$$\xrightarrow[R_3+4R_1]{R_2+(-1)R_1} \left[\begin{array}{ccc|c} 1 & 2 & 7 & -3 \\ 0 & -7 & \boxed{-11} & \boxed{5} \\ 0 & 14 & \boxed{37} & \boxed{-9} \end{array}\right]$$

39. $\begin{cases} x + y = 7 \\ x - y = 1 \end{cases}; x = 4, y = 3$

40. $\begin{cases} 2x + 3y = 23 \\ 6x - 4y = 4 \end{cases}; x = 4, y = 5$

41. $\begin{cases} 3x - 4y = -27 \\ x + 2y = 11 \end{cases}; x = -1, y = 6$

42. $\begin{cases} 4x - 3y = 18 \\ 2x - y = 8 \end{cases}; x = 3, y = -2$

43. $\begin{cases} 2x + y + 3z = 31 \\ x + y - 2z = 3 \\ 4x - 2y + 5z = 17 \end{cases}; x = 3, y = 10, z = 5$

44. $\begin{cases} 2x+7y+4z=1 \\ 3x-8y+9z=20; x=4, y=-1, z=0 \\ 4x+\quad 5z=16 \end{cases}$

45. $\begin{cases} 3x+7y+2z=5 \\ 7x-6y-3z=4; x=1, y=0, z=1 \\ 10x+9y-7z=3 \end{cases}$

46. $\begin{cases} 3x+2y+z=10 \\ 8x-y+6z=16; x=0.5, y=3, z=2.5 \\ 5x+3y-z=9 \end{cases}$

47. $\left[\begin{array}{cc|c} 1 & 9 & 8 \\ 2 & 8 & 6 \end{array}\right]$

$\xrightarrow{R_2+(-2)R_1} \left[\begin{array}{cc|c} 1 & 9 & 8 \\ 0 & -10 & -10 \end{array}\right]$

$\xrightarrow{-\frac{1}{10}R_2} \left[\begin{array}{cc|c} 1 & 9 & 8 \\ 0 & 1 & 1 \end{array}\right]$

$\xrightarrow{R_1+(-9)R_2} \left[\begin{array}{cc|c} 1 & 0 & -1 \\ 0 & 1 & 1 \end{array}\right]$

$x=-1, y=1$

48. $\left[\begin{array}{cc|c} \frac{1}{3} & 2 & 1 \\ -2 & -4 & 6 \end{array}\right] \xrightarrow{3R_1} \left[\begin{array}{cc|c} 1 & 6 & 3 \\ -2 & -4 & 6 \end{array}\right]$

$\xrightarrow{R_2+2R_1} \left[\begin{array}{cc|c} 1 & 6 & 3 \\ 0 & 8 & 12 \end{array}\right]$

$\xrightarrow{\frac{1}{8}R_2} \left[\begin{array}{cc|c} 1 & 6 & 3 \\ 0 & 1 & \frac{3}{2} \end{array}\right]$

$\xrightarrow{R_1+(-6)R_2} \left[\begin{array}{cc|c} 1 & 0 & -6 \\ 0 & 1 & \frac{3}{2} \end{array}\right]$

$x=-6,\ y=\frac{3}{2}$

49. $\left[\begin{array}{ccc|c} 1 & -3 & 4 & 1 \\ 4 & -10 & 10 & 4 \\ -3 & 9 & -5 & -6 \end{array}\right]$

$\xrightarrow{R_2+(-4)R_1} \left[\begin{array}{ccc|c} 1 & -3 & 4 & 1 \\ 0 & 2 & -6 & 0 \\ -3 & 9 & -5 & -6 \end{array}\right]$

$\xrightarrow{R_3+3R_1} \left[\begin{array}{ccc|c} 1 & -3 & 4 & 1 \\ 0 & 2 & -6 & 0 \\ 0 & 0 & 7 & -3 \end{array}\right]$

$\xrightarrow{\frac{1}{2}R_2} \left[\begin{array}{ccc|c} 1 & -3 & 4 & 1 \\ 0 & 1 & -3 & 0 \\ 0 & 0 & 7 & -3 \end{array}\right]$

$\xrightarrow{R_1+3R_2} \left[\begin{array}{ccc|c} 1 & 0 & -5 & 1 \\ 0 & 1 & -3 & 0 \\ 0 & 0 & 7 & -3 \end{array}\right]$

$\xrightarrow{\frac{1}{7}R_3} \left[\begin{array}{ccc|c} 1 & 0 & -5 & 1 \\ 0 & 1 & -3 & 0 \\ 0 & 0 & 1 & -\frac{3}{7} \end{array}\right]$

$\xrightarrow{R_1+5R_3} \left[\begin{array}{ccc|c} 1 & 0 & 0 & -\frac{8}{7} \\ 0 & 1 & -3 & 0 \\ 0 & 0 & 1 & -\frac{3}{7} \end{array}\right]$

$\xrightarrow{R_2+3R_3} \left[\begin{array}{ccc|c} 1 & 0 & 0 & -\frac{8}{7} \\ 0 & 1 & 0 & -\frac{9}{7} \\ 0 & 0 & 1 & -\frac{3}{7} \end{array}\right]$

$x=-\frac{8}{7},\ y=-\frac{9}{7},\ z=-\frac{3}{7}$

50. $\left[\begin{array}{ccc|c} \frac{1}{2} & 1 & 0 & 4 \\ -4 & -7 & 3 & -31 \\ 6 & 14 & 7 & 50 \end{array}\right]$

$\xrightarrow{2R_1}\left[\begin{array}{ccc|c} 1 & 2 & 0 & 8 \\ -4 & -7 & 3 & -31 \\ 6 & 14 & 7 & 50 \end{array}\right]$

$\xrightarrow{R_2+4R_1}\left[\begin{array}{ccc|c} 1 & 2 & 0 & 8 \\ 0 & 1 & 3 & 1 \\ 6 & 14 & 7 & 50 \end{array}\right]$

$\xrightarrow{R_3+(-6)R_1}\left[\begin{array}{ccc|c} 1 & 2 & 0 & 8 \\ 0 & 1 & 3 & 1 \\ 0 & 2 & 7 & 2 \end{array}\right]$

$\xrightarrow{R_1+(-2)R_2}\left[\begin{array}{ccc|c} 1 & 0 & -6 & 6 \\ 0 & 1 & 3 & 1 \\ 0 & 2 & 7 & 2 \end{array}\right]$

$\xrightarrow{R_3+(-2)R_2}\left[\begin{array}{ccc|c} 1 & 0 & -6 & 6 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & 1 & 0 \end{array}\right]$

$\xrightarrow{R_1+6R_3}\left[\begin{array}{ccc|c} 1 & 0 & 0 & 6 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & 1 & 0 \end{array}\right]$

$\xrightarrow{R_2+(-3)R_3}\left[\begin{array}{ccc|c} 1 & 0 & 0 & 6 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{array}\right]$

$x = 6, y = 1, z = 0$

51. $\left[\begin{array}{cc|c} 2 & -2 & -4 \\ 3 & 4 & 1 \end{array}\right]\xrightarrow{\frac{1}{2}R_1}\left[\begin{array}{cc|c} 1 & -1 & -2 \\ 3 & 4 & 1 \end{array}\right]$

$\xrightarrow{R_2+(-3)R_1}\left[\begin{array}{cc|c} 1 & -1 & -2 \\ 0 & 7 & 7 \end{array}\right]$

$\xrightarrow{\frac{1}{7}R_2}\left[\begin{array}{cc|c} 1 & -1 & -2 \\ 0 & 1 & 1 \end{array}\right]$

$\xrightarrow{R_1+1R_2}\left[\begin{array}{cc|c} 1 & 0 & -1 \\ 0 & 1 & 1 \end{array}\right]$

$x = -1, y = 1$

52. $\left[\begin{array}{cc|c} 2 & 3 & 4 \\ -1 & 2 & -2 \end{array}\right]\xrightarrow{\frac{1}{2}R_1}\left[\begin{array}{cc|c} 1 & \frac{3}{2} & 2 \\ -1 & 2 & -2 \end{array}\right]$

$\xrightarrow{R_2+1R_1}\left[\begin{array}{cc|c} 1 & \frac{3}{2} & 2 \\ 0 & \frac{7}{2} & 0 \end{array}\right]$

$\xrightarrow{\frac{2}{7}R_2}\left[\begin{array}{cc|c} 1 & \frac{3}{2} & 2 \\ 0 & 1 & 0 \end{array}\right]$

$\xrightarrow{R_1+\left(-\frac{3}{2}\right)R_2}\left[\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & 0 \end{array}\right]$

$x = 2, y = 0$

53. $\left[\begin{array}{ccc|c} 4 & -4 & 4 & -8 \\ 1 & -2 & -2 & -1 \\ 2 & 1 & 3 & 1 \end{array}\right]$

$\xrightarrow{\frac{1}{4}R_1}\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 1 & -2 & -2 & -1 \\ 2 & 1 & 3 & 1 \end{array}\right]$

$\xrightarrow{R_2+(-1)R_1}\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & -1 & -3 & 1 \\ 2 & 1 & 3 & 1 \end{array}\right]$

$\xrightarrow{R_3+(-2)R_1}\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & -1 & -3 & 1 \\ 0 & 3 & 1 & 5 \end{array}\right]$

$\xrightarrow{(-1)R_2}\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 3 & -1 \\ 0 & 3 & 1 & 5 \end{array}\right]$

$\xrightarrow{R_1+R_2}\left[\begin{array}{ccc|c} 1 & 0 & 4 & -3 \\ 0 & 1 & 3 & -1 \\ 0 & 3 & 1 & 5 \end{array}\right]$

$\xrightarrow{R_3+(-3)R_2}\left[\begin{array}{ccc|c} 1 & 0 & 4 & -3 \\ 0 & 1 & 3 & -1 \\ 0 & 0 & -8 & 8 \end{array}\right]$

$\xrightarrow{\left(-\frac{1}{8}\right)R_3}\left[\begin{array}{ccc|c} 1 & 0 & 4 & -3 \\ 0 & 1 & 3 & -1 \\ 0 & 0 & 1 & -1 \end{array}\right]$

$\xrightarrow{R_1+(-4)R_3}\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 3 & -1 \\ 0 & 0 & 1 & -1 \end{array}\right]$

$$\xrightarrow{R_2+(-3)R_3}\left[\begin{array}{ccc|c}1&0&0&1\\0&1&0&2\\0&0&1&-1\end{array}\right]$$

$x = 1, y = 2, z = -1$

54. $\left[\begin{array}{ccc|c}1&2&2&11\\1&-1&-1&-4\\2&5&9&39\end{array}\right]$

$$\xrightarrow{R_2+(-1)R_1}\left[\begin{array}{ccc|c}1&2&2&11\\0&-3&-3&-15\\2&5&9&39\end{array}\right]$$

$$\xrightarrow{R_3+(-2)R_1}\left[\begin{array}{ccc|c}1&2&2&11\\0&-3&-3&-15\\0&1&5&17\end{array}\right]$$

$$\xrightarrow{\left(-\frac{1}{3}\right)R_2}\left[\begin{array}{ccc|c}1&2&2&11\\0&1&1&5\\0&1&5&17\end{array}\right]$$

$$\xrightarrow{R_1+(-2)R_2}\left[\begin{array}{ccc|c}1&0&0&1\\0&1&1&5\\0&1&5&17\end{array}\right]$$

$$\xrightarrow{R_3+(-1)R_2}\left[\begin{array}{ccc|c}1&0&0&1\\0&1&1&5\\0&0&4&12\end{array}\right]$$

$$\xrightarrow{\frac{1}{4}R_3}\left[\begin{array}{ccc|c}1&0&0&1\\0&1&1&5\\0&0&1&3\end{array}\right]$$

$$\xrightarrow{R_2+(-1)R_3}\left[\begin{array}{ccc|c}1&0&0&1\\0&1&0&2\\0&0&1&3\end{array}\right]$$

$x = 1, y = 2, z = 3$

55. $\left[\begin{array}{cc|c}0.2&0.3&4\\0.6&1.1&15\end{array}\right]$

$$\xrightarrow{5R_1}\left[\begin{array}{cc|c}1&1.5&20\\0.6&1.1&15\end{array}\right]$$

$$\xrightarrow{R_2+(-.6)R_1}\left[\begin{array}{cc|c}1&1.5&20\\0&0.2&3\end{array}\right]$$

$$\xrightarrow{5R_2}\left[\begin{array}{cc|c}1&1.5&20\\0&1&15\end{array}\right]$$

$$\xrightarrow{R_1+(-1.5)R_2}\left[\begin{array}{cc|c}1&0&-2.5\\0&1&15\end{array}\right]$$

$x = -2.5, y = 15$

56. $\left[\begin{array}{cc|c}\frac{3}{2}&6&9\\\frac{1}{2}&-\frac{2}{3}&11\end{array}\right]\xrightarrow{\frac{2}{3}R_1}\left[\begin{array}{cc|c}1&4&6\\\frac{1}{2}&-\frac{2}{3}&11\end{array}\right]$

$$\xrightarrow{R_2+\left(-\frac{1}{2}\right)R_1}\left[\begin{array}{cc|c}1&4&6\\0&-\frac{8}{3}&8\end{array}\right]$$

$$\xrightarrow{\left(-\frac{3}{8}\right)R_2}\left[\begin{array}{cc|c}1&4&6\\0&1&-3\end{array}\right]$$

$$\xrightarrow{R_1+(-4)R_2}\left[\begin{array}{cc|c}1&0&18\\0&1&-3\end{array}\right]$$

$x = 18, y = -3$

57. $\left[\begin{array}{ccc|c}1&1&4&3\\4&1&-2&-6\\-3&0&2&1\end{array}\right]$

$$\xrightarrow{R_2+(-4)R_1}\left[\begin{array}{ccc|c}1&1&4&3\\0&-3&-18&-18\\-3&0&2&1\end{array}\right]$$

$$\xrightarrow{R_3+3R_1}\left[\begin{array}{ccc|c}1&1&4&3\\0&-3&-18&-18\\0&3&14&10\end{array}\right]$$

$$\xrightarrow{\left(-\frac{1}{3}\right)R_2}\left[\begin{array}{ccc|c}1&1&4&3\\0&1&6&6\\0&3&14&10\end{array}\right]$$

$$\xrightarrow{R_1+(-1)R_2}\left[\begin{array}{ccc|c}1&0&-2&-3\\0&1&6&6\\0&3&14&10\end{array}\right]$$

$$\xrightarrow{R_3+(-3)R_2}\left[\begin{array}{ccc|c}1&0&-2&-3\\0&1&6&6\\0&0&-4&-8\end{array}\right]$$

$$\xrightarrow{\left(-\frac{1}{4}\right)R_3}\left[\begin{array}{ccc|c}1&0&-2&-3\\0&1&6&6\\0&0&1&2\end{array}\right]$$

$$\xrightarrow{R_1+2R_3}\left[\begin{array}{ccc|c}1&0&0&1\\0&1&6&6\\0&0&1&2\end{array}\right]$$

$$\xrightarrow{R_2+(-6)R_3}\left[\begin{array}{ccc|c}1&0&0&1\\0&1&0&-6\\0&0&1&2\end{array}\right]$$

$x = 1, y = -6, z = 2$

58. $\left[\begin{array}{ccc|c}-2&-3&2&-2\\1&1&0&3\\-1&-3&5&8\end{array}\right]$

$$\xrightarrow{\left(-\frac{1}{2}\right)R_1}\left[\begin{array}{ccc|c}1&\frac{3}{2}&-1&1\\1&1&0&3\\-1&-3&5&8\end{array}\right]$$

$$\xrightarrow{R_2+(-1)R_1}\left[\begin{array}{ccc|c}1&\frac{3}{2}&-1&1\\0&-\frac{1}{2}&1&2\\-1&-3&5&8\end{array}\right]$$

$$\xrightarrow{R_3+1R_1}\left[\begin{array}{ccc|c}1&\frac{3}{2}&-1&1\\0&-\frac{1}{2}&1&2\\0&-\frac{3}{2}&4&9\end{array}\right]$$

$$\xrightarrow{(-2)R_2}\left[\begin{array}{ccc|c}1&\frac{3}{2}&-1&1\\0&1&-2&-4\\0&-\frac{3}{2}&4&9\end{array}\right]$$

$$\xrightarrow{R_1+\left(-\frac{3}{2}\right)R_2}\left[\begin{array}{ccc|c}1&0&2&7\\0&1&-2&-4\\0&-\frac{3}{2}&4&9\end{array}\right]$$

$$\xrightarrow{R_3+\frac{3}{2}R_2}\left[\begin{array}{ccc|c}1&0&2&7\\0&1&-2&-4\\0&0&1&3\end{array}\right]$$

$$\xrightarrow{R_1+(-2)R_3}\left[\begin{array}{ccc|c}1&0&0&1\\0&1&-2&-4\\0&0&1&3\end{array}\right]$$

$$\xrightarrow{R_2+2R_3}\left[\begin{array}{ccc|c}1&0&0&1\\0&1&0&2\\0&0&1&3\end{array}\right]$$

$x = 1, y = 2, z = 3$

59. $\left[\begin{array}{ccc|c}-1&1&0&-1\\1&0&1&4\\6&-3&2&10\end{array}\right]$

$$\xrightarrow{(-1)R_1}\left[\begin{array}{ccc|c}1&-1&0&1\\1&0&1&4\\6&-3&2&10\end{array}\right]$$

$$\xrightarrow{R_2+(-1)R_1}\left[\begin{array}{ccc|c}1&-1&0&1\\0&1&1&3\\6&-3&2&10\end{array}\right]$$

$$\xrightarrow{R_3+(-6)R_1}\left[\begin{array}{ccc|c}1&-1&0&1\\0&1&1&3\\0&3&2&4\end{array}\right]$$

$$\xrightarrow{R_1+1R_2}\left[\begin{array}{ccc|c}1&0&1&4\\0&1&1&3\\0&3&2&4\end{array}\right]$$

$$\xrightarrow{R_3+(-3)R_2}\left[\begin{array}{ccc|c}1&0&1&4\\0&1&1&3\\0&0&-1&-5\end{array}\right]$$

$$\xrightarrow{(-1)R_3}\left[\begin{array}{ccc|c}1&0&1&4\\0&1&1&3\\0&0&1&5\end{array}\right]$$

$$\xrightarrow{R_1+(-1)R_3}\left[\begin{array}{ccc|c}1&0&0&-1\\0&1&1&3\\0&0&1&5\end{array}\right]$$

$$\xrightarrow{R_2+(-1)R_3}\left[\begin{array}{ccc|c}1&0&0&-1\\0&1&0&-2\\0&0&1&5\end{array}\right]$$

$x = -1, y = -2, z = 5$

60. $\left[\begin{array}{ccc|c} 1 & 0 & 2 & 9 \\ 0 & 1 & 1 & 1 \\ 3 & -2 & 0 & 9 \end{array}\right]$

$\xrightarrow{R_3+(-3)R_1}\left[\begin{array}{ccc|c} 1 & 0 & 2 & 9 \\ 0 & 1 & 1 & 1 \\ 0 & -2 & -6 & -18 \end{array}\right]$

$\xrightarrow{R_3+2R_2}\left[\begin{array}{ccc|c} 1 & 0 & 2 & 9 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & -4 & -16 \end{array}\right]$

$\xrightarrow{\left(-\frac{1}{4}\right)R_3}\left[\begin{array}{ccc|c} 1 & 0 & 2 & 9 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 4 \end{array}\right]$

$\xrightarrow{R_1+(-2)R_3}\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 4 \end{array}\right]$

$\xrightarrow{R_2+(-1)R_3}\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 4 \end{array}\right]$

$x = 1$, $y = -3$, $z = 4$

61. Let x = grams of cheddar cheese
y = grams of potato

$$\begin{cases} x+y=180 \\ 0.25x+0.02y=10.5 \end{cases}$$

$\left[\begin{array}{cc|c} 1 & 1 & 180 \\ 0.25 & 0.02 & 10.5 \end{array}\right]$

$\xrightarrow{R_2+(-0.25)R_1}\left[\begin{array}{cc|c} 1 & 1 & 180 \\ 0 & -0.23 & -34.5 \end{array}\right]$

$\xrightarrow{-\frac{1}{.23}R_2}\left[\begin{array}{cc|c} 1 & 1 & 180 \\ 0 & 1 & 150 \end{array}\right]$

$\xrightarrow{R_1+(-1)R_2}\left[\begin{array}{cc|c} 1 & 0 & 30 \\ 0 & 1 & 150 \end{array}\right]$

30 grams of cheddar cheese

62. Let x = number of brand A calculators
y = number of brand B calculators

$$\begin{cases} x+y=20 \\ 80x+95y=1780 \end{cases}$$

$\left[\begin{array}{cc|c} 1 & 1 & 20 \\ 80 & 95 & 1780 \end{array}\right]$

$\xrightarrow{R_2+(-80)R_1}\left[\begin{array}{cc|c} 1 & 1 & 20 \\ 0 & 15 & 180 \end{array}\right]$

$\xrightarrow{\frac{1}{15}R_2}\left[\begin{array}{cc|c} 1 & 1 & 20 \\ 0 & 1 & 12 \end{array}\right]$

$\xrightarrow{R_1+(-1)R_2}\left[\begin{array}{cc|c} 1 & 0 & 8 \\ 0 & 1 & 12 \end{array}\right]$

8 brand A calculators

63. Let x = cost of golf balls
and y = cost of golf glove.
Then $x + y = 20$.
Using **Statement I**: $y = 3x$

$$x+3x=20$$
$$4x=20$$
$$x=5.$$

Using **Statement II**: $y = 15$

$$x+15=20$$
$$x=5.$$

The box of balls costs $5. Either statement is sufficient, so the answer is (d).

64. Let n = weight of a nickel
and p = weight of a penny.
Then $4n + 3p$ = total weight.
Using **Statement I**: $n = 2p$
Using **Statement II**: $n + 2p = 10$.
Using both statements together:

$$n=2p$$
$$n+2p=10$$
$$2p+2p=10$$
$$4p=10$$
$$p=2.5$$
$$n=5.$$

A nickel weighs 5 grams, a penny weighs 2.5 grams. Then 4 nickels and 3 pennies weigh $4(5) + 3(2.5) = 27.5$ grams.
So the answer is (c).

65. Let x = number of short sleeve shirts
y = number of long sleeve shirts

$$x+y=350$$
$$10x+14y=4300$$

$$\left[\begin{array}{cc|c} 1 & 1 & 350 \\ 10 & 14 & 4300 \end{array}\right]$$

$$\xrightarrow{R_2+(-10)R_1}\left[\begin{array}{cc|c} 1 & 1 & 350 \\ 0 & 4 & 800 \end{array}\right]$$

$$\xrightarrow{\frac{1}{4}R_2}\left[\begin{array}{cc|c} 1 & 1 & 350 \\ 0 & 1 & 200 \end{array}\right]$$

$$\xrightarrow{R_1+(-1)R_2}\left[\begin{array}{cc|c} 1 & 0 & 150 \\ 0 & 1 & 200 \end{array}\right]$$

150 short sleeve, 200 long sleeve

66. Let x = number of bottles of national brand
y = number of bottles of store brand

$$x+y=82$$
$$2.59x+2.09y=194.88$$

$$\left[\begin{array}{cc|c} 1 & 1 & 82 \\ 2.59 & 2.09 & 194.88 \end{array}\right]$$

$$\xrightarrow{R_2+(-2.59)R_1}\left[\begin{array}{cc|c} 1 & 1 & 82 \\ 0 & -0.5 & -17.5 \end{array}\right]$$

$$\xrightarrow{-2R_2}\left[\begin{array}{cc|c} 1 & 1 & 82 \\ 0 & 1 & 35 \end{array}\right]$$

$$\xrightarrow{R_1+(-1)R_2}\left[\begin{array}{cc|c} 1 & 0 & 47 \\ 0 & 1 & 35 \end{array}\right]$$

47 bottles of national brand, 35 bottles of store brand

67. Let x = adults, y = children

$$\begin{cases} x+y=275 \\ 11.25x+8.50y=2860 \end{cases}$$

$$\left[\begin{array}{cc|c} 1 & 1 & 275 \\ 11.25 & 8.50 & 2860 \end{array}\right]$$

$$\xrightarrow{R_2+(-11.25)R_1}\left[\begin{array}{cc|c} 1 & 1 & 275 \\ 0 & -2.75 & -233.75 \end{array}\right]$$

$$\xrightarrow{-\frac{1}{2.75}R_2}\left[\begin{array}{cc|c} 1 & 1 & 275 \\ 0 & 1 & 85 \end{array}\right]$$

$$\xrightarrow{R_1+(-1)R_2}\left[\begin{array}{cc|c} 1 & 0 & 190 \\ 0 & 1 & 85 \end{array}\right]$$

190 adults, 85 children

68. $$\begin{cases} 1000\left(\dfrac{h+1}{n+1}\right)=250 \\ 1000\left(\dfrac{h}{n+1}\right)=187.5 \end{cases}$$

$$\begin{cases} 250n-1000h=750 \\ 187.5n-1000h=-187.5 \end{cases}$$

$$\left[\begin{array}{cc|c} 250 & -1000 & 750 \\ 187.5 & -1000 & -187.5 \end{array}\right]$$

$$\xrightarrow{\frac{1}{250}R_1}\left[\begin{array}{cc|c} 1 & -4 & 3 \\ 187.5 & -1000 & -187.5 \end{array}\right]$$

$$\xrightarrow{R_2+(-187.5)R_1}\left[\begin{array}{cc|c} 1 & -4 & 3 \\ 0 & -250 & -750 \end{array}\right]$$

$$\xrightarrow{-\frac{1}{250}R_2}\left[\begin{array}{cc|c} 1 & -4 & 3 \\ 0 & 1 & 3 \end{array}\right]$$

$$\xrightarrow{R_1+(4)R_2}\left[\begin{array}{cc|c} 1 & 0 & 15 \\ 0 & 1 & 3 \end{array}\right]$$

15 at bats, 3 hits

$$1000\left(\frac{3}{15}\right)=200 \text{ average}$$

69. $$\begin{cases} x+y+z=9.5 \\ -x+y=0.2 \\ x-0.5y=1.75 \end{cases}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 9.5 \\ -1 & 1 & 0 & 0.2 \\ 1 & -0.5 & 0 & 1.75 \end{array}\right]$$

$$\xrightarrow[R_3+(-1)R_1]{R_2+R_1}\left[\begin{array}{ccc|c} 1 & 1 & 1 & 9.5 \\ 0 & 2 & 1 & 9.7 \\ 0 & -1.5 & -1 & -7.75 \end{array}\right]$$

$$\xrightarrow{\frac{1}{2}R_2}\left[\begin{array}{ccc|c} 1 & 1 & 1 & 9.5 \\ 0 & 1 & 0.5 & 4.85 \\ 0 & -1.5 & -1 & -7.75 \end{array}\right]$$

$$\xrightarrow[R_3+1.5R_2]{R_1+(-1)R_2}\left[\begin{array}{ccc|c} 1 & 0 & 0.5 & 4.65 \\ 0 & 1 & 0.5 & 4.85 \\ 0 & 0 & -0.25 & -.475 \end{array}\right]$$

$$\xrightarrow{-4R_3}\left[\begin{array}{ccc|c}1&0&0.5&4.65\\0&1&0.5&4.85\\0&0&1&1.9\end{array}\right]$$

$$\xrightarrow[R_2+(-0.5)R_3]{R_1+(-0.5)R_3}\left[\begin{array}{ccc|c}1&0&0&3.7\\0&1&0&3.9\\0&0&1&1.9\end{array}\right]$$

United States is 3.7 million square miles, Canada is 3.9 million square miles and the other countries are 1.9 million square miles

70. $$\begin{cases}x+y+z=100\\x-y=-6\\2x+2y-z=-4\end{cases}$$

$$\left[\begin{array}{ccc|c}1&1&1&100\\1&-1&0&-6\\2&2&-1&-4\end{array}\right]$$

$$\xrightarrow[R_3+(-2)R_1]{R_2+(-1)R_1}\left[\begin{array}{ccc|c}1&1&1&100\\0&-2&-1&-106\\0&0&-3&-204\end{array}\right]$$

$$\xrightarrow{-\frac{1}{2}R_2}\left[\begin{array}{ccc|c}1&1&1&100\\0&1&0.5&53\\0&0&-3&-204\end{array}\right]$$

$$\xrightarrow{R_1+(-1)R_2}\left[\begin{array}{ccc|c}1&0&0.5&47\\0&1&0.5&53\\0&0&-3&-204\end{array}\right]$$

$$\xrightarrow{-\frac{1}{3}R_3}\left[\begin{array}{ccc|c}1&0&0.5&47\\0&1&0.5&53\\0&0&1&68\end{array}\right]$$

$$\xrightarrow[R_2+(-0.5)R_3]{R_1+(-0.5)R_3}\left[\begin{array}{ccc|c}1&0&0&13\\0&1&0&19\\0&0&1&68\end{array}\right]$$

13 Business majors, 19 Biology majors, and 68 other majors

71. $$\begin{cases}x+y+z=16\\0.6x+0.5y+0.7z=9.70\\x-2y=0\end{cases}$$

$$\left[\begin{array}{ccc|c}1&1&1&16\\0.6&0.5&0.7&9.70\\1&-2&0&0\end{array}\right]$$

$$\xrightarrow[R_3+(-1)R_1]{R_2+(-0.6)R_1}\left[\begin{array}{ccc|c}1&1&1&16\\0&-0.1&0.1&0.1\\0&-3&-1&-16\end{array}\right]$$

$$\xrightarrow{-10R_2}\left[\begin{array}{ccc|c}1&1&1&16\\0&1&-1&-1\\0&-3&-1&-16\end{array}\right]$$

$$\xrightarrow[R_3+3R_2]{R_1+(-1)R_2}\left[\begin{array}{ccc|c}1&0&2&17\\0&1&-1&-1\\0&0&-4&-19\end{array}\right]$$

$$\xrightarrow{-\frac{1}{4}R_3}\left[\begin{array}{ccc|c}1&0&2&17\\0&1&-1&-1\\0&0&1&4.75\end{array}\right]$$

$$\xrightarrow[R_2+R_3]{R_1+(-2)R_3}\left[\begin{array}{ccc|c}1&0&0&7.5\\0&1&0&3.75\\0&0&1&4.75\end{array}\right]$$

7.5 ounces of the Brazilian, 3.75 ounces of the Columbian, and 4.75 ounces of the Peruvian

72. $$\begin{cases}x+y+z=16\\0.7x+0.6y+0.8z=11.10\\x-z=0\end{cases}$$

$$\left[\begin{array}{ccc|c}1&1&1&16\\0.7&0.6&0.8&11.10\\1&0&-1&0\end{array}\right]$$

$$\xrightarrow[R_3+(-1)R_1]{R_2+(-0.7)R_1}\left[\begin{array}{ccc|c}1&1&1&16\\0&-0.1&0.1&-0.1\\0&-1&-2&-16\end{array}\right]$$

$$\xrightarrow{-10R_2}\left[\begin{array}{ccc|c}1&1&1&16\\0&1&-1&1\\0&-1&-2&-16\end{array}\right]$$

$$\xrightarrow[R_3+R_2]{R_1+(-1)R_2}\left[\begin{array}{ccc|c}1&0&2&26\\0&1&-1&1\\0&0&-3&-15\end{array}\right]$$

$$\xrightarrow{-\frac{1}{3}R_3}\left[\begin{array}{ccc|c}1 & 0 & 2 & 26\\0 & 1 & -1 & 1\\0 & 0 & 1 & 5\end{array}\right]$$

$$\xrightarrow[R_2+R_3]{R_1+(-2)R_3}\left[\begin{array}{ccc|c}1 & 0 & 0 & 5\\0 & 1 & 0 & 6\\0 & 0 & 1 & 5\end{array}\right]$$

5 ounces of cashews, 6 ounces of almonds, and 5 ounces of walnuts

73. $\begin{cases} x+y+z=100{,}000\\ 0.08x+0.07y+0.1z=8000\\ x+y-3z=0\end{cases}$

$$\left[\begin{array}{ccc|c}1 & 1 & 1 & 100{,}000\\0.08 & 0.07 & 0.1 & 8000\\1 & 1 & -3 & 0\end{array}\right]$$

$$\xrightarrow{R_2+(-.08)R_1}\left[\begin{array}{ccc|c}1 & 1 & 1 & 100{,}000\\0 & -0.01 & 0.02 & 0\\1 & 1 & -3 & 0\end{array}\right]$$

$$\xrightarrow{R_3+(-1)R_1}\left[\begin{array}{ccc|c}1 & 1 & 1 & 100{,}000\\0 & -0.01 & 0.02 & 0\\0 & 0 & -4 & -100{,}000\end{array}\right]$$

$$\xrightarrow{(-100)R_2}\left[\begin{array}{ccc|c}1 & 1 & 1 & 100{,}000\\0 & 1 & -2 & 0\\0 & 0 & -4 & -100{,}000\end{array}\right]$$

$$\xrightarrow{R_1+(-1)R_2}\left[\begin{array}{ccc|c}1 & 0 & 3 & 100{,}000\\0 & 1 & -2 & 0\\0 & 0 & -4 & -100{,}000\end{array}\right]$$

$$\xrightarrow{\left(-\frac{1}{4}\right)R_3}\left[\begin{array}{ccc|c}1 & 0 & 3 & 100{,}000\\0 & 1 & -2 & 0\\0 & 0 & 1 & 25{,}000\end{array}\right]$$

$$\xrightarrow{R_1+(-3)R_3}\left[\begin{array}{ccc|c}1 & 0 & 0 & 25{,}000\\0 & 1 & -2 & 0\\0 & 0 & 1 & 25{,}000\end{array}\right]$$

$$\xrightarrow{R_2+2R_3}\left[\begin{array}{ccc|c}1 & 0 & 0 & 25{,}000\\0 & 1 & 0 & 50{,}000\\0 & 0 & 1 & 25{,}000\end{array}\right]$$

$x = \$25{,}000$, $y = \$50{,}000$, $z = \$25{,}000$

74. $\begin{cases} 0.1x+0.1y+0.1z=1\\ 0.1x+0.05y+0.25z=1\\ 0.15x+0.1z=1\end{cases}$

$$\left[\begin{array}{ccc|c}0.1 & 0.1 & 0.1 & 1\\0.1 & 0.05 & 0.25 & 1\\0.15 & 0 & 0.1 & 1\end{array}\right]$$

$$\xrightarrow{R_2+(-.1)R_1}\left[\begin{array}{ccc|c}1 & 1 & 1 & 10\\0 & -0.05 & 0.15 & 0\\0.15 & 0 & 0.1 & 1\end{array}\right]$$

$$\xrightarrow{R_3+(-.15)R_1}\left[\begin{array}{ccc|c}1 & 1 & 1 & 10\\0 & -0.05 & 0.15 & 0\\0 & -0.15 & -0.05 & -0.5\end{array}\right]$$

$$\xrightarrow{(-20)R_2}\left[\begin{array}{ccc|c}1 & 1 & 1 & 10\\0 & 1 & -3 & 0\\0 & -0.15 & -0.05 & -0.5\end{array}\right]$$

$$\xrightarrow{R_1+(-1)R_2}\left[\begin{array}{ccc|c}1 & 0 & 4 & 10\\0 & 1 & -3 & 0\\0 & -0.15 & -0.05 & -0.5\end{array}\right]$$

$$\xrightarrow{R_3+(.15)R_2}\left[\begin{array}{ccc|c}1 & 0 & 4 & 10\\0 & 1 & -3 & 0\\0 & 0 & -.5 & -.5\end{array}\right]$$

$$\xrightarrow{(-2)R_3}\left[\begin{array}{ccc|c}1 & 0 & 4 & 10\\0 & 1 & -3 & 0\\0 & 0 & 1 & 1\end{array}\right]$$

$$\xrightarrow{R_1+(-4)R_3}\left[\begin{array}{ccc|c}1 & 0 & 0 & 6\\0 & 1 & -3 & 0\\0 & 0 & 1 & 1\end{array}\right]$$

$$\xrightarrow{R_1+3R_3}\left[\begin{array}{ccc|c}1 & 0 & 0 & 6\\0 & 1 & 0 & 3\\0 & 0 & 1 & 1\end{array}\right]$$

$x = 6$, $y = 3$, $z = 1$

75 Let x = pounds of first type
y = pounds of second type
z = pounds of third type.

$$\begin{aligned} 0.4x+\phantom{0.3y+{}}0.4z&=90\\ 0.6x+0.3y+0.3z&=100\\ 0.7y+0.3z&=120\end{aligned}$$

$$\left[\begin{array}{ccc|c}0.4 & 0 & 0.4 & 90\\0.6 & 0.3 & 0.3 & 100\\0 & 0.7 & 0.3 & 120\end{array}\right]$$

$$\xrightarrow{\frac{1}{0.4}R_1}\left[\begin{array}{ccc|c}1&0&1&225\\0.6&0.3&0.3&100\\0&0.7&0.3&120\end{array}\right]$$

$$\xrightarrow{R_2+(-0.6)R_1}\left[\begin{array}{ccc|c}1&0&1&225\\0&0.3&-0.3&-35\\0&0.7&0.3&120\end{array}\right]$$

$$\xrightarrow{\frac{1}{0.3}R_2}\left[\begin{array}{ccc|c}1&0&1&225\\0&1&-1&-\frac{350}{3}\\0&0.7&0.3&120\end{array}\right]$$

$$\xrightarrow{R_3+(-.7)R_2}\left[\begin{array}{ccc|c}1&0&1&225\\0&1&-1&-\frac{350}{3}\\0&0&1&\frac{605}{3}\end{array}\right]$$

$$\xrightarrow{R_1+(-1)R_3}\left[\begin{array}{ccc|c}1&0&0&\frac{70}{3}\\0&1&-1&\frac{-350}{3}\\0&0&1&\frac{605}{3}\end{array}\right]$$

$$\xrightarrow{R_2+R_3}\left[\begin{array}{ccc|c}1&0&0&\frac{70}{3}\\0&1&0&85\\0&0&1&\frac{605}{3}\end{array}\right]$$

$\frac{70}{3}=23\frac{1}{3}$ pounds of the first type, 85 pounds of the second type, and $\frac{605}{3}=201\frac{2}{3}$ pounds of the third type

76. Let x = amount invested in savings account
y = amount invested in certificate of deposit
z = amount invested in pre-paid college fund.

$$x+y+z=5000$$
$$0.01x+0.036y+0.055z=195$$
$$-x-y+z=0$$

$$\left[\begin{array}{ccc|c}1&1&1&5000\\0.01&0.036&0.055&195\\-1&-1&1&0\end{array}\right]$$

$$\xrightarrow{R_2+(-0.01)R_1}\left[\begin{array}{ccc|c}1&1&1&5000\\0&0.026&0.045&145\\-1&-1&1&0\end{array}\right]$$

$$\xrightarrow{R_3+R_1}\left[\begin{array}{ccc|c}1&1&1&5000\\0&0.026&0.045&145\\0&0&2&5000\end{array}\right]$$

$$\xrightarrow{\frac{1}{0.026}R_2}\left[\begin{array}{ccc|c}1&1&1&5000\\0&1&\frac{45}{26}&\frac{72500}{13}\\0&0&2&5000\end{array}\right]$$

$$\xrightarrow{R_1+(-1)R_2}\left[\begin{array}{ccc|c}1&0&-\frac{19}{26}&-\frac{7500}{13}\\0&1&\frac{45}{26}&\frac{72500}{13}\\0&0&2&5000\end{array}\right]$$

$$\xrightarrow{\frac{1}{2}R_3}\left[\begin{array}{ccc|c}1&0&-\frac{19}{26}&-\frac{7500}{13}\\0&1&\frac{45}{26}&\frac{72500}{13}\\0&0&1&2500\end{array}\right]$$

$$\xrightarrow{R_1+\frac{19}{26}R_3}\left[\begin{array}{ccc|c}1&0&0&1250\\0&1&\frac{45}{26}&\frac{72500}{13}\\0&0&1&2500\end{array}\right]$$

$$\xrightarrow{R_2+(-\frac{45}{26})R_3}\left[\begin{array}{ccc|c}1&0&0&1250\\0&1&0&1250\\0&0&1&2500\end{array}\right]$$

\$1250 in the savings account, \$1250 in the certificate of deposit, and \$2500 in the pre-paid college fund

77. $\left[\begin{array}{cc|c}1&0&-5\\0&1&4\end{array}\right]$

78. $\left[\begin{array}{ccc|c}1&0&0&\frac{58}{13}\\0&1&0&-3\\0&0&1&\frac{-9}{13}\end{array}\right]$

79. $\left[\begin{array}{ccc|c}1&0&0&\frac{175}{54}\\0&1&0&\frac{16}{9}\\0&0&1&\frac{26}{27}\end{array}\right]$

80. $\left[\begin{array}{ccc|c} 1 & 0 & 0 & \frac{109}{11} \\ 0 & 1 & 0 & \frac{-7}{11} \\ 0 & 0 & 1 & \frac{13}{11} \end{array}\right]$

81.

	A	B
1	-2.5	4
2	15	15
3		
4	CELL CONTENT	
5	B1 =.2x+.3y	
6	B2 =.6x+1.1y	

82.

	A	B
1	18	9
2	-3	11
3		
4	CELL CONTENT	
5	B1 =1.5x+6y	
6	B2 =.5x-(2/3)y	

83.

	A	B
1	1	3
2	-6	-6
3	2	1
4		
5	**Cell**	**Content**
6	B1	=x+y+4*z
7	B2	=4*x+y-2*z
8	B3	=-3*x+2*z

84.

	A	B	C
1	1	-2	
2	2	3	
3	3	8	
4			
5	**Cell**	**Content**	
6	B1	=-2*x-3*y+2*z	
7	B2	=x+y	
8	B3	=-x-3*y+5*z	

Exercises 2.2

1. $\begin{bmatrix} 2 & -4 & 6 \\ 3 & 7 & 1 \end{bmatrix} \xrightarrow[R_2+(-3)R_1]{\frac{1}{2}R_1} \begin{bmatrix} 1 & -2 & 3 \\ 0 & 13 & -8 \end{bmatrix}$

2. $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 8 & -12 \end{bmatrix} \xrightarrow[R_1+(-2)R_2]{\frac{1}{8}R_2} \begin{bmatrix} 0 & 0 & 6 \\ \frac{1}{2} & 1 & -\frac{3}{2} \end{bmatrix}$

3. $\begin{bmatrix} 7 & 1 & 4 & 5 \\ -1 & 1 & 2 & 6 \\ 4 & 0 & 2 & 3 \end{bmatrix}$

$\xrightarrow[\substack{R_1+(-4)R_2 \\ R_3+(-2)R_2}]{\frac{1}{2}R_2} \begin{bmatrix} 9 & -1 & 0 & -7 \\ -\frac{1}{2} & \frac{1}{2} & 1 & 3 \\ 5 & -1 & 0 & -3 \end{bmatrix}$

4. $\begin{bmatrix} 5 & 10 & -10 & 12 \\ 4 & 3 & 6 & 12 \\ 4 & -4 & 4 & -16 \end{bmatrix}$

$\xrightarrow[\substack{R_1+(-10)R_3 \\ R_2+(-3)R_3}]{\left(-\frac{1}{4}\right)R_3} \begin{bmatrix} 15 & 0 & 0 & -28 \\ 7 & 0 & 9 & 0 \\ -1 & 1 & -1 & 4 \end{bmatrix}$

5. $\begin{bmatrix} 2 & 3 \\ 6 & 0 \\ 1 & 5 \end{bmatrix} \xrightarrow[\substack{R_2+(-6)R_1 \\ R_3+(-1)R_1}]{\frac{1}{2}R_1} \begin{bmatrix} 1 & \frac{3}{2} \\ 0 & -9 \\ 0 & \frac{7}{2} \end{bmatrix}$

6. $\begin{bmatrix} 2 & 1 \\ -1 & 0 \end{bmatrix} \xrightarrow[R_1+(-2)R_2]{(-1)R_2} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

7. $\begin{bmatrix} 4 & 3 & 0 \\ \frac{2}{3} & 0 & -2 \\ 1 & 3 & 6 \end{bmatrix} \xrightarrow[R_2+2R_3]{\frac{1}{6}R_3} \begin{bmatrix} 4 & 3 & 0 \\ 1 & 1 & 0 \\ \frac{1}{6} & \frac{1}{2} & 1 \end{bmatrix}$

8. $\begin{bmatrix} 1 & 0 & 2 \\ -1 & 1 & -2 \\ 1 & 2 & 6 \end{bmatrix} \xrightarrow[\substack{R_1+(-2)R_2 \\ R_3+(-6)R_2}]{-\frac{1}{2}R_2} \begin{bmatrix} 0 & 1 & 0 \\ \frac{1}{2} & -\frac{1}{2} & 1 \\ -2 & 5 & 0 \end{bmatrix}$

9. $\begin{cases} x+y+4z=6 \\ 2x+y+z=10 \end{cases}$

z = any value, $y=2-7z, x=4+3z$

10. $\begin{cases} 2x-2y+z=2 \\ -6x+6y-3z=5 \end{cases}$

no solution

11. $\begin{cases} -5x+15y-10z=5 \\ x-3y+2z=0 \end{cases}$

no solution

12. $\begin{cases} 2x-6y-4z=0 \\ -3x+9y+6z=0 \end{cases}$

y = any value, z = any value, $x=3y+2z$

13. $\begin{cases} 2x-y+5z=12 \\ -x-4y+2z=3 \\ 8x+5y+11z=30 \end{cases}$

z = any value, $y=z-2, x=5-2z$

14. $\begin{cases} 2x-y+2z=4 \\ 3x+y+z=-2 \\ x+2y-z=5 \end{cases}$

no solution

15. $\begin{cases} x+2y+3z-w=4 \\ 2x+3y+w=-3 \\ 4x+7y+6z-w=5 \end{cases}$

z = any value, w = any value,
$y=11-6z+3w, x=9z-5w-18$

16. $\begin{cases} x+y+z=-1 \\ x+2y-z=-6 \\ 2x+y+4z=3 \end{cases}$

z = any value, $y=2z-5, x=4-3z$

17. $\left[\begin{array}{rr|r} \underline{2} & -4 & 6 \\ -1 & 2 & -3 \end{array}\right]$

$\left[\begin{array}{rr|r} 1 & -2 & 3 \\ 0 & 0 & 0 \end{array}\right]$

$\begin{cases} x-2y=3 \\ 0=0 \end{cases}$

y = any value, $x = 3 + 2y$

18. $\left[\begin{array}{rr|r} \underline{-\frac{1}{2}} & 1 & \frac{3}{2} \\ -3 & 6 & 10 \end{array}\right]$

$\left[\begin{array}{rr|r} 1 & -2 & -3 \\ 0 & 0 & 1 \end{array}\right]$

$\begin{cases} x-2y=-3 \\ 0=1 \end{cases}$

No solution

19. $\left[\begin{array}{rr|r} \underline{-1} & 3 & 11 \\ 3 & -9 & -30 \end{array}\right]$

$\left[\begin{array}{rr|r} 1 & -3 & 0 \\ 0 & 0 & 1 \end{array}\right]$

$\begin{cases} x-3y=0 \\ 0=1 \end{cases}$

No solution

20. $\left[\begin{array}{rr|r} \underline{.25} & -.75 & 1 \\ -2 & 6 & -8 \end{array}\right]$

$\left[\begin{array}{rr|r} 1 & -3 & 4 \\ 0 & 0 & 0 \end{array}\right]$

$\begin{cases} x-3y=4 \\ 0=0 \end{cases}$

y = any value, $x=4+3y$

21. $\left[\begin{array}{rr|r} \underline{1} & 2 & 5 \\ 3 & -1 & 1 \\ -1 & 3 & 5 \end{array}\right]$

$\left[\begin{array}{rr|r} 1 & 2 & 5 \\ 0 & \underline{-7} & -14 \\ 0 & 5 & 10 \end{array}\right]$

$\left[\begin{array}{rr|r} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{array}\right]$

$x=1, y=2$

22. $\left[\begin{array}{rr|r} \underline{1} & -6 & 12 \\ -\frac{1}{2} & 3 & -6 \\ \frac{1}{3} & -2 & 4 \end{array}\right]$

$\left[\begin{array}{rr|r} 1 & -6 & 12 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right]$

$\begin{cases} x-6y=12 \\ 0=0 \\ 0=0 \end{cases}$

y = any value, $x = 6y + 12$

23. $\left[\begin{array}{rr|r} \underline{4} & 5 & 3 \\ 3 & 6 & 1 \\ 2 & -3 & 7 \end{array}\right]$

$\left[\begin{array}{rr|r} 1 & \frac{5}{4} & \frac{3}{4} \\ 0 & \underline{\frac{9}{4}} & -\frac{5}{4} \\ 0 & -\frac{11}{2} & \frac{11}{2} \end{array}\right]$

$\left[\begin{array}{rr|r} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array}\right]$

no solution

24. $\left[\begin{array}{rr|r} \underline{2} & 3 & 12 \\ 2 & -3 & 0 \\ 5 & -1 & 13 \end{array}\right]$

$\left[\begin{array}{rr|r} 1 & \frac{3}{2} & 6 \\ 0 & \underline{-6} & -12 \\ 0 & -\frac{17}{2} & -17 \end{array}\right]$

$\left[\begin{array}{rr|r} 1 & 0 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{array}\right]$

$x = 3, y = 2$

25. $\left[\begin{array}{rrr|r} \underline{1} & -1 & 3 & 3 \\ -2 & 3 & -11 & -4 \\ 1 & -2 & 8 & 6 \end{array}\right]$

$\left[\begin{array}{rrr|r} 1 & -1 & 3 & 3 \\ 0 & \underline{1} & -5 & 2 \\ 0 & -1 & 5 & 3 \end{array}\right]$

$\left[\begin{array}{rrr|r} 1 & 0 & -2 & 5 \\ 0 & 1 & -5 & 2 \\ 0 & 0 & 0 & 5 \end{array}\right]$

$\begin{cases} x-2z=5 \\ y-5z=2 \\ 0=5 \end{cases}$

No solution

26. $\left[\begin{array}{rrr|r} \underline{1} & -3 & 1 & 5 \\ -2 & 7 & -6 & -9 \\ 1 & -2 & -3 & 6 \end{array}\right]$

$\left[\begin{array}{rrr|r} 1 & -3 & 1 & 5 \\ 0 & \underline{1} & -4 & 1 \\ 0 & 1 & -4 & 1 \end{array}\right]$

$\left[\begin{array}{rrr|r} 1 & 0 & -11 & 8 \\ 0 & 1 & -4 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right]$

$\begin{cases} x-11z=8 \\ y-4z=1 \\ 0=0 \end{cases}$

z = any value, $x = 11z + 8$, $y = 4z + 1$

27. $\left[\begin{array}{rrr|r} \underline{1} & 1 & 1 & -1 \\ 2 & 3 & 2 & 3 \\ 2 & 1 & 2 & -7 \end{array}\right]$

$\left[\begin{array}{rrr|r} 1 & 1 & 1 & -1 \\ 0 & \underline{1} & 0 & 5 \\ 0 & -1 & 0 & -5 \end{array}\right]$

$\left[\begin{array}{rrr|r} 1 & 0 & 1 & -6 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 0 & 0 \end{array}\right]$

$\begin{cases} x+z=-6 \\ y=5 \\ 0=0 \end{cases}$

z = any value, $x = -z - 6$, $y = 5$

28. $\left[\begin{array}{rrr|r} \underline{1} & -3 & 2 & 10 \\ -1 & 3 & -1 & -6 \\ -1 & 3 & 2 & 6 \end{array}\right]$

$\left[\begin{array}{rrr|r} 1 & -3 & 2 & 10 \\ 0 & 0 & \underline{1} & 4 \\ 0 & 0 & 4 & 16 \end{array}\right]$

$\left[\begin{array}{rrr|r} 1 & -3 & 0 & 2 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 \end{array}\right]$

$\begin{cases} x-3y=2 \\ z=4 \\ 0=0 \end{cases}$

y = any value, $x = 3y + 2$, $z = 4$

29. $\left[\begin{array}{rrr|r} \underline{6} & -2 & 2 & 4 \\ 3 & -1 & 2 & 2 \\ -12 & 4 & -8 & 8 \end{array}\right]$

$\left[\begin{array}{rrr|r} 1 & -\frac{1}{3} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]$

$\begin{cases} x-\frac{1}{3}y=0 \\ z=0 \\ 0=1 \end{cases}$

No solution

30. $\left[\begin{array}{rrr|r} \underline{1} & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 1 & 2 & 3 & 5 \end{array}\right]$

$\left[\begin{array}{rrr|r} 1 & 0 & -1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]$

$\begin{cases} x-z=0 \\ y+2z=0 \\ 0=1 \end{cases}$

No solution

31. $\left[\begin{array}{rrr|r} 1 & 2 & 8 & 1 \\ 3 & -1 & 4 & 10 \\ -1 & 5 & 10 & -8 \\ 1 & 1 & 1 & 3 \end{array}\right]$

$\left[\begin{array}{rrr|r} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]$

$\begin{cases} x=0 \\ y=0 \\ z=0 \\ 0=1 \end{cases}$

no solution

32. $\left[\begin{array}{rrr|r} 2 & 6 & 6 & 0 \\ -3 & -10 & 1 & 1 \\ -1 & -4 & 3 & 1 \\ 5 & 6 & 8 & 9 \end{array}\right]$

$\left[\begin{array}{rrr|r} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$

$\begin{cases} x=3 \\ y=-1 \\ z=0 \\ 0=0 \end{cases}$

$x=3,\ y=-1,\ z=0$

33. $\left[\begin{array}{rrrr|r} \underline{1} & 1 & -2 & 2 & 5 \\ 2 & 1 & -4 & 1 & 5 \\ 3 & 4 & -6 & 9 & 20 \\ 4 & 4 & -8 & 8 & 20 \end{array}\right]$

$\left[\begin{array}{rrrr|r} 1 & 1 & -2 & 2 & 5 \\ 0 & \underline{-1} & 0 & -3 & -5 \\ 0 & 1 & 0 & 3 & 5 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$

$\left[\begin{array}{rrrr|r} 1 & 0 & -2 & -1 & 0 \\ 0 & 1 & 0 & 3 & 5 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$

$$\begin{cases} x-2z-w=0 \\ y+3w=5 \\ 0=0 \\ 0=0 \end{cases}$$

z = any value, w = any value, $x = 2z + w$, $y = 5 - 3w$

34. $\left[\begin{array}{rrrr|r} 0 & 2 & 1 & -1 & 1 \\ 1 & -1 & 1 & 1 & 14 \\ -1 & -9 & -1 & 4 & 11 \\ 1 & 1 & 1 & 0 & 9 \end{array}\right]$

$$\left[\begin{array}{rrrr|r} \underline{1} & -1 & 1 & 1 & 14 \\ 0 & 2 & 1 & -1 & 1 \\ -1 & -9 & -1 & 4 & 11 \\ 1 & 1 & 1 & 0 & 9 \end{array}\right]$$

$$\left[\begin{array}{rrrr|r} 1 & -1 & 1 & 1 & 14 \\ 0 & \underline{2} & 1 & -1 & 1 \\ 0 & -10 & 0 & 5 & 25 \\ 0 & 2 & 0 & -1 & -5 \end{array}\right]$$

$$\left[\begin{array}{rrrr|r} 1 & 0 & \frac{3}{2} & \frac{1}{2} & \frac{29}{2} \\ 0 & 1 & \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ 0 & 0 & \underline{5} & 0 & 30 \\ 0 & 0 & -1 & 0 & -6 \end{array}\right]$$

$$\left[\begin{array}{rrrr|r} 1 & 0 & 0 & \frac{1}{2} & \frac{11}{2} \\ 0 & 1 & 0 & -\frac{1}{2} & -\frac{5}{2} \\ 0 & 0 & 1 & 0 & 6 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$$\begin{cases} x+\frac{1}{2}w=\frac{11}{2} \\ y-\frac{1}{2}w=-\frac{5}{2} \\ z=6 \\ 0=0 \end{cases}$$

w = any value, $x=-\frac{1}{2}w+\frac{11}{2},\ y=\frac{1}{2}w-\frac{5}{2},$
$z=6$

35. $\left[\begin{array}{rrrr|r} \underline{1} & -1 & 1 & 1 & 1 \\ 0 & 1 & 3 & 2 & -7 \\ 0 & 1 & -1 & -3 & 1 \\ 1 & 0 & 4 & 3 & 0 \end{array}\right]$

$$\left[\begin{array}{rrrr|r} 1 & 0 & 0 & -2 & 0 \\ 0 & 1 & 0 & -1.75 & 0 \\ 0 & 0 & 1 & 1.25 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{array}\right]$$

$$\begin{cases} x-2w=0 \\ y-\frac{7}{4}w=0 \\ z+\frac{5}{4}w=0 \\ 0=1 \end{cases}$$

no solution

36. $\left[\begin{array}{rrrr|r} \underline{1} & 1 & 0 & 2 & 4 \\ 1 & 2 & 2 & 0 & 7 \\ 2 & 1 & -1 & 6 & 5 \\ -1 & 1 & 2 & -6 & 2 \end{array}\right]$

$$\left[\begin{array}{rrrr|r} 1 & 0 & 0 & 4 & 1 \\ 0 & 1 & 0 & -2 & 3 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$$\begin{cases} x+4w=1 \\ y-2w=3 \\ z=0 \\ 0=0 \end{cases}$$

w = any value, $x=1-4w,\ y=2w+3,\ z=0$

37. $\left[\begin{array}{ccc|c} \underline{1} & 2 & 1 & 5 \\ 0 & 1 & 3 & 9 \end{array}\right]$

$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 5 \\ 0 & \underline{1} & 3 & 9 \end{array}\right]$

$\left[\begin{array}{ccc|c} 1 & 0 & -5 & -13 \\ 0 & 1 & 3 & 9 \end{array}\right]$

$\begin{cases} x-5z=-13 \\ y+3z=9 \end{cases}$

z = any value, $x = 5z - 13$, $y = -3z + 9$
Possible answers: $z = 0$, $x = -13$, $y = 9$;
$z = 1$, $x = -8$, $y = 6$; $z = 2$, $x = -3$, $y = 3$

38. $\left[\begin{array}{ccc|c} \underline{1} & 5 & 3 & 9 \\ 2 & 9 & 7 & 5 \end{array}\right]$

$\left[\begin{array}{ccc|c} 1 & 5 & 3 & 9 \\ 0 & \underline{-1} & 1 & -13 \end{array}\right]$

$\left[\begin{array}{ccc|c} 1 & 0 & 8 & -56 \\ 0 & 1 & -1 & 13 \end{array}\right]$

$\begin{cases} x+8z=-56 \\ y-z=13 \end{cases}$

z = any value, $x = -8z - 56$, $y = z + 13$
Possible answers: $z = 0$, $x = -56$, $y = 13$;
$z = 1$, $x = -64$, $y = 14$; $z = 2$, $x = -72$, $y = 15$

39. $\left[\begin{array}{ccc|c} \underline{1} & 7 & -3 & 8 \\ 0 & 0 & 1 & 5 \end{array}\right]$

$\left[\begin{array}{ccc|c} 1 & 7 & -3 & 8 \\ 0 & 0 & \underline{1} & 5 \end{array}\right]$

$\left[\begin{array}{ccc|c} 1 & 7 & 0 & 23 \\ 0 & 0 & 1 & 5 \end{array}\right]$

$\begin{cases} x+7y=23 \\ z=5 \end{cases}$

y = any value, $x = -7y + 23$, $z = 5$
Possible answers: $y = 0$, $x = 23$, $z = 5$;
$y = 1$, $x = 16$, $z = 5$; $y = 2$, $x = 9$, $z = 5$

40. $\left[\begin{array}{ccc|c} \underline{1} & 0 & 0 & 4 \\ 0 & 1 & -3 & 7 \end{array}\right]$

$\begin{cases} x=4 \\ y-3z=7 \end{cases}$

z = any value, $y = 3z + 7$, $x = 4$
Possible answers: $z = 0$, $y = 7$, $x = 4$;
$z = 1$, $y = 10$, $x = 4$; $z = 2$, $y = 13$, $x = 4$

41. $\left[\begin{array}{ccc|c} 2 & 4 & 6 & 1000 \\ 3 & 7 & 10 & 1600 \\ 5 & 9 & 14 & 2400 \end{array}\right]$

$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 300 \\ 0 & 1 & 1 & 100 \\ 0 & 0 & 0 & 0 \end{array}\right]$

$\begin{cases} x+z=300 \\ y+z=100 \\ 0=0 \end{cases}$

Food 3: z = any value between 0 and 100, Food 2: $y = 100 - z$, Food 1: $x = 300 - z$

42. $\left[\begin{array}{ccc|c} 2 & 4 & 6 & 1000 \\ 3 & 7 & 10 & 1600 \\ 5 & 9 & 14 & 2000 \end{array}\right]$

$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]$

$\begin{cases} x+z=0 \\ y+z=0 \\ 0=1 \end{cases}$

There would be no solution

43. $\left[\begin{array}{ccc|c} 3 & 10 & 15 & 72 \\ 4 & 12 & 8 & 68 \\ 5 & 14 & 1 & 60 \end{array}\right]$

$\left[\begin{array}{ccc|c} 1 & 0 & -25 & 0 \\ 0 & 1 & 9 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]$

$\begin{cases} x-25z=0 \\ y+9z=0 \\ 0=1 \end{cases}$

There would be no solution

44. $\left[\begin{array}{ccc|c} 3 & 10 & 15 & 72 \\ 4 & 12 & 8 & 68 \\ 5 & 14 & 1 & 64 \end{array}\right]$

$\left[\begin{array}{ccc|c} 1 & 0 & -25 & -46 \\ 0 & 1 & 9 & 21 \\ 0 & 0 & 0 & 0 \end{array}\right]$

$\begin{cases} x-25z=-46 \\ y+9z=21 \\ 0=0 \end{cases}$

Possible answers: 4 grams of food A, 3 grams of food B, 2 grams of food C; 1.5 grams of food A, 3.9 grams of food B, 1.9 grams of food C.

45. $\left[\begin{array}{ccc|c} \underline{1} & 3 & 6 & 380 \\ 3 & 6 & 3 & 450 \end{array}\right]$

$\left[\begin{array}{ccc|c} 1 & 0 & -9 & -310 \\ 0 & 1 & 5 & 230 \end{array}\right]$

$\begin{cases} x-9z=-310 \\ y+5z=230 \end{cases}$

Possible answers: 50 ottomans, 30 sofas, 40 chairs; 5 ottomans, 55 sofas, 35 chairs; 95 ottomans, 5 sofas, 45 chairs

46. $\left[\begin{array}{ccc|c} \underline{1} & 1 & 1 & 15 \\ 1000 & 100 & 400 & 10200 \end{array}\right]$

$\left[\begin{array}{ccc|c} 1 & 0 & \frac{1}{3} & \frac{29}{3} \\ 0 & 1 & \frac{2}{3} & \frac{16}{3} \end{array}\right]$

$\begin{cases} x+\frac{1}{3}z=\frac{29}{3} \\ y+\frac{2}{3}z=\frac{16}{3} \end{cases}$

Possible answers: 9 computers, 4 printers, 2 scanners; 8 computers, 2 printers, 5 scanners

47. $\begin{cases} g+b+f=8\times 12 \\ g+b-15f=0 \\ 3g+3b+5f=300 \end{cases}$

$\left[\begin{array}{ccc|c} \underline{1} & 1 & 1 & 96 \\ 1 & 1 & -15 & 0 \\ 3 & 3 & 5 & 300 \end{array}\right]$

$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 96 \\ 0 & 0 & \underline{-16} & -96 \\ 0 & 0 & 2 & 12 \end{array}\right]$

$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 90 \\ 0 & 0 & 1 & 6 \\ 0 & 0 & 0 & 0 \end{array}\right]$

$\begin{cases} g+b=90 \\ f=6 \\ 0=0 \end{cases}$

6 floral squares, the other 90 any mix of solid green and blue

48. Let x = \$7 plants, y = \$10 plants, and z = \$13 plants.

$\left[\begin{array}{ccc|c} \underline{7} & 10 & 13 & 150 \\ 1 & 1 & 1 & 15 \end{array}\right]$

$\left[\begin{array}{ccc|c} 1 & \frac{10}{7} & \frac{13}{7} & \frac{150}{7} \\ 0 & \underline{-\frac{3}{7}} & -\frac{6}{7} & -\frac{45}{7} \end{array}\right]$

$\left[\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 1 & 2 & 15 \end{array}\right]$

$\begin{cases} x-z=0 \\ y+2z=15 \end{cases}$

The same number of \$7 and \$13 plants, up to 7 of each type, the rest \$10 plants

49. $\left[\begin{array}{cc|c} 2 & -3 & 4 \\ -6 & 9 & k \end{array}\right]$

$\left[\begin{array}{cc|c} 1 & -\frac{3}{2} & 2 \\ 0 & 0 & 12+k \end{array}\right]$

No solution if $0 \neq 12 + k$, which happens when $k \neq -12$.

Infinitely many if $0 = 12 + k$, which happens when $k = -12$.

50. $\left[\begin{array}{cc|c} \underline{2} & 6 & 4 \\ 1 & 7 & 10 \\ k & 8 & 4 \end{array}\right]$

$\left[\begin{array}{cc|c} 1 & 3 & 2 \\ 0 & \underline{4} & 8 \\ 0 & -3k+8 & -2k+4 \end{array}\right]$

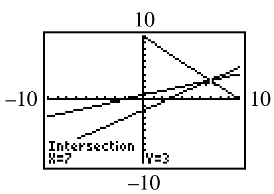
$\left[\begin{array}{cc|c} 1 & 0 & -4 \\ 0 & 1 & 2 \\ 0 & 0 & 4k-12 \end{array}\right]$

$0 = 4k - 12$, so $k = 3$.

51. None; There is no point that satisfies all three equations at the same time.

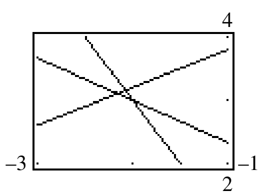
52. No; A system that has more equations than unknowns (3 equations, 2 unknowns) will have a row of zeros but may have a specific solution.

53.



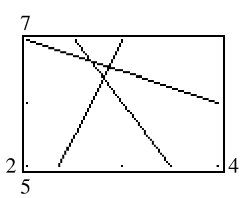
One solution when $x = 7$ and $y = 3$

54.



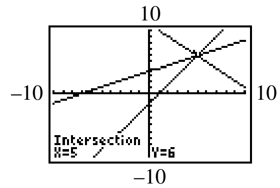
No Solution

55.



No Solution

56.



One solution when $x = 5$ and $y = 6$

57. There has been a pivot about the bottom right entry.

58. There is no difference.

Exercises 2.3

1. 2 by 3

2. 2 by 1, column matrix

3. 1 by 3, row matrix

4. 2 by 2, square identity matrix

5. 2 by 2, square matrix

6. 1 by 1, square, column, and row matrix

7. $a_{12} = -4;\ a_{21} = 0$

8. $a_{23} = -1;\ a_{11} = 2$

9. $i = 1;\ \mathrm{j} = 3$

10. $i = 2;\ \mathrm{j} = 2$

11. $\begin{bmatrix} 4+5 & -2+5 \\ 3+4 & 0+(-1) \end{bmatrix} = \begin{bmatrix} 9 & 3 \\ 7 & -1 \end{bmatrix}$

12. $\begin{bmatrix} 8+5 \\ -3+6 \end{bmatrix} = \begin{bmatrix} 13 \\ 3 \end{bmatrix}$

13. $\begin{bmatrix} 1.3+0.7 & 5-1 & 2.3+0.2 \\ -6+0.5 & 0+1 & 0.7+0.5 \end{bmatrix} = \begin{bmatrix} 2 & 4 & 2.5 \\ -5.5 & 1 & 1.2 \end{bmatrix}$

14. $\begin{bmatrix} \frac{5}{6}+\frac{2}{3} & 10-7 & \frac{1}{2}+\frac{3}{2} \end{bmatrix} = \begin{bmatrix} \frac{3}{2} & 3 & 2 \end{bmatrix}$

15. $\begin{bmatrix} 2-1 & 8-5 \\ \frac{4}{3}-\frac{1}{3} & 4-2 \\ 1-(-3) & -2-0 \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 1 & 2 \\ 4 & -2 \end{bmatrix}$

16. $\begin{bmatrix} 1-0.8 & 0-0.5 \\ 0-0.2 & 1-0.5 \end{bmatrix} = \begin{bmatrix} 0.2 & -0.5 \\ -0.2 & 0.5 \end{bmatrix}$

17. $\begin{bmatrix} -5-2 \\ \frac{1}{2}-\frac{1}{3} \end{bmatrix} = \begin{bmatrix} -7 \\ \frac{1}{6} \end{bmatrix}$

18. $\begin{bmatrix} 1.4-0.6 & 0-(-1) & 3-3 \\ 0.5-0.1 & -1.2-0.4 & 2.5-1 \end{bmatrix}$

$= \begin{bmatrix} 0.8 & 1 & 0 \\ 0.4 & -1.6 & 1.5 \end{bmatrix}$

19. $[5\cdot 1+3\cdot 2] = [11]$

20. $\left[1\cdot \frac{1}{2}+0\cdot 6+0\cdot 2\right] = \left[\frac{1}{2}\right]$

21. $\left[6\cdot \frac{1}{2}+1(-3)+5\cdot 2\right] = [10]$

22. $[0\cdot 5+0\cdot(-3)] = [0]$

23. $\begin{bmatrix} \frac{2}{3}\cdot 6 & \frac{2}{3}\cdot 0 & \frac{2}{3}\cdot -1 \\ \frac{2}{3}\cdot -9 & \frac{2}{3}\cdot\frac{3}{4} & \frac{2}{3}\cdot\frac{1}{2} \end{bmatrix} = \begin{bmatrix} 4 & 0 & -\frac{2}{3} \\ -6 & \frac{1}{2} & \frac{1}{3} \end{bmatrix}$

24. $\begin{bmatrix} 1.5\cdot 4 & 1.5\cdot .5 \\ 1.5\cdot 0 & 1.5\cdot 1.2 \end{bmatrix} = \begin{bmatrix} 6 & .75 \\ 0 & 1.8 \end{bmatrix}$

25. $\begin{bmatrix} 2\cdot .5 & 2\cdot -1 \\ 2\cdot 4 & 2\cdot 0 \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ 8 & 0 \end{bmatrix}$

$\begin{bmatrix} 3\cdot \frac{2}{3} & 3\cdot 7 \\ 3\cdot 5 & 3\cdot 1 \end{bmatrix} = \begin{bmatrix} 2 & 21 \\ 15 & 3 \end{bmatrix}$

$\begin{bmatrix} 1+2 & -2+21 \\ 8+15 & 0+3 \end{bmatrix} = \begin{bmatrix} 3 & 19 \\ 23 & 3 \end{bmatrix}$

26. $\left[\frac{3}{5}\cdot 10 \quad \frac{3}{5}\cdot 25\right] = [6 \quad 15]$

$[6-6 \quad 15-(-3)] = [0 \quad 18]$

27. Yes, columns of A = rows of B; 4, therefore the product will be size 3×5

28. Yes, columns of A = rows of B; 3, therefore the product will be size 3×4

29. No, columns of $A \neq$ rows of B

30. Yes, columns of A = rows of B; 1, therefore the product will be size 1×1.

31. Yes, columns of A = rows of B; 3, therefore the product will be size 3×1

32. No, columns of $A \neq$ rows of B.

33. $\begin{bmatrix} 3\cdot 1+1\cdot 3 & 3\cdot 4+1\cdot 5 \\ 0\cdot 1+2\cdot 3 & 0\cdot 4+2\cdot 5 \end{bmatrix} = \begin{bmatrix} 6 & 17 \\ 6 & 10 \end{bmatrix}$

34. $\begin{bmatrix} 4\cdot 3+(-1)2 \\ 2\cdot 3+\frac{1}{2}\cdot 2 \end{bmatrix} = \begin{bmatrix} 10 \\ 7 \end{bmatrix}$

35. $\begin{bmatrix} 4\cdot 5+1\cdot 1+0\cdot 2 \\ -2\cdot 5+0\cdot 1+3\cdot 2 \\ 1\cdot 5+5\cdot 1+(-1)2 \end{bmatrix} = \begin{bmatrix} 21 \\ -4 \\ 8 \end{bmatrix}$

36. Multiplication by zero matrix: $\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$

37. Multiplication by identity matrix: $\begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$

38. $\begin{bmatrix} 1\cdot 3+2(-1) & 1(-2)+2\cdot 1 \\ 1\cdot 3+3(-1) & 1(-2)+3\cdot 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

39. $\begin{bmatrix} (0.6)(0.6)+(0.3)(0.4) & (0.6)(0.3)+(0.3)(0.7) \\ (0.4)(0.6)+(0.7)(0.4) & (0.4)(0.3)+(0.7)(0.7) \end{bmatrix}$

$= \begin{bmatrix} 0.48 & 0.39 \\ 0.52 & 0.61 \end{bmatrix}$

40. $\begin{bmatrix} 0\cdot 3+1\cdot 0+2\cdot 4 & 0(-1)+1\cdot 2+2(-6) & 0\cdot 5+1\cdot 2+2\cdot 0 \\ -1\cdot 3+4\cdot 0+\frac{1}{2}\cdot 4 & (-1)(-1)+4\cdot 2+\frac{1}{2}(-6) & -1\cdot 5+4\cdot 2+\frac{1}{2}\cdot 0 \\ 1\cdot 3+3\cdot 0+0\cdot 4 & 1(-1)+3\cdot 2+0(-6) & 1\cdot 5+3\cdot 2+0\cdot 0 \end{bmatrix} = \begin{bmatrix} 8 & -10 & 2 \\ -1 & 6 & 3 \\ 3 & 5 & 11 \end{bmatrix}$

41. $\begin{bmatrix} 2\cdot 4+(-1)3+4\cdot 5 & 2\cdot 8+(-1)(-1)+4\cdot 0 & 2\cdot 0+(-1)2+4\cdot 1 \\ 0\cdot 4+1\cdot 3+0\cdot 5 & 0\cdot 8+1(-1)+0\cdot 0 & 0\cdot 0+1\cdot 2+0\cdot 1 \\ \frac{1}{2}\cdot 4+3\cdot 3+(-2)5 & \frac{1}{2}\cdot 8+3(-1)+(-2)0 & \frac{1}{2}\cdot 0+3\cdot 2+(-2)\cdot 1 \end{bmatrix} = \begin{bmatrix} 25 & 17 & 2 \\ 3 & -1 & 2 \\ 1 & 1 & 4 \end{bmatrix}$

42. Multiplication by identity matrix: $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

43. $\begin{bmatrix} (\frac{1}{3})(\frac{1}{3})+(\frac{2}{3})(\frac{1}{3}) & (\frac{1}{3})(\frac{2}{3})+(\frac{2}{3})(\frac{2}{3}) \\ (\frac{1}{3})(\frac{1}{3})+(\frac{2}{3})(\frac{1}{3}) & (\frac{1}{3})(\frac{2}{3})+(\frac{2}{3})(\frac{2}{3}) \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & \frac{2}{3} \\ \frac{1}{3} & \frac{2}{3} \end{bmatrix}$

44. $\begin{bmatrix} 0.4\cdot 0.4+0.4\cdot 0.4+0.4\cdot 0.2 & 0.4\cdot 0.4+0.4\cdot 0.4+0.4\cdot 0.2 & 0.4\cdot 0.4+0.4\cdot 0.4+0.4\cdot 0.2 \\ 0.4\cdot 0.4+0.4\cdot 0.4+0.4\cdot 0.2 & 0.4\cdot 0.4+0.4\cdot 0.4+0.4\cdot 0.2 & 0.4\cdot 0.4+0.4\cdot 0.4+0.4\cdot 0.2 \\ 0.2\cdot 0.4+0.2\cdot 0.4+0.2\cdot 0.2 & 0.2\cdot 0.4+0.2\cdot 0.4+0.2\cdot 0.2 & 0.2\cdot 0.4+0.2\cdot 0.4+0.2\cdot 0.2 \end{bmatrix} = \begin{bmatrix} 0.4 & 0.4 & 0.4 \\ 0.4 & 0.4 & 0.4 \\ 0.2 & 0.2 & 0.2 \end{bmatrix}$

45. $\begin{bmatrix} 2(0)+5(6) & 2(3)+5(7) \end{bmatrix} = \begin{bmatrix} 30 & 41 \end{bmatrix}$

46. $\begin{bmatrix} 4(2)+0(4)+1(0) & 4(3)+0(5)+1(6) \end{bmatrix} = \begin{bmatrix} 8 & 18 \end{bmatrix}$

47. $\begin{bmatrix} 2(5)+0(0) & 2(0)+0(5) \\ 0(5)+3(0) & 0(0)+3(5) \end{bmatrix} = \begin{bmatrix} 10 & 0 \\ 0 & 15 \end{bmatrix}$

48. $\begin{bmatrix} \frac{1}{3}(6)+0(0) & \frac{1}{3}(0)+0(8) \\ 0(6)+\frac{1}{4}(0) & 0(0)+\frac{1}{4}(8) \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$

49. $\begin{bmatrix} 0(2.34)+0(-3.7) & 0(5.6)+0(0.08) \\ 0(2.34)+0(-3.7) & 0(5.6)+0(0.08) \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

50. $\begin{bmatrix} -78(0)+56(0) & -78(0)+56(0) \\ 312(0)+23(0) & 312(0)+23(0) \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

51. $\begin{bmatrix} 23(1)+24(0) & 23(0)+24(1) \\ 25(1)+26(0) & 25(0)+26(1) \end{bmatrix} = \begin{bmatrix} 23 & 24 \\ 25 & 26 \end{bmatrix}$

52. $\begin{bmatrix} 1(2.4)+0(7.8) & 1(5.6)+0(9.9) \\ 0(2.4)+1(7.8) & 0(5.6)+1(9.9) \end{bmatrix} = \begin{bmatrix} 2.4 & 5.6 \\ 7.8 & 9.9 \end{bmatrix}$

53. $\begin{cases} 2x+3y=6 \\ 4x+5y=7 \end{cases}$

54. $\begin{cases} -3x+4y=1 \\ \qquad\quad y=1 \end{cases}$

55. $\begin{cases} x+2y+3z=10 \\ 4x+5y+6z=11 \\ 7x+8y+9z=12 \end{cases}$

56. $\begin{cases} x=1 \\ y=2 \\ z=3 \end{cases}$

57. $\begin{bmatrix} 3 & 2 \\ 7 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} -1 \\ 2 \end{bmatrix}$

58. $\begin{bmatrix} 5 & -2 \\ -2 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} 6 \\ 0 \end{bmatrix}$

59. $\begin{bmatrix} 1 & -2 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} 5 \\ 6 \\ 2 \end{bmatrix}$

60. $\begin{bmatrix} -2 & 4 & -1 \\ 1 & 6 & 3 \\ 7 & 0 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} 5 \\ -1 \\ 8 \end{bmatrix}$

61. $\left(\begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}+\begin{bmatrix} 3 & -2 \\ 4 & 5 \end{bmatrix}\right)\begin{bmatrix} 1 & 6 \\ 2 & 0 \end{bmatrix}=\begin{bmatrix} 4 & 0 \\ 4 & 8 \end{bmatrix}\begin{bmatrix} 1 & 6 \\ 2 & 0 \end{bmatrix}=\begin{bmatrix} 4 & 24 \\ 20 & 24 \end{bmatrix}$

$\begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} 1 & 6 \\ 2 & 0 \end{bmatrix}+\begin{bmatrix} 3 & -2 \\ 4 & 5 \end{bmatrix}\begin{bmatrix} 1 & 6 \\ 2 & 0 \end{bmatrix}=\begin{bmatrix} 5 & 6 \\ 6 & 0 \end{bmatrix}+\begin{bmatrix} -1 & 18 \\ 14 & 24 \end{bmatrix}=\begin{bmatrix} 4 & 24 \\ 20 & 24 \end{bmatrix}$

62. $\left(\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}+\begin{bmatrix} 2 & 1 & 3 \\ 0 & 5 & -1 \\ 3 & 6 & 0 \end{bmatrix}\right)\begin{bmatrix} 0 \\ 3 \\ -4 \end{bmatrix}=\begin{bmatrix} 3 & 1 & 3 \\ 0 & 6 & -1 \\ 3 & 6 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ 3 \\ -4 \end{bmatrix}=\begin{bmatrix} -9 \\ 22 \\ 14 \end{bmatrix}$

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ 3 \\ -4 \end{bmatrix}+\begin{bmatrix} 2 & 1 & 3 \\ 0 & 5 & -1 \\ 3 & 6 & 0 \end{bmatrix}\begin{bmatrix} 0 \\ 3 \\ -4 \end{bmatrix}=\begin{bmatrix} 0 \\ 3 \\ -4 \end{bmatrix}+\begin{bmatrix} -9 \\ 19 \\ 18 \end{bmatrix}=\begin{bmatrix} -9 \\ 22 \\ 14 \end{bmatrix}$

63. $\begin{bmatrix} 3\cdot 1+(-1)2 & 3\cdot 2+(-1)6 \\ -1\cdot 1+\frac{1}{2}\cdot 2 & -1\cdot 2+\frac{1}{2}\cdot 6 \end{bmatrix}=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\begin{bmatrix} 1\cdot 3+2(-1) & 1(-1)+(2)\frac{1}{2} \\ 2\cdot 3+6(-1) & 2(-1)+(6)\frac{1}{2} \end{bmatrix}=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

64. $$\begin{bmatrix} 2\cdot 1+8\cdot 4+(-11)3 & 2\cdot 2+8\cdot 5+(-11)4 & 2\cdot 1+8(-3)+(-11)(-2) \\ -1\cdot 1+(-5)4+7\cdot 3 & -1\cdot 2+(-5)5+7\cdot 4 & -1\cdot 1+(-5)(-3)+7(-2) \\ 1\cdot 1+2\cdot 4+(-3)3 & 1\cdot 2+2\cdot 5+(-3)4 & 1\cdot 1+2(-3)+(-3)(-2) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1\cdot 2+2(-1)+(1)1 & 1\cdot 8+2(-5)+(1)2 & 1(-11)+2(7)+(1)(-3) \\ 4\cdot 2+5(-1)+(-3)1 & 4\cdot 8+5(-5)+(-3)2 & 4(-11)+5(7)+(-3)(-3) \\ 3\cdot 2+4(-1)+(-2)1 & 3\cdot 8+4(-5)+(-2)2 & 3(-11)+4(7)+(-2)(-3) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

65. a. $\begin{bmatrix} 6 & 8 & 2 \\ 2 & 5 & 3 \end{bmatrix}\begin{bmatrix} 20 \\ 15 \\ 50 \end{bmatrix} = \begin{bmatrix} 340 \\ 265 \end{bmatrix}$

b. Mike's clothes are worth \$340; Don's clothes are worth \$265.

c. $\begin{bmatrix} 1.25\cdot 20 \\ 1.25\cdot 15 \\ 1.25\cdot 50 \end{bmatrix} = \begin{bmatrix} 25 \\ 18.75 \\ 62.50 \end{bmatrix}$

d. The matrix represents the costs of the three items of clothing after a 25% increase.

66. a. $\begin{bmatrix} 250 & 80 & 60 \end{bmatrix}\begin{bmatrix} 40 & 35 \\ 30 & 35 \\ 50 & 75 \end{bmatrix}$

$= \begin{bmatrix} 15{,}400 & 16{,}050 \end{bmatrix}$

b. The monthly sales for Store 1 were \$15,400 and for Store 2 were \$16,050.

c. $\begin{bmatrix} 1.1\cdot 250 & 1.1\cdot 80 & 1.1\cdot 60 \end{bmatrix} = \begin{bmatrix} 275 & 88 & 66 \end{bmatrix}$

d. The matrix represents the retail prices after a 10% increase.

67. a. $\begin{bmatrix} 210 & 175 & 135 \end{bmatrix}\begin{bmatrix} 3 & 3 & 5.8 \\ 2.5 & 3.5 & 6 \\ 9 & 8 & 9.5 \end{bmatrix}$

$= \begin{bmatrix} 2282.50 & 2322.50 & 3550.50 \end{bmatrix}$

The total value of the store's plain items was \$2282.50, of the milk chocolate-covered items was \$2322.50, and of the dark chocolate covered items was \$3550.50.

b. $\begin{bmatrix} 3 & 3 & 5.8 \\ 2.5 & 3.5 & 6 \\ 9 & 8 & 9.5 \end{bmatrix}\begin{bmatrix} 105 \\ 390 \\ 285 \end{bmatrix} = \begin{bmatrix} 3138.00 \\ 3337.50 \\ 6772.50 \end{bmatrix}$

The store's weekly sales of peanuts was \$3138.00, of raisins was \$3337.50, and of espresso beans was \$6772.50.

c. $\begin{bmatrix} .9\cdot 105 \\ .9\cdot 390 \\ .9\cdot 285 \end{bmatrix} = \begin{bmatrix} 94.50 \\ 351 \\ 256.50 \end{bmatrix}$

The store's weekly sales if there were a 10% reduction in the number of pound sold.

68. a. $WN = \begin{bmatrix} 18{,}500 & 21{,}750 & 24{,}250 \end{bmatrix}$,
November wholesale costs for each of the three stores

b. $WD = \begin{bmatrix} 18{,}000 & 26{,}500 & 27{,}500 \end{bmatrix}$,
December wholesale costs for each of the three stores

c. $RN = \begin{bmatrix} 31{,}500 & 37{,}250 & 40{,}750 \end{bmatrix}$,
November revenue for each of the three stores

d. $RD = \begin{bmatrix} 31{,}000 & 44{,}500 & 46{,}500 \end{bmatrix}$,
December revenue for each of the three stores

e. $R - W = \begin{bmatrix} 200 & 200 & 300 \end{bmatrix}$, profits for each of the three appliances

f. $(R - W)N = \begin{bmatrix} 13{,}000 & 15{,}500 & 16{,}500 \end{bmatrix}$,
November profits for each of the three stores

g. $(R-W)D=\begin{bmatrix}13{,}000 & 18{,}000 & 19{,}000\end{bmatrix}$, December profits for each of the three stores

h. $N+D=\begin{bmatrix}50 & 90 & 50\\50 & 40 & 30\\20 & 25 & 65\end{bmatrix}$, quantities of each of the appliances sold during November and December

i. $(R-W)(N+D)$
$=\begin{bmatrix}26{,}000 & 33{,}500 & 35{,}500\end{bmatrix}$, combined November and December profits for each of the three stores

j. $0.95R=\begin{bmatrix}0.95\cdot 500 & 0.95\cdot 450 & 0.95\cdot 750\end{bmatrix}$
$=\begin{bmatrix}475 & 427.5 & 712.5\end{bmatrix}$,
retail prices after a 5 % decrease.

69. a. $\begin{bmatrix}0.25 & 0.35 & 0.30 & 0.10 & 0\\0.10 & 0.20 & 0.40 & 0.20 & 0.10\\0.05 & 0.10 & 0.20 & 0.40 & 0.25\end{bmatrix}\begin{bmatrix}4\\3\\2\\1\\0\end{bmatrix}=\begin{bmatrix}2.75\\2.00\\1.30\end{bmatrix}$

I: 2.75; II: 2, III: 1.3

b. $[240\ 120\ 40]\begin{bmatrix}0.25 & 0.35 & 0.30 & .10 & 0\\0.10 & 0.20 & 0.40 & .20 & .10\\0.05 & 0.10 & 0.20 & .40 & .25\end{bmatrix}$
$=[74\ 112\ 128\ 64\ 22]$
A: 74, B: 112, C: 128, D: 64, F: 22

70. $\begin{bmatrix}0.10 & 0.10 & 0.30 & 0.50\\0.10 & 0.20 & 0.30 & 0.40\\0.15 & 0.15 & 0.35 & 0.35\end{bmatrix}\begin{bmatrix}97\\72\\83\\75\end{bmatrix}=\begin{bmatrix}79.3\\79\\80.65\end{bmatrix}$

Scheme III, with 80.65 points

71. $[6000\ 8000\ 4000]\begin{bmatrix}0.65 & 0.35\\0.55 & 0.45\\0.45 & 0.55\end{bmatrix}=[10{,}100\ 7900]$

10,100 voting Democratic, 7900 voting Republican

72. a. A Democratic victory, with $\dfrac{10{,}100}{10{,}100+7900}\approx 56.1\%$

b. $[2000\ 4000\ 12{,}000]\begin{bmatrix}0.65 & 0.35\\0.55 & 0.45\\0.45 & 0.55\end{bmatrix}$
$=[8900\ 9100]$
A Republican victory, with $\dfrac{9100}{8900+9100}\approx 50.6\%$

73. $\begin{bmatrix}50 & 20 & 10\\30 & 30 & 15\\20 & 20 & 5\end{bmatrix}\begin{bmatrix}20\\30\\40\end{bmatrix}=\begin{bmatrix}2000\\2100\\1200\end{bmatrix}$

Carpenters: \$2000, Bricklayers: \$2100, Plumbers: \$1200

74. a. $\begin{bmatrix}0.70 & 0.70 & 0.60\\0.10 & 0.20 & 0.30\\0.20 & 0.10 & 0.10\end{bmatrix}\begin{bmatrix}60{,}000 & 65{,}000\\100{,}000 & 110{,}000\\200{,}000 & 230{,}000\end{bmatrix}$

$=\begin{bmatrix}232{,}000 & 260{,}500\\86{,}000 & 97{,}500\\42{,}000 & 47{,}000\end{bmatrix}$

b. 86,000

c. 47,000

75. a. $BN=\begin{bmatrix}162 & 150 & 143\end{bmatrix}$, number of units of each nutrient consumed at breakfast

b. $LN=\begin{bmatrix}186 & 200 & 239\end{bmatrix}$, number of units of each nutrient consumed at lunch

c. $DN=\begin{bmatrix}288 & 300 & 344\end{bmatrix}$, number of units of each nutrient consumed at dinner

d. $B+L+D=\begin{bmatrix}5 & 8\end{bmatrix}$, total number of ounces of each food that Mikey eats during a day

e. $(B+L+D)N=\begin{bmatrix}636 & 650 & 726\end{bmatrix}$, number of units of each nutrient consumed per day

76. a. $RM = \begin{bmatrix} 100 & 115 & 85 & 75 \end{bmatrix}$, units of each ingredient needed to fill the order

b. $MN = \begin{bmatrix} 108 \\ 102 \\ 182 \end{bmatrix}$, cost to make each type of cookie

c. $RMN = \begin{bmatrix} 5850 \end{bmatrix}$, total cost to fill order

d. $S - MN = \begin{bmatrix} 67 \\ 48 \\ 43 \end{bmatrix}$, profit for each type of cookie

e. $R(S - MN) = \begin{bmatrix} 2275 \end{bmatrix}$, total profit for the order

f. $RS = \begin{bmatrix} 8125 \end{bmatrix}$, total price of the order

77. a. $AP = \begin{bmatrix} 720 \\ 646 \end{bmatrix}$, the average amount taken in daily by the pool and the weight room

b. \$720

78. a.

$$T = \begin{array}{c} \begin{array}{cc} \text{DVD} & \text{TV} \end{array} \\ \begin{bmatrix} 3 & 5 \\ \frac{1}{2} & 1 \end{bmatrix} \end{array} \begin{array}{l} \\ \text{Assembly} \\ \text{Packaging} \end{array}$$

b. $S = \begin{bmatrix} 30 \\ 20 \end{bmatrix} \begin{array}{l} \text{DVDs} \\ \text{TVs} \end{array}$

$TS = \begin{bmatrix} 190 \\ 35 \end{bmatrix}$

190 hours of assembly, 35 hours of packaging.

79. a.

$$T = \begin{array}{c} \begin{array}{cc} \text{Boston Cream Pie} & \text{Carrot Cake} \end{array} \\ \begin{bmatrix} 30 & 45 \\ 30 & 50 \\ 15 & 10 \end{bmatrix} \end{array} \begin{array}{l} \\ \text{Preparation} \\ \text{Baking} \\ \text{Finishing} \end{array}$$

b. $S = \begin{bmatrix} 20 \\ 8 \end{bmatrix} \begin{array}{l} \text{Boston Cream Pie} \\ \text{Carrot Cake} \end{array}$

$TS = \begin{bmatrix} 960 \\ 1000 \\ 380 \end{bmatrix} \begin{array}{l} \text{Preparation} \\ \text{Baking} \\ \text{Finishing} \end{array}$

c. Total baking time is 1000 minutes or $16\frac{2}{3}$ hours. Total finishing time is 380 minutes or $6\frac{1}{3}$ hours.

80. a.

$$T = \begin{array}{c} \begin{array}{ccc} \text{Preparation} & \text{Lacquering} & \text{Drying} \end{array} \\ \begin{bmatrix} 20 & 5 & 15 \\ 30 & 5 & 20 \end{bmatrix} \end{array} \begin{array}{l} \\ \text{Manicure} \\ \text{Pedicure} \end{array}$$

b.
$$S = \begin{array}{cc} \text{Manicure} & \text{Pedicure} \\ \left[\ 15\right. & \left. 9\ \right] \end{array}$$

$$ST = \begin{array}{ccc} \text{Preparation} & \text{Lacquering} & \text{Drying} \\ \left[\ 570\right. & 120 & \left. 405\ \right] \end{array}$$

c. Total drying time is 405 minutes or $6\frac{3}{4}$ hours.

81. a.
$$T = \begin{array}{c} \begin{array}{ccc} \text{Cutting} & \text{Sewing} & \text{Finishing} \end{array} \\ \begin{bmatrix} 2 & 3 & 2 \\ 1.5 & 2 & 1 \end{bmatrix} \end{array} \begin{array}{l} \text{Huge One} \\ \text{Regular Joe} \end{array}$$

b. $S = \begin{bmatrix} 32 \\ 24 \end{bmatrix} \begin{array}{l} \text{Huge One} \\ \text{Regular Joe} \end{array}$

c.
$$A = \begin{array}{cc} \text{Huge One} & \text{Regular Joe} \\ \left[\ 27\right. & \left. 56\ \right] \end{array}$$

$$AT = \begin{array}{ccc} \text{Cutting} & \text{Sewing} & \text{Finishing} \\ \left[\ 138\right. & 193 & \left. 110\ \right] \end{array}$$

$$AS = [2208]$$

d. 193 hours are needed for sewing.

e. The total revenue would be \$2208.

82. a. $BC = \begin{bmatrix} 40 & 75 & 150 \end{bmatrix} \begin{bmatrix} 25 \\ 16 \\ 32 \end{bmatrix} = [7000]$; The total revenue for these three MP3 players for one week is \$7000.

b. $AC = \begin{bmatrix} 4 & 8 & 16 \\ 25 & 30 & 30 \\ 1 & 1.9 & 1.1 \end{bmatrix} \begin{bmatrix} 25 \\ 16 \\ 32 \end{bmatrix} = \begin{bmatrix} 740 \\ 2065 \\ 90.6 \end{bmatrix}$; The total storage is 740 GB, the total battery life is 2065 hours, and the total weight is 90.6 ounces.

c. 2065; The total battery life for the MP3 players sold is 2065 hours.

83. answers will vary.

84. $AB = \begin{bmatrix} 3a-2 & 3b+6 & 0 \\ a-1 & b+3 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$a = 1$, $b = -2$

85. $(A+B)-A = \begin{bmatrix} 3 & -2 & 1 \\ -5 & 6 & 7 \end{bmatrix}$

86. $A-(A-B) = \begin{bmatrix} 5 & 4 & -3 \\ 0 & -1 & 2 \end{bmatrix}$

87. 4×4

88. 3×3

89. The matrix product:

$$\begin{bmatrix} 9257 & 57,718 & 89,389 \end{bmatrix}\begin{bmatrix} 13.9 \\ 14.9 \\ 14.2 \end{bmatrix}$$ gives the total number of pupils in the three states.

90. The matrix product

$$\begin{bmatrix} 155,959 & 95,997 & 66,544 \end{bmatrix}\begin{bmatrix} 250.0 \\ 42.0 \\ 107.8 \end{bmatrix}$$ gives the total population of the three states.

91. $A+B=\begin{bmatrix} 6.4 & -2 & -2.7 \\ 20.5 & 22.5 & -2.4 \\ -14 & 17.6 & 16 \end{bmatrix}$

92. $B-A=\begin{bmatrix} 5.6 & -16 & 3.3 \\ -17.5 & 21.5 & -5.6 \\ 4 & -4.4 & 12 \end{bmatrix}$

93. $BA=\begin{bmatrix} -171.3 & 40.8 & -31.8 \\ 454.6 & -22.5 & 22.7 \\ -2.6 & 122.3 & 53.56 \end{bmatrix}$

94. $AB=\begin{bmatrix} 27.9 & 130.6 & -69.88 \\ 106.75 & -149.44 & 26.1 \\ -47.5 & 336.2 & -18.7 \end{bmatrix}$

95. $3A=\begin{bmatrix} 1.2 & 21 & -9 \\ 57 & 1.5 & 4.8 \\ -27 & 33 & 6 \end{bmatrix}$

96. $I_3+2A=\begin{bmatrix} 1.8 & 14 & -6 \\ 38 & 2 & 3.2 \\ -18 & 22 & 5 \end{bmatrix}$

97. Answer may vary. One possibility is with the message ERR:DIM MISMATCH

98. Answers may vary. One possibility is with the message ERR:DIM MISMATCH

Exercises 2.4

1. $\begin{bmatrix} 1 & -2 \\ -\frac{1}{2} & 2 \end{bmatrix}\begin{bmatrix} 4 \\ 1 \end{bmatrix}=\begin{bmatrix} 2 \\ 0 \end{bmatrix}$

$x = 2, y = 0$

2. $\begin{bmatrix} 1 & -2 \\ -\frac{1}{2} & 2 \end{bmatrix}\begin{bmatrix} 14 \\ 4 \end{bmatrix}=\begin{bmatrix} 6 \\ 1 \end{bmatrix}$

$x = 6, y = 1$

3. $D=7\cdot 1-3\cdot 2=1$

$\begin{bmatrix} \frac{1}{1} & -\frac{2}{1} \\ -\frac{3}{1} & \frac{7}{1} \end{bmatrix}=\begin{bmatrix} 1 & -2 \\ -3 & 7 \end{bmatrix}$

4. $D=2\cdot 7-5\cdot 3=-1$

$\begin{bmatrix} -\frac{7}{1} & -\frac{3}{-1} \\ -\frac{5}{-1} & -\frac{2}{1} \end{bmatrix}=\begin{bmatrix} -7 & 3 \\ 5 & -2 \end{bmatrix}$

5. $D=6\cdot 2-5\cdot 2=2$

$\begin{bmatrix} \frac{2}{2} & -\frac{2}{2} \\ -\frac{5}{2} & \frac{6}{2} \end{bmatrix}=\begin{bmatrix} 1 & -1 \\ -\frac{5}{2} & 3 \end{bmatrix}$

6. $D=1\cdot .5-0\cdot 5=.5$

$\begin{bmatrix} \frac{.5}{.5} & -\frac{.5}{.5} \\ -\frac{0}{.5} & \frac{1}{.5} \end{bmatrix}=\begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix}$

7. $D=0.7\cdot 0.8-0.3\cdot 0.2=0.5$

$\begin{bmatrix} \frac{.8}{.5} & -\frac{.2}{.5} \\ -\frac{.3}{.5} & \frac{.7}{.5} \end{bmatrix}=\begin{bmatrix} 1.6 & -.4 \\ -.6 & 1.4 \end{bmatrix}$

8. $D=0\cdot 0-1\cdot 1=-1$

$\begin{bmatrix} \frac{0}{-1} & -\frac{1}{-1} \\ -\frac{1}{-1} & \frac{0}{-1} \end{bmatrix}=\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

9. For a 1×1 matrix $[a]$ $(a\neq 0)$, $[a]^{-1}=\begin{bmatrix} \frac{1}{a} \end{bmatrix}$.

$\begin{bmatrix} \frac{1}{3} \end{bmatrix}$

10. For 1×1 matrix $[a]$ $(a \neq 0)$, $[a]^{-1} = \left[\frac{1}{a}\right]$.

[5]

11. $\begin{bmatrix} 1 & 2 \\ 2 & 6 \end{bmatrix}^{-1} \begin{bmatrix} 3 \\ 5 \end{bmatrix} = \begin{bmatrix} 3 & -1 \\ -1 & \frac{1}{2} \end{bmatrix} \begin{bmatrix} 3 \\ 5 \end{bmatrix} = \begin{bmatrix} 4 \\ -\frac{1}{2} \end{bmatrix}$

$x = 4,\ y = -\frac{1}{2}$

12. $\begin{bmatrix} 5 & 3 \\ 7 & 4 \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} -4 & 3 \\ 7 & -5 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \end{bmatrix}$

$x = 2, y = -3$

13. $\begin{bmatrix} \frac{1}{2} & 2 \\ 3 & 16 \end{bmatrix}^{-1} \begin{bmatrix} 4 \\ 0 \end{bmatrix} = \begin{bmatrix} 8 & -1 \\ -\frac{3}{2} & \frac{1}{4} \end{bmatrix} \begin{bmatrix} 4 \\ 0 \end{bmatrix} = \begin{bmatrix} 32 \\ -6 \end{bmatrix}$

$x = 32, y = -6$

14. $\begin{bmatrix} 0.8 & 0.6 \\ 0.2 & 0.4 \end{bmatrix}^{-1} \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 & -3 \\ -1 & 4 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

$x = 1, y = 2$

15. a. $\begin{bmatrix} 0.8 & 0.3 \\ 0.2 & 0.7 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} m \\ s \end{bmatrix}$

b. $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0.8 & 0.3 \\ 0.2 & 0.7 \end{bmatrix}^{-1} \begin{bmatrix} m \\ s \end{bmatrix}$

$= \begin{bmatrix} 1.4 & -0.6 \\ -0.4 & 1.6 \end{bmatrix} \begin{bmatrix} m \\ s \end{bmatrix}$

c. $\begin{bmatrix} 1.4 & -0.6 \\ -0.4 & 1.6 \end{bmatrix} \begin{bmatrix} 100{,}000 \\ 50{,}000 \end{bmatrix} = \begin{bmatrix} 110{,}000 \\ 40{,}000 \end{bmatrix}$

110,000 married; 40,000 single

d. $\begin{bmatrix} 1.4 & -0.6 \\ -0.4 & 1.6 \end{bmatrix} \begin{bmatrix} 110{,}000 \\ 40{,}000 \end{bmatrix} = \begin{bmatrix} 130{,}000 \\ 20{,}000 \end{bmatrix}$

130,000 married; 20,000 single

16. a. $\begin{bmatrix} \frac{1}{3} & \frac{1}{4} \\ \frac{2}{3} & \frac{3}{4} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} s \\ w \end{bmatrix}$

b. $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & \frac{1}{4} \\ \frac{2}{3} & \frac{3}{4} \end{bmatrix}^{-1} \begin{bmatrix} s \\ w \end{bmatrix} = \begin{bmatrix} 9 & -3 \\ -8 & 4 \end{bmatrix} \begin{bmatrix} s \\ w \end{bmatrix}$

c. 48,000 – 13,000 = 35,000 well people

$\begin{bmatrix} 9 & -3 \\ -8 & 4 \end{bmatrix} \begin{bmatrix} 13{,}000 \\ 35{,}000 \end{bmatrix} = \begin{bmatrix} 12{,}000 \\ 36{,}000 \end{bmatrix}$

12,000

d. $\begin{bmatrix} 9 & -3 \\ -8 & 4 \end{bmatrix} \begin{bmatrix} 14{,}000 \\ 34{,}000 \end{bmatrix} = \begin{bmatrix} 24{,}000 \\ 24{,}000 \end{bmatrix}$

24,000

17. a. $\begin{bmatrix} 0.7 & 0.1 \\ 0.3 & 0.9 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} u \\ v \end{bmatrix}$

b. $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0.7 & 0.1 \\ 0.3 & 0.9 \end{bmatrix}^{-1} \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} \frac{3}{2} & -\frac{1}{6} \\ -\frac{1}{2} & \frac{7}{6} \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix}$

c. $\begin{bmatrix} \frac{3}{2} & -\frac{1}{6} \\ -\frac{1}{2} & \frac{7}{6} \end{bmatrix} \begin{bmatrix} 6000 \\ 3000 \end{bmatrix} = \begin{bmatrix} 8500 \\ 500 \end{bmatrix}$

$\begin{bmatrix} 0.7 & 0.1 \\ 0.3 & 0.9 \end{bmatrix} \begin{bmatrix} 6000 \\ 3000 \end{bmatrix} = \begin{bmatrix} 4500 \\ 4500 \end{bmatrix}$

8500; 4500

18. a. $\begin{cases} 0.8x + 0.5y = u \\ 0.2x + 0.5y = v \end{cases}$

$\begin{bmatrix} 0.8 & 0.5 \\ 0.2 & 0.5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} u \\ v \end{bmatrix}$

b. $\begin{bmatrix} 0.8 & 0.5 \\ 0.2 & 0.5 \end{bmatrix} \begin{bmatrix} 25 \\ 8 \end{bmatrix} = \begin{bmatrix} 24 \\ 9 \end{bmatrix}$

$\begin{bmatrix} 0.8 & 0.5 \\ 0.2 & 0.5 \end{bmatrix}^{-1} \begin{bmatrix} 25 \\ 8 \end{bmatrix} = \begin{bmatrix} \frac{5}{3} & -\frac{5}{3} \\ -\frac{2}{3} & \frac{8}{3} \end{bmatrix} \begin{bmatrix} 25 \\ 8 \end{bmatrix} = \begin{bmatrix} 28\frac{1}{3} \\ 4\frac{2}{3} \end{bmatrix}$

24; 28

19. $\begin{bmatrix} 5 & -2 & -2 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix} = \begin{bmatrix} 9 \\ -2 \\ -2 \end{bmatrix}$

$x = 9, y = -2, z = -2$

20. $\begin{bmatrix} 5 & -2 & -2 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 5 \\ -1 \\ -1 \end{bmatrix}$

$x = 5, y = -1, z = -1$

21. $\begin{bmatrix}1 & 2 & 2\\1 & 3 & 2\\1 & 2 & 3\end{bmatrix}\begin{bmatrix}3\\4\\5\end{bmatrix}=\begin{bmatrix}21\\25\\26\end{bmatrix}$

$x = 21, y = 25, z = 26$

22. $\begin{bmatrix}1 & 2 & 2\\1 & 3 & 2\\1 & 2 & 3\end{bmatrix}\begin{bmatrix}0\\1\\2\end{bmatrix}=\begin{bmatrix}6\\7\\8\end{bmatrix}$

$x = 6, y = 7, z = 8$

23. $\begin{bmatrix}1 & 0 & -2 & 0\\0 & 1 & 0 & -5\\-4 & 0 & 9 & 0\\0 & 2 & 1 & -9\end{bmatrix}\begin{bmatrix}1\\0\\0\\-1\end{bmatrix}=\begin{bmatrix}1\\5\\-4\\9\end{bmatrix}$

$x = 1, y = 5, z = -4, w = 9$

24. $\begin{bmatrix}1 & 0 & -2 & 0\\0 & 1 & 0 & -5\\-4 & 0 & 9 & 0\\0 & 2 & 1 & 9\end{bmatrix}\begin{bmatrix}2\\1\\3\\0\end{bmatrix}=\begin{bmatrix}-4\\1\\19\\5\end{bmatrix}$

$x = -4, y = 1, z = 19, w = 5$

25. $\begin{bmatrix}9 & 0 & 2 & 0\\-20 & -9 & -5 & 5\\4 & 0 & 1 & 0\\-4 & -2 & -1 & 1\end{bmatrix}\begin{bmatrix}0\\1\\2\\0\end{bmatrix}=\begin{bmatrix}4\\-19\\2\\-4\end{bmatrix}$

$x = 4, y = -19, z = 2, w = -4$

26. $\begin{bmatrix}9 & 0 & 2 & 0\\-20 & -9 & -5 & 5\\4 & 0 & 1 & 0\\-4 & -2 & -1 & 1\end{bmatrix}\begin{bmatrix}-1\\0\\0\\1\end{bmatrix}=\begin{bmatrix}-9\\25\\-4\\5\end{bmatrix}$

$x = -9, y = 25, z = -4, w = 5$

27. $D = a\cdot b - 0\cdot 0 = ab$

$$A^{-1}=\frac{1}{D}\begin{bmatrix}b & 0\\0 & a\end{bmatrix}=\begin{bmatrix}\frac{b}{ab} & -\frac{0}{ab}\\-\frac{0}{ab} & \frac{a}{ab}\end{bmatrix}=\begin{bmatrix}\frac{1}{a} & 0\\0 & \frac{1}{b}\end{bmatrix}$$

28. True; Once one of the inverse equations is confirmed, the other equation will also be satisfied.

29. a. $\begin{cases}x+2y=a\\ .9x=b\end{cases}$

$\begin{bmatrix}1 & 2\\.9 & 0\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}a\\b\end{bmatrix}$

b. $\begin{bmatrix}1 & 2\\.9 & 0\end{bmatrix}\begin{bmatrix}450,000\\360,000\end{bmatrix}=\begin{bmatrix}1,170,000\\405,000\end{bmatrix}$

After 1 year:
1,170,000 in group I,
405,000 in group II

$\begin{bmatrix}1 & 2\\.9 & 0\end{bmatrix}\begin{bmatrix}1,170,000\\405,000\end{bmatrix}=\begin{bmatrix}1,980,000\\1,053,000\end{bmatrix}$

After 2 years:
1,980,000 in group I,
1,053,000 in group II

c. $\begin{bmatrix}1 & 2\\.9 & 0\end{bmatrix}^{-1}\begin{bmatrix}810,000\\630,000\end{bmatrix}=\begin{bmatrix}700,000\\55,000\end{bmatrix}$

700,000 in group I, 55,000 in group II

30. $A^2A = A^3$, so $(A^2)^{-1}A^3 = A$.

$$\begin{bmatrix}-2 & -1\\2 & -1\end{bmatrix}^{-1}\begin{bmatrix}-2 & 1\\-2 & -3\end{bmatrix}=\begin{bmatrix}-\frac{1}{4} & \frac{1}{4}\\-\frac{1}{2} & -\frac{1}{2}\end{bmatrix}\begin{bmatrix}-2 & 1\\-2 & -3\end{bmatrix}$$
$$=\begin{bmatrix}0 & -1\\2 & 1\end{bmatrix}$$

31. If AB = 0 (zero matrix) and A has an inverse the B is a matrix of all zeros:
Proof : Assume AB = 0 and A has an inverse.

$A^{-1}(AB)=A^{-1}(0)\rightarrow$

$(A^{-1}A)B=A^{-1}(0)\rightarrow$

$(I_n)B=0\rightarrow$

$B=0$

32. If A = $\begin{bmatrix}3 & 1\\5 & 2\end{bmatrix}$ $B=\begin{bmatrix}6 & 2\\5 & 2\end{bmatrix}$ show that

$(AB)^{-1}=B^{-1}A^{-1}$

$$AB=\begin{bmatrix}3 & 1\\5 & 2\end{bmatrix}\begin{bmatrix}6 & 2\\5 & 2\end{bmatrix}=\begin{bmatrix}23 & 8\\40 & 14\end{bmatrix}$$

$$(AB)^{-1}=\begin{bmatrix}\frac{14}{2} & \frac{-8}{2}\\\frac{-40}{2} & \frac{23}{2}\end{bmatrix}=\begin{bmatrix}7 & -4\\-20 & \frac{23}{2}\end{bmatrix}$$

$$B^{-1}=\begin{bmatrix}\frac{2}{2} & \frac{-2}{2}\\ \frac{-5}{2} & \frac{6}{2}\end{bmatrix}=\begin{bmatrix}1 & -1\\ -\frac{5}{2} & 3\end{bmatrix}$$

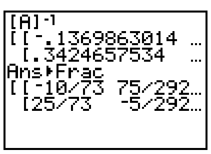
$$A^{-1}=\begin{bmatrix}2 & -1\\ -5 & 3\end{bmatrix}$$

$$B^{-1}A^{-1}=\begin{bmatrix}1 & -1\\ -\frac{5}{2} & 3\end{bmatrix}\begin{bmatrix}2 & -1\\ -5 & 3\end{bmatrix}=\begin{bmatrix}7 & -4\\ -20 & \frac{23}{2}\end{bmatrix}=(AB)^{-1}$$

33. One example:
$AX=B$
Let $A=\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix}$ $X=\begin{bmatrix}x\\ y\end{bmatrix}$ $B=\begin{bmatrix}2\\ 3\end{bmatrix}$
The system $\begin{cases}x+y=2\\ x+y=3\end{cases}$ has no solution.

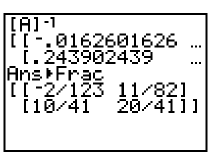
34. One example:
$AX=B$
Let $A=\begin{bmatrix}2 & 3\\ 4 & 6\end{bmatrix}$ $X=\begin{bmatrix}x\\ y\end{bmatrix}$ $B=\begin{bmatrix}6\\ 12\end{bmatrix}$
The system $\begin{cases}2x+3y=6\\ 4x+6y=12\end{cases}$ has infinitely many solutions.

35



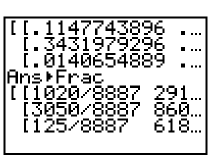
$$\begin{bmatrix}-\frac{10}{73} & \frac{75}{292}\\ \frac{25}{73} & -\frac{5}{292}\end{bmatrix}$$

36.



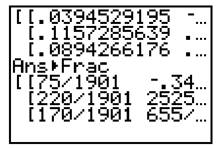
$$\begin{bmatrix}-\frac{2}{123} & \frac{11}{82}\\ \frac{10}{41} & \frac{20}{41}\end{bmatrix}$$

37.



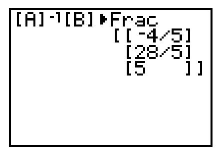
$$\begin{bmatrix}\frac{1020}{8887} & \frac{2910}{8887} & -\frac{500}{8887}\\ \frac{3050}{8887} & \frac{860}{8887} & \frac{1990}{8887}\\ \frac{125}{8887} & \frac{618}{8887} & \frac{810}{8887}\end{bmatrix}$$

38.



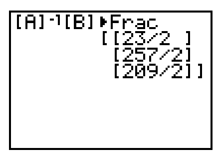
$$\begin{bmatrix}\frac{75}{1901} & -.3414\ldots & -\frac{2305}{5703}\\ \frac{220}{1901} & \frac{2525}{5703} & \frac{2110}{5703}\\ \frac{170}{1901} & \frac{655}{5703} & \frac{1112}{5703}\end{bmatrix}$$

39.



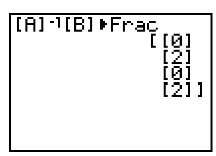
$x=-\frac{4}{5},\ y=\frac{28}{5},\ z=5$

40.



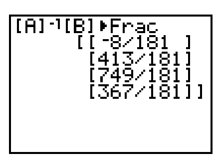
$x=\frac{23}{2},\ y=\frac{257}{2},\ z=\frac{209}{2}$

41.



$x=0,\ y=2,\ z=0,\ w=2$

42.



$x=-\frac{8}{181},\ y=\frac{413}{181},\ z=\frac{749}{181},\ w=\frac{367}{181}$

43. Answer may vary. One possibility is with the message ERR:INVALID DIM

Exercises 2.5

1. $\left[\begin{array}{cc|cc} \underline{7} & 3 & 1 & 0 \\ 5 & 2 & 0 & 1 \end{array}\right]$

$\left[\begin{array}{cc|cc} 1 & \frac{3}{7} & \frac{1}{7} & 0 \\ 0 & \underline{-\frac{1}{7}} & -\frac{5}{7} & 1 \end{array}\right]$

$\left[\begin{array}{cc|cc} 1 & 0 & -2 & 3 \\ 0 & 1 & 5 & -7 \end{array}\right]$

$\left[\begin{array}{cc} -2 & 3 \\ 5 & -7 \end{array}\right]$

2. $\left[\begin{array}{cc|cc} \underline{5} & -2 & 1 & 0 \\ 6 & 2 & 0 & 1 \end{array}\right]$

$\left[\begin{array}{cc|cc} 1 & -\frac{2}{5} & \frac{1}{5} & 0 \\ 0 & \underline{\frac{22}{5}} & -\frac{6}{5} & 1 \end{array}\right]$

$\left[\begin{array}{cc|cc} 1 & 0 & \frac{1}{11} & \frac{1}{11} \\ 0 & 1 & -\frac{3}{11} & \frac{5}{22} \end{array}\right]$

$\left[\begin{array}{cc} \frac{1}{11} & \frac{1}{11} \\ -\frac{3}{11} & \frac{5}{22} \end{array}\right]$

3. $\left[\begin{array}{cc|cc} \underline{2} & 3 & 1 & 0 \\ -4 & -7 & 0 & 1 \end{array}\right]$

$\left[\begin{array}{cc|cc} 1 & \frac{3}{2} & \frac{1}{2} & 0 \\ 0 & \underline{-1} & 2 & 1 \end{array}\right]$

$\left[\begin{array}{cc|cc} 1 & 0 & \frac{7}{2} & \frac{3}{2} \\ 0 & 1 & -2 & -1 \end{array}\right]$

$\left[\begin{array}{cc} \frac{7}{2} & \frac{3}{2} \\ -2 & -1 \end{array}\right]$

4. $\left[\begin{array}{cc|cc} \underline{1} & -3 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{array}\right]$

$\left[\begin{array}{cc|cc} 1 & -3 & 1 & 0 \\ 0 & \underline{1} & 0 & 1 \end{array}\right]$

$\left[\begin{array}{cc|cc} 1 & 0 & 1 & 3 \\ 0 & 1 & 0 & 1 \end{array}\right]$

$\left[\begin{array}{cc} 1 & 3 \\ 0 & 1 \end{array}\right]$

5. $\left[\begin{array}{cc|cc} \underline{2} & -4 & 1 & 0 \\ -1 & 2 & 0 & 1 \end{array}\right]$

$\left[\begin{array}{cc|cc} 1 & -2 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & 1 \end{array}\right]$

No inverse

6. $\left[\begin{array}{ccc|ccc} \underline{1} & 3 & 1 & 1 & 0 & 0 \\ -1 & 2 & 0 & 0 & 1 & 0 \\ 2 & 11 & 3 & 0 & 0 & 1 \end{array}\right]$

$\left[\begin{array}{ccc|ccc} 1 & 3 & 1 & 1 & 0 & 0 \\ 0 & \underline{5} & 1 & 1 & 1 & 0 \\ 0 & 5 & 1 & -2 & 0 & 1 \end{array}\right]$

$\left[\begin{array}{ccc|ccc} 1 & 0 & \frac{2}{5} & \frac{2}{5} & -\frac{3}{5} & 0 \\ 0 & 1 & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & 0 \\ 0 & 0 & 0 & -3 & -1 & 1 \end{array}\right]$

No inverse

7. $\left[\begin{array}{ccc|ccc} \underline{1} & 2 & -2 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$

$\left[\begin{array}{ccc|ccc} 1 & 2 & -2 & 1 & 0 & 0 \\ 0 & \underline{-1} & 3 & -1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$

$\left[\begin{array}{ccc|ccc} 1 & 0 & 4 & -1 & 2 & 0 \\ 0 & 1 & -3 & 1 & -1 & 0 \\ 0 & 0 & \underline{1} & 0 & 0 & 1 \end{array}\right]$

$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & 2 & -4 \\ 0 & 1 & 0 & 1 & -1 & 3 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$

$\left[\begin{array}{ccc} -1 & 2 & -4 \\ 1 & -1 & 3 \\ 0 & 0 & 1 \end{array}\right]$

8. $\left[\begin{array}{ccc|ccc} \underline{2} & 2 & 0 & 1 & 0 & 0 \\ 0 & -2 & 0 & 0 & 1 & 0 \\ 3 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$

$\left[\begin{array}{ccc|ccc} 1 & 1 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & \underline{-2} & 0 & 0 & 1 & 0 \\ 0 & -3 & 1 & -\frac{3}{2} & 0 & 1 \end{array}\right]$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 1 & 0 & 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 1 & -\frac{3}{2} & -\frac{3}{2} & 1 \end{array}\right]$$

$$\begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & -\frac{1}{2} & 0 \\ -\frac{3}{2} & -\frac{3}{2} & 1 \end{bmatrix}$$

9. $$\left[\begin{array}{ccc|ccc} \underline{-2} & 5 & 2 & 1 & 0 & 0 \\ 1 & -3 & -1 & 0 & 1 & 0 \\ -1 & 2 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & -\frac{5}{2} & -1 & -\frac{1}{2} & 0 & 0 \\ 0 & \underline{-\frac{1}{2}} & 0 & \frac{1}{2} & 1 & 0 \\ 0 & -\frac{1}{2} & 0 & -\frac{1}{2} & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -1 & -3 & -5 & 0 \\ 0 & 1 & 0 & -1 & -2 & 0 \\ 0 & 0 & 0 & -1 & -1 & 1 \end{array}\right]$$

No inverse

10. $$\left[\begin{array}{ccc|ccc} \underline{1} & 0 & 0 & 1 & 0 & 0 \\ 2 & 1 & -2 & 0 & 1 & 0 \\ -1 & 2 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & \underline{1} & -2 & -2 & 1 & 0 \\ 0 & 2 & 1 & 1 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & -2 & -2 & 1 & 0 \\ 0 & 0 & \underline{5} & 5 & -2 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & \frac{1}{5} & \frac{2}{5} \\ 0 & 0 & 1 & 1 & -\frac{2}{5} & \frac{1}{5} \end{array}\right]$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{5} & \frac{2}{5} \\ 1 & -\frac{2}{5} & \frac{1}{5} \end{bmatrix}$$

11. $$\left[\begin{array}{cccc|cccc} \underline{1} & 6 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 5 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 4 & 2 & 0 & 0 & 1 & 0 \\ 0 & 0 & 50 & 2 & 0 & 0 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{cccc|cccc} 1 & 6 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & \underline{-1} & 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 4 & 2 & 0 & 0 & 1 & 0 \\ 0 & 0 & 50 & 2 & 0 & 0 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & -5 & 6 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & \underline{4} & 2 & 0 & 0 & 1 & 0 \\ 0 & 0 & 50 & 2 & 0 & 0 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & -5 & 6 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & \frac{1}{2} & 0 & 0 & \frac{1}{4} & 0 \\ 0 & 0 & 0 & \underline{-23} & 0 & 0 & -\frac{25}{2} & 1 \end{array}\right]$$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & -5 & 6 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & -\frac{1}{46} & \frac{1}{46} \\ 0 & 0 & 0 & 1 & 0 & 0 & \frac{25}{46} & -\frac{1}{23} \end{array}\right]$$

$$\begin{bmatrix} -5 & 6 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & -\frac{1}{46} & \frac{1}{46} \\ 0 & 0 & \frac{25}{46} & -\frac{1}{23} \end{bmatrix}$$

12. $$\left[\begin{array}{cccc|cccc} \underline{6} & 0 & 2 & 0 & 1 & 0 & 0 & 0 \\ -6 & 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ -9 & 0 & -1 & 1 & 0 & 0 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & \frac{1}{3} & 0 & \frac{1}{6} & 0 & 0 & 0 \\ 0 & 1 & 2 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & \underline{\frac{2}{3}} & 0 & -\frac{1}{6} & 0 & 1 & 0 \\ 0 & 0 & 2 & 1 & \frac{3}{2} & 0 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & \frac{1}{4} & 0 & -\frac{1}{2} & 0 \\ 0 & 1 & 0 & 1 & \frac{3}{2} & 1 & -3 & 0 \\ 0 & 0 & 1 & 0 & -\frac{1}{4} & 0 & \frac{3}{2} & 0 \\ 0 & 0 & 0 & \underline{1} & 2 & 0 & -3 & 1 \end{array}\right]$$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & \frac{1}{4} & 0 & -\frac{1}{2} & 0 \\ 0 & 1 & 0 & 0 & -\frac{1}{2} & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 & -\frac{1}{4} & 0 & \frac{3}{2} & 0 \\ 0 & 0 & 0 & 1 & 2 & 0 & -3 & 1 \end{array}\right]$$

$$\begin{bmatrix} \frac{1}{4} & 0 & -\frac{1}{2} & 0 \\ -\frac{1}{2} & 1 & 0 & -1 \\ -\frac{1}{4} & 0 & \frac{3}{2} & 0 \\ 2 & 0 & -3 & 1 \end{bmatrix}$$

13. Find the inverse of $\begin{bmatrix} 1 & 1 & 2 \\ 3 & 2 & 2 \\ 1 & 1 & 3 \end{bmatrix}$.

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 2 & 1 & 0 & 0 \\ 3 & 2 & 2 & 0 & 1 & 0 \\ 1 & 1 & 3 & 0 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 2 & 1 & 0 & 0 \\ 0 & -1 & -4 & -3 & 1 & 0 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -2 & -2 & 1 & 0 \\ 0 & 1 & 4 & 3 & -1 & 0 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -4 & 1 & 2 \\ 0 & 1 & 0 & 7 & -1 & -4 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array}\right]$$

$$\begin{bmatrix} 1 & 1 & 2 \\ 3 & 2 & 2 \\ 1 & 1 & 3 \end{bmatrix}^{-1} \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix} = \begin{bmatrix} -4 & 1 & 2 \\ 7 & -1 & -4 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \\ 2 \end{bmatrix}$$

$x = 2, y = -3, z = 2$

14. $\left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 3 & 5 & 5 & 0 & 1 & 0 \\ 2 & 4 & 2 & 0 & 0 & 1 \end{array}\right]$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & -1 & -4 & -3 & 1 & 0 \\ 0 & 0 & -4 & -2 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -5 & -5 & 2 & 0 \\ 0 & 1 & 4 & 3 & -1 & 0 \\ 0 & 0 & -4 & -2 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -\frac{5}{2} & 2 & -\frac{5}{4} \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & \frac{1}{2} & 0 & -\frac{1}{4} \end{array}\right]$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 3 & 5 & 5 \\ 2 & 4 & 2 \end{bmatrix}^{-1} \begin{bmatrix} 4 \\ 3 \\ 4 \end{bmatrix} = \begin{bmatrix} -\frac{5}{2} & 2 & -\frac{5}{4} \\ 1 & -1 & 1 \\ \frac{1}{2} & 0 & -\frac{1}{4} \end{bmatrix} \begin{bmatrix} 4 \\ 3 \\ 4 \end{bmatrix}$$

$$= \begin{bmatrix} -9 \\ 5 \\ 1 \end{bmatrix}$$

$x = -9, y = 5, z = 1$

15. Find the inverse of $\begin{bmatrix} 1 & 4 & 3 \\ 1 & 3 & 4 \\ 2 & 3 & 3 \end{bmatrix}$.

$$\left[\begin{array}{ccc|ccc} 1 & 4 & 3 & 1 & 0 & 0 \\ 1 & 3 & 4 & 0 & 1 & 0 \\ 2 & 3 & 3 & 0 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 4 & 3 & 1 & 0 & 0 \\ 0 & -1 & 1 & -1 & 1 & 0 \\ 0 & -5 & -3 & -2 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 7 & -3 & 4 & 0 \\ 0 & 1 & -1 & 1 & -1 & 0 \\ 0 & 0 & -8 & 3 & -5 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -\frac{3}{8} & -\frac{3}{8} & \frac{7}{8} \\ 0 & 1 & 0 & \frac{5}{8} & -\frac{3}{8} & -\frac{1}{8} \\ 0 & 0 & 1 & -\frac{3}{8} & \frac{5}{8} & -\frac{1}{8} \end{array}\right]$$

$$\begin{bmatrix}1&4&3\\1&3&4\\2&3&3\end{bmatrix}^{-1}\begin{bmatrix}15\\17\\16\end{bmatrix}=\begin{bmatrix}-\frac{3}{8}&-\frac{3}{8}&\frac{7}{8}\\\frac{5}{8}&-\frac{3}{8}&-\frac{1}{8}\\-\frac{3}{8}&\frac{5}{8}&-\frac{1}{8}\end{bmatrix}\begin{bmatrix}15\\17\\16\end{bmatrix}=\begin{bmatrix}2\\1\\3\end{bmatrix}$$

$x = 2, y = 1, z = 3$

16. $\left[\begin{array}{ccc|ccc}0&1&2&1&0&0\\2&1&3&0&1&0\\1&1&2&0&0&1\end{array}\right]$

$$\left[\begin{array}{ccc|ccc}1&1&2&0&0&1\\2&1&3&0&1&0\\0&1&2&1&0&0\end{array}\right]$$

$$\left[\begin{array}{ccc|ccc}1&1&2&0&0&1\\0&-1&-1&0&1&-2\\0&1&2&1&0&0\end{array}\right]$$

$$\left[\begin{array}{ccc|ccc}1&0&1&0&1&-1\\0&1&1&0&-1&2\\0&0&1&1&1&-2\end{array}\right]$$

$$\left[\begin{array}{ccc|ccc}1&0&0&-1&0&1\\0&1&0&-1&-2&4\\0&0&1&1&1&-2\end{array}\right]$$

$$\begin{bmatrix}0&1&2\\2&1&3\\1&1&2\end{bmatrix}^{-1}\begin{bmatrix}1\\2\\3\end{bmatrix}=\begin{bmatrix}-1&0&1\\-1&-2&4\\1&1&-2\end{bmatrix}\begin{bmatrix}1\\2\\3\end{bmatrix}$$

$$=\begin{bmatrix}2\\7\\-3\end{bmatrix}$$

$x = 2, y = 7, z = -3$

17. Find the inverse of $\begin{bmatrix}1&0&-2&-2\\0&1&0&-5\\-4&0&9&9\\0&2&1&-8\end{bmatrix}$.

$$\left[\begin{array}{cccc|cccc}1&0&-2&-2&1&0&0&0\\0&1&0&-5&0&1&0&0\\-4&0&9&9&0&0&1&0\\0&2&1&-8&0&0&0&1\end{array}\right]$$

$$\left[\begin{array}{cccc|cccc}1&0&-2&-2&1&0&0&0\\0&1&0&-5&0&1&0&0\\0&0&1&1&4&0&1&0\\0&2&1&-8&0&0&0&1\end{array}\right]$$

$$\left[\begin{array}{cccc|cccc}1&0&-2&-2&1&0&0&0\\0&1&0&-5&0&1&0&0\\0&0&1&1&4&0&1&0\\0&0&1&2&0&-2&0&1\end{array}\right]$$

$$\left[\begin{array}{cccc|cccc}1&0&0&0&9&0&2&0\\0&1&0&-5&0&1&0&0\\0&0&1&1&4&0&1&0\\0&0&0&1&-4&-2&-1&1\end{array}\right]$$

$$\left[\begin{array}{cccc|cccc}1&0&0&0&9&0&2&0\\0&1&0&0&-20&-9&-5&5\\0&0&1&0&8&2&2&-1\\0&0&0&1&-4&-2&-1&1\end{array}\right]$$

$$\begin{bmatrix}1&0&-2&-2\\0&1&0&-5\\-4&0&9&9\\0&2&1&-8\end{bmatrix}^{-1}\begin{bmatrix}0\\1\\2\\3\end{bmatrix}$$

$$=\begin{bmatrix}9&0&2&0\\-20&-9&-5&5\\8&2&2&-1\\-4&-2&-1&1\end{bmatrix}\begin{bmatrix}0\\1\\2\\3\end{bmatrix}$$

$$=\begin{bmatrix}4\\-4\\3\\-1\end{bmatrix}$$

$x = 4, y = -4, z = 3, w = -1$

18. Find the inverse of $\begin{bmatrix}1&2&-1&1\\2&-1&1&1\\2&0&1&2\\3&2&0&-1\end{bmatrix}$.

$$\left[\begin{array}{cccc|cccc}1&2&-1&1&1&0&0&0\\2&-1&1&1&0&1&0&0\\2&0&1&2&0&0&1&0\\3&2&0&-1&0&0&0&1\end{array}\right]$$

$$\left[\begin{array}{cccc|cccc}1&2&-1&1&1&0&0&0\\0&-5&3&-1&-2&1&0&0\\0&-4&3&0&-2&0&1&0\\0&-4&3&-4&-3&0&0&1\end{array}\right]$$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & \frac{1}{5} & \frac{3}{5} & \frac{1}{5} & \frac{2}{5} & 0 & 0 \\ 0 & 1 & -\frac{3}{5} & \frac{1}{5} & \frac{2}{5} & -\frac{1}{5} & 0 & 0 \\ 0 & 0 & \frac{3}{5} & \frac{4}{5} & -\frac{2}{5} & -\frac{4}{5} & 1 & 0 \\ 0 & 0 & \frac{3}{5} & -\frac{16}{5} & -\frac{7}{5} & -\frac{4}{5} & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & \frac{1}{3} & \frac{1}{3} & \frac{2}{3} & -\frac{1}{3} & 0 \\ 0 & 1 & 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & \frac{4}{3} & -\frac{2}{3} & -\frac{4}{3} & \frac{5}{3} & 0 \\ 0 & 0 & 0 & -4 & -1 & 0 & -1 & 1 \end{array}\right]$$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & \frac{1}{4} & \frac{2}{3} & -\frac{5}{12} & \frac{1}{12} \\ 0 & 1 & 0 & 0 & -\frac{1}{4} & -1 & \frac{3}{4} & \frac{1}{4} \\ 0 & 0 & 1 & 0 & -1 & -\frac{4}{3} & \frac{4}{3} & \frac{1}{3} \\ 0 & 0 & 0 & 1 & \frac{1}{4} & 0 & \frac{1}{4} & -\frac{1}{4} \end{array}\right]$$

$$\begin{bmatrix} 1 & 2 & -1 & 1 \\ 2 & -1 & 1 & 1 \\ 2 & 0 & 1 & 2 \\ 3 & 2 & 0 & -1 \end{bmatrix}^{-1} \begin{bmatrix} 15 \\ 12 \\ 18 \\ 21 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{4} & \frac{2}{3} & -\frac{5}{12} & \frac{1}{12} \\ -\frac{1}{4} & -1 & \frac{3}{4} & \frac{1}{4} \\ -1 & -\frac{4}{3} & \frac{4}{3} & \frac{1}{3} \\ \frac{1}{4} & 0 & \frac{1}{4} & -\frac{1}{4} \end{bmatrix} \begin{bmatrix} 15 \\ 12 \\ 18 \\ 21 \end{bmatrix}$$

$$= \begin{bmatrix} 6 \\ 3 \\ 0 \\ 3 \end{bmatrix}$$

$x = 6, y = 3, z = 0, w = 3$

19. $A \cdot \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 4 & 2 \end{bmatrix}$

$$A = A \cdot \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}^{-1}$$

$$= \begin{bmatrix} -1 & 0 \\ 4 & 2 \end{bmatrix} \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}^{-1}$$

$$= \begin{bmatrix} -1 & 0 \\ 4 & 2 \end{bmatrix} \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} -3 & 5 \\ 10 & -16 \end{bmatrix}$$

20. $\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix} \cdot A = \begin{bmatrix} -1 & 0 \\ 4 & 2 \end{bmatrix}$

$$A = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}^{-1} \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix} \cdot A$$

$$= \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}^{-1} \begin{bmatrix} -1 & 0 \\ 4 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 4 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} -23 & -10 \\ 9 & 4 \end{bmatrix}$$

21. $\begin{cases} x + y + z = 100 \\ x - 2y = 0 \\ x + y - z = 26 \end{cases}$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & -2 & 0 & 0 & 1 & 0 \\ 1 & 1 & -1 & 0 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & -3 & -1 & -1 & 1 & 0 \\ 0 & 0 & -2 & -1 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & \frac{1}{3} & \frac{1}{3} & -\frac{1}{3} & 0 \\ 0 & 0 & -2 & -1 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & \frac{2}{3} & \frac{2}{3} & \frac{1}{3} & 0 \\ 0 & 1 & \frac{1}{3} & \frac{1}{3} & -\frac{1}{3} & 0 \\ 0 & 0 & -2 & -1 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & \frac{2}{3} & \frac{2}{3} & \frac{1}{3} & 0 \\ 0 & 1 & \frac{1}{3} & \frac{1}{3} & -\frac{1}{3} & 0 \\ 0 & 0 & 1 & \frac{1}{2} & 0 & -\frac{1}{2} \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ 0 & 1 & 0 & \frac{1}{6} & -\frac{1}{3} & \frac{1}{6} \\ 0 & 0 & 1 & \frac{1}{2} & 0 & -\frac{1}{2} \end{array}\right]$$

$$\begin{bmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{6} & -\frac{1}{3} & \frac{1}{6} \\ \frac{1}{2} & 0 & -\frac{1}{2} \end{bmatrix} \begin{bmatrix} 100 \\ 0 \\ 26 \end{bmatrix} = \begin{bmatrix} 42 \\ 21 \\ 37 \end{bmatrix}$$

$x = 42, y = 21, z = 37$

22. $\begin{cases} x+y+z=100 \\ x-y-z=16 \\ x-y+z=46 \end{cases}$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & -1 & -1 & 0 & 1 & 0 \\ 1 & -1 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & -2 & -2 & -1 & 1 & 0 \\ 0 & -2 & 0 & -1 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0.5 & -0.5 & 0 \\ 0 & -2 & 0 & -1 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0.5 & 0.5 & 0 \\ 0 & 1 & 1 & 0.5 & -0.5 & 0 \\ 0 & 0 & 2 & 0 & -1 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0.5 & 0.5 & 0 \\ 0 & 1 & 1 & 0.5 & -0.5 & 0 \\ 0 & 0 & 1 & 0 & -0.5 & 0.5 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0.5 & 0.5 & 0 \\ 0 & 1 & 0 & 0.5 & 0 & -0.5 \\ 0 & 0 & 1 & 0 & -0.5 & 0.5 \end{array}\right]$$

$$\begin{bmatrix} 0.5 & 0.5 & 0 \\ 0.5 & 0 & -0.5 \\ 0 & -0.5 & 0.5 \end{bmatrix}\begin{bmatrix} 100 \\ 16 \\ 46 \end{bmatrix} = \begin{bmatrix} 58 \\ 27 \\ 15 \end{bmatrix}$$

$x = 58, y = 27, z = 15$

23. $\begin{cases} x+y+z=100 \\ x-5y+3z=0 \\ 6x-29y+z=0 \end{cases}$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & -5 & 3 & 0 & 1 & 0 \\ 6 & -29 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & -6 & 2 & -1 & 1 & 0 \\ 0 & -35 & -5 & -6 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & -\frac{1}{3} & \frac{1}{6} & -\frac{1}{6} & 0 \\ 0 & -35 & -5 & -6 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & \frac{4}{3} & \frac{5}{6} & \frac{1}{6} & 0 \\ 0 & 1 & -\frac{1}{3} & \frac{1}{6} & -\frac{1}{6} & 0 \\ 0 & 0 & -\frac{50}{3} & -\frac{1}{6} & -\frac{35}{6} & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & \frac{4}{3} & \frac{5}{6} & \frac{1}{6} & 0 \\ 0 & 1 & -\frac{1}{3} & \frac{1}{6} & -\frac{1}{6} & 0 \\ 0 & 0 & 1 & \frac{1}{100} & \frac{7}{20} & -\frac{3}{50} \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{41}{50} & -\frac{3}{10} & \frac{2}{25} \\ 0 & 1 & 0 & \frac{17}{100} & -\frac{1}{20} & -\frac{1}{50} \\ 0 & 0 & 1 & \frac{1}{100} & \frac{7}{20} & -\frac{3}{50} \end{array}\right]$$

$$\begin{bmatrix} \frac{41}{50} & -\frac{3}{10} & \frac{2}{25} \\ \frac{17}{100} & -\frac{1}{20} & -\frac{1}{50} \\ \frac{1}{100} & \frac{7}{20} & -\frac{3}{50} \end{bmatrix}\begin{bmatrix} 100 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 82 \\ 17 \\ 1 \end{bmatrix}$$

$x = 82, y = 17, z = 1$

24. $\begin{cases} x+y+z=82 \\ \quad\;\; y-2z=2 \\ x \quad\;\; -z=-8 \end{cases}$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & -2 & 0 & 1 & 0 \\ 1 & 0 & -1 & 0 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & -2 & 0 & 1 & 0 \\ 0 & -1 & -2 & -1 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 3 & 1 & -1 & 0 \\ 0 & 1 & -2 & 0 & 1 & 0 \\ 0 & 0 & -4 & -1 & 1 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 3 & 1 & -1 & 0 \\ 0 & 1 & -2 & 0 & 1 & 0 \\ 0 & 0 & 1 & \frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{1}{4} & -\frac{1}{4} & \frac{3}{4} \\ 0 & 1 & 0 & \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\ 0 & 0 & 1 & \frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} \end{array}\right]$$

$$\begin{bmatrix} \frac{1}{4} & -\frac{1}{4} & \frac{3}{4} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\ \frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} \end{bmatrix}\begin{bmatrix} 82 \\ 2 \\ -8 \end{bmatrix} = \begin{bmatrix} 14 \\ 46 \\ 22 \end{bmatrix}$$

$x = 14, y = 46, z = 22$

Exercises 2.6

1. $A_{21} = 0.2,$ 20 cents of energy are required to produce \$1 worth of manufactured goods.

2. $A_{23} = 0.15,$ 15 cents of energy are required to produce \$1 worth of services.

3. $A_{31} = 0.1, A_{32} = 0.2, A_{33} = 0.15$
   $A_{32} > A_{33} > A_{31}$
   The energy sector uses the greatest amount of services in order to produce \$1 worth of output.

4. $A_{11} = 0.3, A_{12} = 0.1, A_{13} = 0.2$
   $A_{12} < A_{13} < A_{11}$
   The energy sector uses the least amount of manufacturing in order to produce \$1 worth of output

5. $[10{,}000{,}000][0.2 \quad 0.25 \quad 0.15]$
   $= [2{,}000{,}000 \quad 2{,}500{,}000 \quad 1{,}500{,}000]$
   \$2 million for Manufacturing, \$2.5 million for Energy, \$1.5 million for Services for a total of \$6 million.

6. $[10{,}000{,}000][0.1 \quad 0.2 \quad 0.15]$
   $= [1{,}000{,}000 \quad 2{,}000{,}000 \quad 1{,}500{,}000]$
   \$1 million for Manufacturing, \$2 million for Energy, \$1.5 million for Services for a total of \$4.5 million.

7. $A_{13} = 0.2, A_{23} = 0.15, A_{33} = 0.15$
   $A_{13} > A_{23} = A_{33}$
   The services sector is most dependent on manufacturing.

8. $A_{11} = 0.3, A_{21} = 0.2, A_{31} = 0.1$
   $A_{13} < A_{21} < A_{11}$
   The manufacturing sector is least dependent on services.

9. The initial consumption matrix is

$$\begin{bmatrix} 0.3 & 0.1 & 0.2 \\ 0.2 & 0.25 & 0.15 \\ 0.1 & 0.2 & 0.15 \end{bmatrix}\begin{bmatrix} 10 \\ 20 \\ 30 \end{bmatrix} = \begin{bmatrix} 11.00 \\ 11.50 \\ 9.50 \end{bmatrix}$$

10. $$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 0.3 & 0.1 & 0.2 \\ 0.2 & 0.25 & 0.15 \\ 0.1 & 0.2 & 0.15 \end{bmatrix} = \begin{bmatrix} 0.7 & -0.1 & -0.2 \\ -0.2 & 0.75 & -0.15 \\ -0.1 & -0.2 & 0.85 \end{bmatrix}$$

11. The production matrix is

$$\begin{bmatrix} 1.58 & 0.33 & 0.43 \\ 0.48 & 1.5 & 0.38 \\ 0.3 & 0.39 & 1.32 \end{bmatrix}\begin{bmatrix} 5 \\ 6 \\ 7 \end{bmatrix} = \begin{bmatrix} 12.89 \\ 14.06 \\ 13.08 \end{bmatrix}$$

12. The production matrix is

$$\begin{bmatrix} 1.58 & 0.33 & 0.43 \\ 0.48 & 1.5 & 0.38 \\ 0.3 & 0.39 & 1.32 \end{bmatrix}\begin{bmatrix} 10 \\ 5 \\ 20 \end{bmatrix} = \begin{bmatrix} 26.05 \\ 19.90 \\ 31.35 \end{bmatrix}$$

13. $$D_{\text{new}} = \begin{bmatrix} 4 \\ 1.5 \\ 9 \end{bmatrix}$$

$$(I - A)^{-1} D_{new} = \begin{bmatrix} 1.01 & 0.20 & 0.50 \\ 0.02 & 1.05 & 0.23 \\ 0.01 & 0.09 & 1.08 \end{bmatrix}\begin{bmatrix} 4 \\ 1.5 \\ 9 \end{bmatrix} = \begin{bmatrix} 8.84 \\ 3.725 \\ 9.895 \end{bmatrix}$$

Coal: \$8.84 billion, steel: \$3.725 billion, electricity: \$9.895 billion

14. $D_{\text{new}} = \begin{bmatrix} 4.5 \\ 2 \\ 1 \end{bmatrix}$

$$(I-A)^{-1}D_{\text{new}} = \begin{bmatrix} 1.04 & 0.02 & 0.10 \\ 0.21 & 1.01 & 0.02 \\ 0.11 & 0.02 & 1.02 \end{bmatrix}\begin{bmatrix} 4.5 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 4.82 \\ 2.985 \\ 1.555 \end{bmatrix}$$

Computers: \$482 million,
semiconductors: \$298.5 million,
business forms: \$155.5 million

15. $D_{\text{new}} = \begin{bmatrix} 3 \\ 1 \\ 4 \end{bmatrix}$

$$(I-A)^{-1}D_{\text{new}} = \begin{bmatrix} 1.04 & 0.02 & 0.10 \\ 0.21 & 1.01 & 0.02 \\ 0.11 & 0.02 & 1.02 \end{bmatrix}\begin{bmatrix} 3 \\ 1 \\ 4 \end{bmatrix} = \begin{bmatrix} 3.54 \\ 1.72 \\ 4.43 \end{bmatrix}$$

Computers: \$354 million,
semiconductors: \$172 million

16. $D_{\text{new}} = \begin{bmatrix} 2.4 \\ 1 \\ 3 \end{bmatrix}$

$$(I-A)^{-1}D_{\text{new}} = \begin{bmatrix} 1.01 & 0.2 & 0.5 \\ 0.02 & 1.05 & 0.23 \\ 0.01 & 0.09 & 1.08 \end{bmatrix}\begin{bmatrix} 2.4 \\ 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 4.124 \\ 1.788 \\ 3.354 \end{bmatrix}$$

Coal: \$4.124 billion, steel: \$1.788 billion, electricity: \$3.354 billion

17. $D_{\text{new}} = \begin{bmatrix} 4 \\ 2 \\ 5 \end{bmatrix}$

$$(I-A)D_{\text{new}} = \begin{bmatrix} 1 & -0.15 & -0.43 \\ -0.02 & 0.97 & -0.20 \\ -0.01 & -0.08 & 0.95 \end{bmatrix}\begin{bmatrix} 4 \\ 2 \\ 5 \end{bmatrix} = \begin{bmatrix} 1.55 \\ 0.86 \\ 4.55 \end{bmatrix}$$

\$1.55 billion worth of coal; \$.86 billion worth of steel; \$4.55 billion worth of electricity

18. $D_{\text{new}} = \begin{bmatrix} 4 \\ 2 \\ 3 \end{bmatrix}$

$$(I-A)D_{\text{new}} = \begin{bmatrix} 0.98 & -0.02 & -0.10 \\ -0.20 & 0.99 & 0 \\ -0.10 & -0.02 & 0.99 \end{bmatrix}\begin{bmatrix} 4 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 3.58 \\ 1.18 \\ 2.53 \end{bmatrix}$$

\$358,000,000 worth of computers; \$118, 000, 000 worth of semiconductors \$253,000, 000 worth of business forms.

19. **a.** $$A = \begin{array}{c} \begin{array}{cc} T & E \end{array} \\ \begin{bmatrix} 0.25 & 0.30 \\ 0.20 & 0.15 \end{bmatrix} \end{array} \begin{array}{c} \\ T \\ E \end{array}$$

b. $$(I - A)^{-1} = \left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0.25 & 0.30 \\ 0.20 & 0.15 \end{bmatrix} \right)^{-1} \approx \begin{bmatrix} 1.47 & 0.52 \\ 0.35 & 1.30 \end{bmatrix}$$

c. $$(I - A)^{-1} D \approx \begin{bmatrix} 1.47 & 0.52 \\ 0.35 & 1.30 \end{bmatrix} \begin{bmatrix} 5 \\ 3 \end{bmatrix} = \begin{bmatrix} 8.91 \\ 5.65 \end{bmatrix}$$

Transportation should produce \$8.91 billion worth of output and Energy should produce \$5.65 billion.

d. $$\begin{bmatrix} 0.25 & 0.30 \\ 0.20 & 0.15 \end{bmatrix} \begin{bmatrix} 8.91 \\ 5.65 \end{bmatrix} = \begin{bmatrix} 3.92 \\ 2.63 \end{bmatrix}$$

Transportation should use \$3.92 billion worth of output and Energy should use \$2.63 billion.

20. **a.** $$A = \begin{array}{c} \begin{array}{cc} T & E \end{array} \\ \begin{bmatrix} 0.25 & 0.30 \\ 0.20 & 0.15 \end{bmatrix} \end{array} \begin{array}{c} \\ T \\ E \end{array}$$

b. $$(I - A)^{-1} = \left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0.25 & 0.30 \\ 0.20 & 0.15 \end{bmatrix} \right)^{-1} \approx \begin{bmatrix} 1.47 & 0.52 \\ 0.35 & 1.30 \end{bmatrix}$$

c. $$(I - A)^{-1} D \approx \begin{bmatrix} 1.47 & 0.52 \\ 0.35 & 1.30 \end{bmatrix} \begin{bmatrix} 4 \\ 7 \end{bmatrix} = \begin{bmatrix} 9.52 \\ 10.50 \end{bmatrix}$$

Transportation should produce \$9.52 billion worth of output and Energy should produce \$10.50 billion.

d. $$\begin{bmatrix} 0.25 & 0.30 \\ 0.20 & 0.15 \end{bmatrix} \begin{bmatrix} 9.52 \\ 10.50 \end{bmatrix} = \begin{bmatrix} 5.53 \\ 3.48 \end{bmatrix}$$

Transportation should use \$5.53 billion worth of output and Energy should use \$3.48 billion

21. $$A = \begin{array}{c} \begin{array}{cc} P & I \end{array} \\ \begin{bmatrix} 0.02 & 0.01 \\ 0.10 & 0.05 \end{bmatrix} \end{array} \begin{array}{c} \\ P \\ I \end{array}$$

$$D = \begin{bmatrix} 930 \\ 465 \end{bmatrix}$$

$$(I - A)^{-1} D = \begin{bmatrix} 955 \\ 590 \end{bmatrix}$$

Plastics: \$955,000,
industrial equipment: \$590,000

22. $D_{\text{new}} = \begin{bmatrix} 1.86 \\ 2.79 \end{bmatrix}$

$(I - A)^{-1} D_{\text{new}} = \begin{bmatrix} 1.93 \\ 3.14 \end{bmatrix}$

Plastics: $1.93 million,
industrial equipment: $3.14 million

23. a.
$$A = \begin{array}{c} \\ \text{w} \\ \text{s} \\ \text{c} \end{array}\!\!\begin{array}{c} \begin{array}{ccc} \text{w} & \text{s} & \text{c} \end{array} \\ \begin{bmatrix} 0.3 & 0 & 0.1 \\ 0.2 & 0.3 & 0.2 \\ 0.1 & 0.2 & 0.05 \end{bmatrix} \end{array}, D = \begin{bmatrix} 1 \\ 4 \\ 2 \end{bmatrix}$$

b. $$(I - A)^{-1} = \left(\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 0.3 & 0 & 0.1 \\ 0.2 & 0.3 & 0.2 \\ 0.1 & 0.2 & 0.05 \end{bmatrix} \right)^{-1} \approx \begin{bmatrix} 1.47 & 0.05 & 0.16 \\ 0.49 & 1.54 & 0.38 \\ 0.26 & 0.33 & 1.15 \end{bmatrix}$$

c. $$(I - A)^{-1} D = \begin{bmatrix} 1.47 & 0.05 & 0.16 \\ 0.49 & 1.54 & 0.38 \\ 0.26 & 0.33 & 1.15 \end{bmatrix} \begin{bmatrix} 1 \\ 4 \\ 2 \end{bmatrix} = \begin{bmatrix} 1.99 \\ 7.41 \\ 3.88 \end{bmatrix}$$

Wood: $1.99, steel: $7.41, coal $3.88

d. $$\begin{bmatrix} 0.30 & 0 & 0.10 \\ 0.20 & 0.30 & 0.20 \\ 0.10 & 0.20 & 0.05 \end{bmatrix} \begin{bmatrix} 1.99 \\ 7.41 \\ 3.88 \end{bmatrix} = \begin{bmatrix} 0.99 \\ 3.40 \\ 1.88 \end{bmatrix}$$

Wood should use $0.99 worth of output, steel should use $3.40, and coal should use $1.88.

24. a.
$$A = \begin{array}{c} \\ \text{w} \\ \text{s} \\ \text{c} \end{array}\!\!\begin{array}{c} \begin{array}{ccc} \text{w} & \text{s} & \text{c} \end{array} \\ \begin{bmatrix} 0.3 & 0 & 0.1 \\ 0.2 & 0.3 & 0.2 \\ 0.1 & 0.2 & 0.05 \end{bmatrix} \end{array}, D = \begin{bmatrix} 1 \\ 4 \\ 2 \end{bmatrix}$$

b. $$(I - A)^{-1} = \left(\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 0.3 & 0 & 0.1 \\ 0.2 & 0.3 & 0.2 \\ 0.1 & 0.2 & 0.05 \end{bmatrix} \right)^{-1} \approx \begin{bmatrix} 1.47 & 0.05 & 0.16 \\ 0.49 & 1.54 & 0.38 \\ 0.26 & 0.33 & 1.15 \end{bmatrix}$$

c. $$(I - A)^{-1} D = \begin{bmatrix} 1.47 & 0.05 & 0.16 \\ 0.49 & 1.54 & 0.38 \\ 0.26 & 0.33 & 1.15 \end{bmatrix} \begin{bmatrix} 200 \\ 300 \\ 800 \end{bmatrix} = \begin{bmatrix} 437 \\ 864 \\ 1071 \end{bmatrix}$$

Wood: $437, steel: $864, coal $1071

d. $$\begin{bmatrix} 0.30 & 0 & 0.10 \\ 0.20 & 0.30 & 0.20 \\ 0.10 & 0.20 & 0.05 \end{bmatrix} \begin{bmatrix} 437 \\ 864 \\ 1071 \end{bmatrix} = \begin{bmatrix} 238.20 \\ 560.80 \\ 270.05 \end{bmatrix}$$

Wood should use $238.20 worth of output, steel should use $560.80, and coal should use $270.05

25. $(I-A)^{-1}\begin{bmatrix}100\\80\\200\end{bmatrix}\approx\begin{bmatrix}398\\313\\452\end{bmatrix}$

manufacturing: \$398 million,
transportation: \$313 million,
agriculture: \$452 million

26. a.

$$A=\begin{array}{c} \begin{array}{ccc} A & E & M \end{array} \\ \begin{bmatrix}0.08 & 0.15 & 0.25\\0.10 & 0.14 & 0.12\\0.20 & 0.10 & 0.05\end{bmatrix} \end{array}\begin{array}{c} \\ A\\E\\M\end{array}$$

b. $(I-A)^{-1}=\left(\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}-\begin{bmatrix}0.08 & 0.15 & 0.25\\0.10 & 0.14 & 0.12\\0.20 & 0.10 & 0.05\end{bmatrix}\right)^{-1}\approx\begin{bmatrix}1.19 & 0.25 & 0.34\\0.18 & 1.22 & 0.20\\0.27 & 0.18 & 1.15\end{bmatrix}$

c. $(I-A)^{-1}D\approx\begin{bmatrix}1.19 & 0.25 & 0.34\\0.18 & 1.22 & 0.20\\0.27 & 0.18 & 1.15\end{bmatrix}\begin{bmatrix}4\\3\\2\end{bmatrix}=\begin{bmatrix}6.19\\4.78\\3.91\end{bmatrix}$

Agriculture: \$6.19 billion, Energy: \$4.78 billion, Manufacturing: \$3.91 billion

d. $\begin{bmatrix}0.08 & 0.15 & 0.25\\0.10 & 0.14 & 0.12\\0.20 & 0.10 & 0.05\end{bmatrix}\begin{bmatrix}6.19\\4.78\\3.91\end{bmatrix}=\begin{bmatrix}2.18\\1.76\\1.91\end{bmatrix}$

Agriculture should use \$2.18 billion worth of output, energy should use \$1.76 billion, and manufacturing should use \$1.91 billion.

27.

$$A=\begin{array}{c} \begin{array}{ccc} M & B & F \end{array} \\ \begin{bmatrix}0 & 0.50 & 0.30\\0.30 & 0.10 & 0.20\\0.40 & 0.30 & 0.30\end{bmatrix} \end{array}\begin{array}{c} \\ M\\B\\F\end{array}$$

$D=\begin{bmatrix}20\\15\\18\end{bmatrix}$

$(I-A)^{-1}D\approx\begin{bmatrix}85\\68\\103\end{bmatrix}$

Merchant: \$85,000, baker: \$68,000,
farmer: \$103,000

28 $\left(\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}-\begin{bmatrix}0.02 & 0 & 0.02\\0.01 & 0.03 & 0.01\\0.03 & 0 & 0.01\end{bmatrix}\right)^{-1}\begin{bmatrix}800\\300\\1400\end{bmatrix}\approx\begin{bmatrix}846\\333\\1440\end{bmatrix}$

U.S.: \$846 million, Canada: \$333 million, England: \$1440 million

29. $(I-A)^{-1}\begin{bmatrix}3\\1\\3\end{bmatrix}=(I-A)^{-1}\left(\begin{bmatrix}2\\1\\3\end{bmatrix}+\begin{bmatrix}1\\0\\0\end{bmatrix}\right)$ When the matrix $(I-A)^{-1}$ is multiplied by the column matrix $\begin{bmatrix}1\\0\\0\end{bmatrix}$, the entries give the first column of the matrix $(I-A)^{-1}$, which represents the additional amounts that must be produced by the three industries.

$$=(I-A)^{-1}D_{new}-(I-A)^{-1}D_{old}$$

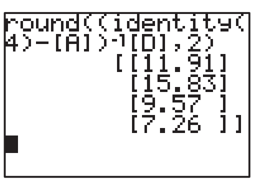
$$=(I-A)^{-1}\left(\begin{bmatrix}3\\1\\3\end{bmatrix}-\begin{bmatrix}2\\1\\3\end{bmatrix}\right)$$

$$=(I-A)^{-1}\left(\begin{bmatrix}1\\0\\0\end{bmatrix}\right)$$

$$=\begin{bmatrix}1.01 & .20 & .50\\ .02 & 1.05 & .23\\ .01 & .09 & 1.08\end{bmatrix}\begin{bmatrix}1\\0\\0\end{bmatrix}=\begin{bmatrix}1.01\\ .02\\ .01\end{bmatrix}$$

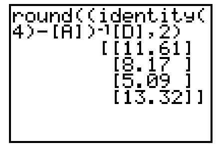
30. The second and third columns of $=(I-A)^{-1}$ represent the increased production levels required by \$1 billion increases in the final demand for steel and electricity, respectively.

31.



$$\begin{bmatrix}11.91\\15.83\\9.57\\7.26\end{bmatrix}$$

32.



$$\begin{bmatrix}11.61\\8.17\\5.09\\13.32\end{bmatrix}$$

Chapter 2 Fundamental Concept Check

1. Values of $x, y, z, \ldots$ that satisfy each equation in the system

2. Rectangular array of numbers

3. (a) interchange any two equations (or rows) (b) multiply an equation (or row) by a nonzero number (c) change an equation (or row) by adding to it a multiple of another equation (or row)

4. System of equations: $x=c_1;\ y=c_2;\ \cdots$

matrix: all entries on the main diagonal are 1; all entries off the main diagonal are zero.

5. Use elementary row operations to make the entry have value 1 and make the other entries in the column have value 0

6. (a) create a matrix corresponding to the system of linear equations (b) attempt to put the matrix into diagonal form as described in the box following Example 1 of Section 2.2 (c) if the matrix cannot be put into diagonal form follow the first step in the box following Example 3 of Section 2.2 (d) write the system of linear equations corresponding to the matrix and read off the solution(s)

7. Row matrix: a matrix consisting of a single row (that is, a $1\times n$ matrix)
column matrix: a matrix consisting of a single column (that is, a $m\times 1$ matrix)
square matrix: a matrix having the same number of columns as rows (that is, a $n\times n$ matrix)
identity matrix: a square matrix having 1s on the main diagonal and 0s elsewhere

8. The entry in the i^{th} row and the j^{th} column

9. For two matrices of the same size, the sum (difference) is the matrix obtained by adding (subtracting) the corresponding entries of the two matrices

10. For two matrices *A* and *B*, where the number of columns of *A* is the same as the number of rows of *B*, the matrix *AB* is the matrix having the same number of rows as *A* and the same number of columns as *B* whose ij^{th} entry is obtained by adding the products of the corresponding entries of the i^{th} row of *A* with the j^{th} column of *B*.

11. The scalar product of the number *c* and the matrix *A* is the matrix obtained by multiplying each element of *A* by *c.*

12. The inverse of the square matrix *A* is the matrix whose product with *A* is an identity matrix

13. The inverse of $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is $\dfrac{1}{D}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$, where $D = ad - bc$ and $D \neq 0$

14. Write the matrix form ($AX = B$) of the system of linear equations. If the matrix *A* has an inverse, then the solution of the system of linear equations is given by the entries of the matrix $A^{-1}B$.

15. Adjoin an identity matrix to the right of the invertible matrix *A* and then apply the Gauss-Jordan elimination method to the entire matrix until its left side is an identity matrix. The new right side of the matrix will be the inverse of *A*.

16. A square matrix whose ij^{th} entry is the amount of input from the i^{th} industry required to produce one unit of the j^{th} industry; a column matrix whose i^{th} element is the amount of units demanded from the i^{th} industry

17. If *A* is an input-output matrix and *D* is a consumer-demand matrix, then the i^{th} entry of the matrix $(I - A)^{-1} D$ gives the amount of input required from the i^{th} industry to meet the final demand

Chapter 2 Review Exercises

1. $\begin{bmatrix} \underline{3} & -6 & 1 \\ 2 & 4 & 6 \end{bmatrix} \xrightarrow{\frac{1}{3}R_1} \begin{bmatrix} 1 & -2 & \frac{1}{3} \\ 2 & 4 & 6 \end{bmatrix}$

$\xrightarrow{R_2+(-2)R_1} \begin{bmatrix} 1 & -2 & \frac{1}{3} \\ 0 & 8 & \frac{16}{3} \end{bmatrix}$

2. $\begin{bmatrix} -5 & -3 & 1 \\ 4 & \underline{2} & 0 \\ 0 & 6 & 7 \end{bmatrix} \xrightarrow{\frac{1}{2}R_2} \begin{bmatrix} -5 & -3 & 1 \\ 2 & 1 & 0 \\ 0 & 6 & 7 \end{bmatrix}$

$\xrightarrow{R_3+(-6)R_2} \begin{bmatrix} 1 & 0 & 1 \\ 2 & 1 & 0 \\ -12 & 0 & 7 \end{bmatrix}$

3. $\left[\begin{array}{cc|c} \underline{\frac{1}{2}} & -1 & -3 \\ 4 & -5 & -9 \end{array}\right]$

$\left[\begin{array}{cc|c} 1 & -2 & -6 \\ 0 & \underline{3} & 15 \end{array}\right]$

$\left[\begin{array}{cc|c} 1 & 0 & 4 \\ 0 & 1 & 5 \end{array}\right]$

$x = 4, y = 5$

4. $\left[\begin{array}{ccc|c} \underline{3} & 0 & 9 & 42 \\ 2 & 1 & 6 & 30 \\ -1 & 3 & -2 & -20 \end{array}\right]$

$\left[\begin{array}{ccc|c} 1 & 0 & 3 & 14 \\ 0 & \underline{1} & 0 & 2 \\ 0 & 3 & 1 & -6 \end{array}\right]$

$$\left[\begin{array}{ccc|c} 1 & 0 & 3 & 14 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & \underline{1} & -12 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 50 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -12 \end{array}\right]$$

$x = 50, y = 2, z = -12$

5. $$\left[\begin{array}{ccc|c} 3 & -6 & 6 & -5 \\ -2 & 3 & -5 & \frac{7}{3} \\ 1 & 1 & 10 & 3 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & -2 & 2 & -\frac{5}{3} \\ 0 & -1 & -1 & -1 \\ 0 & 3 & 8 & \frac{14}{3} \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 4 & \frac{1}{3} \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 5 & \frac{5}{3} \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & \frac{2}{3} \\ 0 & 0 & 1 & \frac{1}{3} \end{array}\right]$$

$x = -1,\ y = \dfrac{2}{3},\ z = \dfrac{1}{3}$

6. $$\left[\begin{array}{ccc|c} \underline{3} & 6 & -9 & 1 \\ 2 & 4 & -6 & 1 \\ 3 & 4 & 5 & 0 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 2 & -3 & \frac{1}{3} \\ 0 & 0 & 0 & \frac{1}{3} \\ 0 & -2 & 14 & -1 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 2 & -3 & \frac{1}{3} \\ 0 & \underline{-2} & 14 & -1 \\ 0 & 0 & 0 & \frac{1}{3} \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 11 & -\frac{2}{3} \\ 0 & 1 & -7 & \frac{1}{2} \\ 0 & 0 & 0 & \frac{1}{3} \end{array}\right]$$

No solution

7. $$\left[\begin{array}{cccc|c} \underline{1} & 2 & -5 & 3 & 16 \\ -5 & -7 & 13 & -9 & -50 \\ -1 & 1 & -7 & 2 & 9 \\ 3 & 4 & -7 & 6 & 33 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & 2 & -5 & 3 & 16 \\ 0 & \underline{3} & -12 & 6 & 30 \\ 0 & 3 & -12 & 5 & 25 \\ 0 & -2 & 8 & -3 & -15 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 3 & -1 & -4 \\ 0 & 1 & -4 & 2 & 10 \\ 0 & 0 & 0 & \underline{-1} & -5 \\ 0 & 0 & 0 & 1 & 5 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 3 & 0 & 1 \\ 0 & 1 & -4 & 0 & 0 \\ 0 & 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$$\begin{cases} x + 3z = 1 \\ y - 4z = 0 \\ \quad w = 5 \\ \quad 0 = 0 \end{cases}$$

z = any value, $x = 1 - 3z$, $y = 4z$, $w = 5$

8. $$\left[\begin{array}{cc|c} 5 & -10 & 5 \\ 3 & -8 & -3 \\ -3 & 7 & 0 \end{array}\right]$$

$$\left[\begin{array}{cc|c} 1 & -2 & 1 \\ 0 & -2 & -6 \\ 0 & 1 & 3 \end{array}\right]$$

$$\left[\begin{array}{cc|c} 1 & 0 & 7 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{array}\right]$$

$x = 7, y = 3$

9. $$\begin{bmatrix} 2+3 \\ -1+4 \\ 0+7 \end{bmatrix} = \begin{bmatrix} 5 \\ 3 \\ 7 \end{bmatrix}$$

10. $$\begin{bmatrix} 1\cdot 3 + 3\cdot 1 + (-2)0 & 1\cdot 5 + 3\cdot 0 + (-2)(-6) \\ 4\cdot 3 + 0\cdot 1 + (-1)0 & 4\cdot 5 + 0\cdot 0 + (-1)(-6) \end{bmatrix}$$

$$= \begin{bmatrix} 6 & 17 \\ 12 & 26 \end{bmatrix}$$

11. $\begin{bmatrix} \frac{3}{4}\cdot 8 & \frac{3}{4}\cdot(-6) \\ \frac{3}{4}\cdot\frac{2}{3} & \frac{3}{4}\cdot 0 \end{bmatrix} = \begin{bmatrix} 6 & \frac{-9}{2} \\ \frac{1}{2} & 0 \end{bmatrix}$

12. $\begin{bmatrix} 1.4-.8 & -3-7 \\ 8.2-1.6 & 0-(-2) \\ 4-0 & 5.5-(-5.5) \end{bmatrix} = \begin{bmatrix} .6 & -10 \\ 6.6 & 2 \\ 4 & 11 \end{bmatrix}$

13. $AB = \begin{bmatrix} 23 & -10+5k \\ -9 & 15+k \end{bmatrix}$

$BA = \begin{bmatrix} 23 & 15 \\ 6-3k & 15+k \end{bmatrix}$

Therefore,

$$\begin{aligned} -10+5k &= 15 \\ 5k &= 25 \\ k &= 5 \\ 6-3k &= -9 \\ -3k &= -15 \\ k &= 5 \end{aligned}$$

Since any value of k will work in the $a_{2,2}$ position $k = 5$ is the only solution.

14. $AB = \begin{bmatrix} 3k+5 & -10+k \\ 9 & 17 \end{bmatrix}$

$BA = \begin{bmatrix} 17 & -10+k \\ 9 & 3k+5 \end{bmatrix}$

Therefore,

$$\begin{aligned} 3k+5 &= 17 \\ 3k &= 12 \\ k &= 4 \\ 3k+5 &= 17 \\ 3k &= 12 \\ k &= 4 \end{aligned}$$

Since any value of k will work in the $a_{1,2}$ position, $k = 4$ is the only solution

15. $\begin{bmatrix} 3 & 2 \\ 5 & 4 \end{bmatrix}^{-1} = \begin{bmatrix} \frac{4}{2} & -\frac{2}{2} \\ -\frac{5}{2} & \frac{3}{2} \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ -\frac{5}{2} & \frac{3}{2} \end{bmatrix}$

$\begin{bmatrix} 2 & -1 \\ -\frac{5}{2} & \frac{3}{2} \end{bmatrix}\begin{bmatrix} 0 \\ 2 \end{bmatrix} = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$

$x = -2, y = 3$

16. a. $\begin{bmatrix} 4 & -2 & 3 \\ 8 & -3 & 5 \\ 7 & -2 & 4 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix} = \begin{bmatrix} 13 \\ 23 \\ 19 \end{bmatrix}$

$x = 13, y = 23, z = 19$

b. $\begin{bmatrix} -2 & 2 & -1 \\ 3 & -5 & 4 \\ 5 & -6 & 4 \end{bmatrix}\begin{bmatrix} 0 \\ -1 \\ 2 \end{bmatrix} = \begin{bmatrix} -4 \\ 13 \\ 14 \end{bmatrix}$

$x = -4, y = 13, z = 14$

17. $\left[\begin{array}{cc|cc} \underline{2} & 6 & 1 & 0 \\ 1 & 2 & 0 & 1 \end{array}\right]$

$\left[\begin{array}{cc|cc} 1 & 3 & \frac{1}{2} & 0 \\ 0 & \underline{-1} & -\frac{1}{2} & 1 \end{array}\right]$

$\left[\begin{array}{cc|cc} 1 & 0 & -1 & 3 \\ 0 & 1 & \frac{1}{2} & -1 \end{array}\right]$

$\begin{bmatrix} -1 & 3 \\ \frac{1}{2} & -1 \end{bmatrix}$

18. $\left[\begin{array}{ccc|ccc} \underline{1} & 1 & 1 & 1 & 0 & 0 \\ 3 & 4 & 3 & 0 & 1 & 0 \\ 1 & 1 & 2 & 0 & 0 & 1 \end{array}\right]$

$\left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & \underline{1} & 0 & -3 & 1 & 0 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array}\right]$

$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 4 & -1 & 0 \\ 0 & 1 & 0 & -3 & 1 & 0 \\ 0 & 0 & \underline{1} & -1 & 0 & 1 \end{array}\right]$

$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 5 & -1 & -1 \\ 0 & 1 & 0 & -3 & 1 & 0 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array}\right]$

$\begin{bmatrix} 5 & -1 & -1 \\ -3 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$

19. $\begin{cases} c+w+s=1000 \\ 357c+137w+181s=269{,}000 \\ c-w-s=0 \end{cases}$

$$\begin{bmatrix} 1 & 1 & 1 \\ 357 & 137 & 181 \\ 1 & -1 & -1 \end{bmatrix}^{-1} \begin{bmatrix} 1000 \\ 269{,}000 \\ 0 \end{bmatrix} = \begin{bmatrix} 500 \\ 0 \\ 500 \end{bmatrix}$$

Corn: 500 acres, wheat: 0 acres, soybeans: 500 acres

20. a. $AC = \begin{bmatrix} 15 & 20 & 8 \\ 10 & 17 & 12 \end{bmatrix} \begin{bmatrix} 165 \\ 65 \\ 210 \end{bmatrix} = \begin{bmatrix} 5455 \\ 5275 \end{bmatrix}$

The total cost to make the equipment in the first store was \$5455 and in the second store \$5275.

b. $AS = \begin{bmatrix} 15 & 20 & 8 \\ 10 & 17 & 12 \end{bmatrix} \begin{bmatrix} 200 \\ 80 \\ 250 \end{bmatrix} = \begin{bmatrix} 6600 \\ 6360 \end{bmatrix}$

The total revenue of the equipment in the first store was \$6600 and in the second store \$6360.

c. $S - C = \begin{bmatrix} 200 \\ 80 \\ 250 \end{bmatrix} - \begin{bmatrix} 165 \\ 65 \\ 210 \end{bmatrix} = \begin{bmatrix} 35 \\ 15 \\ 40 \end{bmatrix}$

The entries represent the profit per unit of each item.

d. $A(S-C) = \begin{bmatrix} 15 & 20 & 8 \\ 10 & 17 & 12 \end{bmatrix} \begin{bmatrix} 35 \\ 15 \\ 40 \end{bmatrix} = \begin{bmatrix} 1145 \\ 1085 \end{bmatrix}$

The total profit for the first store was \$1145 and for the second store \$1085.

21. a. $BA = \begin{bmatrix} 5000 & 8000 & 10{,}000 \end{bmatrix} \begin{bmatrix} 0.50 & 0.43 & 0.07 \\ 0.45 & 0.26 & 0.29 \\ 0.40 & 0.40 & 0.20 \end{bmatrix} = \begin{bmatrix} 10{,}100 & 8230 & 4670 \end{bmatrix}$

Total amount invested in bonds, stocks , and the conservative fixed income fund, respectively.

b. $BC = \begin{bmatrix} 5000 & 8000 & 10{,}000 \end{bmatrix} \begin{bmatrix} 0.0032 & 0.1119 \\ 0.0233 & 0.0976 \\ 0.0320 & 0.0467 \end{bmatrix} = \begin{bmatrix} 522.40 & 1807.30 \end{bmatrix}$

total return on the investments for one year and five years, respectively.

c. $2B = \begin{bmatrix} 2(5000) & 2(8000) & 2(10{,}000) \end{bmatrix} = \begin{bmatrix} 10{,}000 & 16{,}000 & 20{,}000 \end{bmatrix}$ The result of doubling the amounts invested

d. \$8230 is the total amount invested in stocks.

e. \$522.40 is the total return after one year.

22. a. $AB = \begin{bmatrix} 11 & 7 & 12 \\ 9 & 5 & 16 \\ 13 & 8 & 9 \\ 13 & 7 & 10 \end{bmatrix} \begin{bmatrix} 11 \\ 9 \\ 12 \end{bmatrix} = \begin{array}{l} \begin{bmatrix} 328 \\ 336 \\ 323 \\ 326 \end{bmatrix} \end{array} \begin{array}{l} \textit{Sara} \\ \textit{Quinn} \\ \textit{Tamia} \\ \textit{Zack} \end{array}$

total amount earned by each person for the week.

b. Quinn earned the most and Tamia earned the least.

c. $AB = \begin{bmatrix} 11 & 7 & 12 \\ 9 & 5 & 16 \\ 13 & 8 & 9 \\ 13 & 7 & 10 \end{bmatrix} \begin{bmatrix} 12 \\ 9 \\ 11 \end{bmatrix} = \begin{bmatrix} 327 \\ 329 \\ 327 \\ 329 \end{bmatrix} \begin{matrix} \textit{Sara} \\ \textit{Quinn} \\ \textit{Tamia} \\ \textit{Zack} \end{matrix}$

Quinn and Zack both earn \$329.

d. Sara worked 11 hours in concessions, 7 hours at the front desk, and 12 hours cleaning for a total of 30 hours.

23. Let x = number of apples
y = number of bananas
z = number of oranges

$$x + y + z = 18$$
$$0.85x + 0.20y + 0.76z = 9$$
$$-x + y - z = 0$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 18 \\ 0.85 & 0.20 & 0.76 & 9 \\ -1 & 1 & -1 & 0 \end{array}\right]$$

$$\xrightarrow{R_3 + R_1} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 18 \\ 0.85 & 0.20 & 0.76 & 9 \\ 0 & 2 & 0 & 18 \end{array}\right]$$

$$\xrightarrow{R_2 + (-0.85)R_1} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 18 \\ 0 & -0.65 & -0.09 & -6.3 \\ 0 & 2 & 0 & 18 \end{array}\right]$$

$$\xrightarrow{R_2 \leftrightarrow R_3} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 18 \\ 0 & 2 & 0 & 18 \\ 0 & -0.65 & -0.09 & -6.3 \end{array}\right]$$

$$\xrightarrow{\frac{1}{2}R_2} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 18 \\ 0 & 1 & 0 & 9 \\ 0 & -0.65 & -0.09 & -6.3 \end{array}\right]$$

$$\xrightarrow{R_1 + (-1)R_2} \left[\begin{array}{ccc|c} 1 & 0 & 1 & 9 \\ 0 & 1 & 0 & 9 \\ 0 & -0.65 & -0.09 & -6.3 \end{array}\right]$$

$$\xrightarrow{R_3 + (0.65)R_2} \left[\begin{array}{ccc|c} 1 & 0 & 1 & 9 \\ 0 & 1 & 0 & 9 \\ 0 & 0 & -0.09 & -0.45 \end{array}\right]$$

$$\xrightarrow{-\frac{1}{0.09}R_3} \left[\begin{array}{ccc|c} 1 & 0 & 1 & 9 \\ 0 & 1 & 0 & 9 \\ 0 & 0 & 1 & 5 \end{array}\right]$$

$$\xrightarrow{R_1 + (-1)R_3} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & 9 \\ 0 & 0 & 1 & 5 \end{array}\right]$$

4 apples, 9 bananas, 5 oranges

24. $x = A$'s current stockpile,
$y = B$'s current stockpile,
$a = A$'s next-year's stockpile,
$b = B$'s next-year's stockpile
$0.8x + 0.2y = a$
$0.1x + 0.9y = b$

a. Next year: $\begin{bmatrix} 0.8 & 0.2 \\ 0.1 & 0.9 \end{bmatrix} \begin{bmatrix} 10,000 \\ 7000 \end{bmatrix} = \begin{bmatrix} 9400 \\ 7300 \end{bmatrix}$

Two years: $\begin{bmatrix} 0.8 & 0.2 \\ 0.1 & 0.9 \end{bmatrix}^2 \begin{bmatrix} 10,000 \\ 7000 \end{bmatrix} = \begin{bmatrix} 8980 \\ 7510 \end{bmatrix}$

A: 9400, 8980; B: 7300, 7510

b. Previous year:

$$\begin{bmatrix} 0.8 & 0.2 \\ 0.1 & 0.9 \end{bmatrix}^{-1} \begin{bmatrix} 10,000 \\ 7000 \end{bmatrix} \approx \begin{bmatrix} 10,857 \\ 6571 \end{bmatrix}$$

Two years ago:

$$\left(\begin{bmatrix} 0.8 & 0.2 \\ 0.1 & 0.9 \end{bmatrix}^{-1}\right)^2 \begin{bmatrix} 10,000 \\ 7000 \end{bmatrix} \approx \begin{bmatrix} 12,082 \\ 5959 \end{bmatrix}$$

A: 10,857, 12,082; B: 6571, 5959

c. $a - b = (0.8x + 0.2y) - (0.1x + 0.9y)$
$= 0.7x - 0.7y = 0.7(x - y)$

Thus, the "missile gap" of the following year is 70% of the previous, so the "missile gap" decreases 30% each year.

$a + b = 0.9x + 1.1y < x + y$
when $0.1y < 0.1x$ or $y < x$.
$a + b = 0.9x + 1.1y > x + y$ when $y > x$.

25. $\left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0.4 & 0.2 \\ 0.1 & 0.3 \end{bmatrix}\right)^{-1} \begin{bmatrix} 8 \\ 12 \end{bmatrix} = \begin{bmatrix} 20 \\ 20 \end{bmatrix}$

Industry I: 20; industry II: 20

26. Let x = number of nickels
y = number of dimes
z = number of quarters

$$\begin{cases} x+y+z=30 \\ 0.05x+0.10y+0.25z=3.30 \\ y=5z \end{cases}$$

$$\left[\begin{array}{ccc|c} \underline{1} & 1 & 1 & 30 \\ 0.05 & 0.10 & 0.25 & 3.30 \\ 0 & 1 & -5 & 0 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 30 \\ 0 & \underline{0.05} & 0.20 & 1.8 \\ 0 & 1 & -5 & 0 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 30 \\ 0 & \underline{1} & 4 & 36 \\ 0 & 1 & -5 & 0 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -3 & -6 \\ 0 & 1 & 4 & 36 \\ 0 & 0 & \underline{-9} & -36 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -3 & -6 \\ 0 & 1 & 4 & 36 \\ 0 & 0 & \underline{1} & 4 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 6 \\ 0 & 1 & 0 & 20 \\ 0 & 0 & 1 & 4 \end{array}\right]$$

Joe has 4 quarters.

27. **a.** True; a system of equations has no solution, exactly one solution, or infinitely many solutions.

b. False; a system of equations could have two or more equations that are multiples of each other.

c. True; At least one variable must be dependent on another when there are less equations than variables.

28. a. True; The matrix and itself will have the same dimensions and therefore can be added.

b. False; In order to be multiplied by itself, the number of rows must be the same as the number of columns. In other words, only a square matrix can be multiplied by itself.

29. One possible system with infinitely many solutions is: $\begin{cases} 2x+3y=4 \\ 4x+6y=8 \end{cases}$.

30. One possible system with no solution is: $\begin{cases} 2x+3y=4 \\ 2x+3y=5 \end{cases}$.

31. Answers may vary, but two possible matrices are: $A=\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ and $B=\begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}$.

32. If the matrix has no inverse, there will be a row of zeros in the resulting Gauss-Jordan matrix.

33. The column values should add to less than one in an input-output matrix because they reflect the amount of input required from each industry and all the industries are dependent on each other.

Chapter 2 Project

1. 117

2. The total population of females decreased during the year.

3. **a.** 33

b. 18

c. 71, 94

4. Using a graphing calculator, we get $X_2=\begin{bmatrix} 698 \\ 125 \\ 2072 \end{bmatrix}$ $X_3=\begin{bmatrix} 684 \\ 126 \\ 2036 \end{bmatrix}$ $X_4=\begin{bmatrix} 672 \\ 123 \\ 2003 \end{bmatrix}$ $X_5=\begin{bmatrix} 661 \\ 121 \\ 1971 \end{bmatrix}$.

5.

Year	0	1	2	3	4	5
Total	2950	2926	2895	2846	2798	2753

The total female population is decreasing.

6. Answers may vary. *Sample answer*:
The original population column matrix must be multiplied by A one time for each year to get the population column matrices for subsequent years. Thus, the matrix after 50 years is given by X_{50} = [A]^50 * [B].

7. $X_{50} = \begin{bmatrix} 314 \\ 57 \\ 936 \end{bmatrix} X_{100} = \begin{bmatrix} 137 \\ 25 \\ 409 \end{bmatrix} X_{150} = \begin{bmatrix} 60 \\ 11 \\ 179 \end{bmatrix}$

The spotted owl will eventually become extinct.

8. $X_1 = \begin{bmatrix} 693 \\ 169 \\ 2116 \end{bmatrix} X_2 = \begin{bmatrix} 698 \\ 180 \\ 2109 \end{bmatrix} X_3 = \begin{bmatrix} 696 \\ 182 \\ 2110 \end{bmatrix} X_4 = \begin{bmatrix} 696 \\ 181 \\ 2113 \end{bmatrix} X_5 = \begin{bmatrix} 697 \\ 181 \\ 2114 \end{bmatrix}$

$X_{50} = \begin{bmatrix} 723 \\ 188 \\ 2194 \end{bmatrix} X_{100} = \begin{bmatrix} 754 \\ 196 \\ 2285 \end{bmatrix} X_{150} = \begin{bmatrix} 785 \\ 204 \\ 2381 \end{bmatrix}$

Extinction will be avoided.