Math 141 Spring 2025	Midterm Exam 5	April 22, 2025

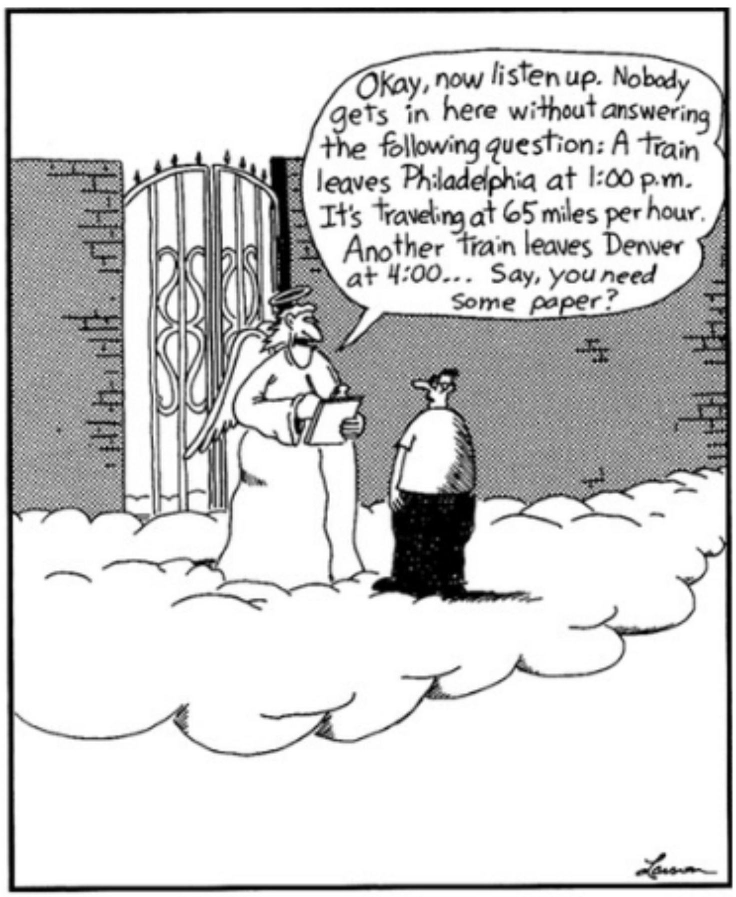
Name: Solutions

Problem	1 / 2	3	4 / 5	6 / 7 / 8	9	Total
Possible	16	20	20	30	14	100
Received						

DO NOT OPEN YOUR EXAM UNTIL TOLD TO DO SO.

You may use a 3 x 5 card (both sides) of handwritten notes and a calculator.

FOR FULL CREDIT, SHOW YOUR WORK FOR TO FINDING EACH SOLUTION.



Math phobic's nightmare

10 points **1.** A college English department purchased two types of tablets for their students to use. Type One costs \$50 each and Type Two costs \$60 each. They purchased of total of 35 tablets for a total of \$2000. How many of each tablet type did they purchase? Solve the system of two equations and two unknowns using either Gauss-Jordan Elimination or using matrices.

$$x + y = 35$$
$$50x + 60y = 2000$$

$$\left[\begin{array}{cc|c} 1 & 1 & 35 \\ 50 & 60 & 2000 \end{array}\right]$$

$$\xrightarrow{R2+(-50)R1} \left[\begin{array}{cc|c} 1 & 1 & 35 \\ 0 & 10 & 250 \end{array}\right] \xrightarrow{\frac{1}{10}R2} \left[\begin{array}{cc|c} 1 & 1 & 35 \\ 0 & 1 & 25 \end{array}\right]$$

$$\xrightarrow{R1-R2} \left[\begin{array}{cc|c} 1 & 0 & 10 \\ 0 & 1 & 25 \end{array}\right]$$

Type I
Type II

6 points **2.** A dietician wishes to plan a meal around two foods.

Each ounce of food I contains 10% of the daily requirements for carbohydrates, 20% for protein, and 30% for vitamin C.

Each ounce of food II contains 30% of the daily requirements for carbohydrates, 40% for protein, and 50% for vitamin C.

Set up (but **<u>DO NOT SOLVE</u>**) three equations with two unknowns (the two food types) that correspond to this information.

Carbs: $10x + 30y = 100$ ← 100% of daily requirements

Prot: $20x + 40y = 100$

Vit. C: $30x + 50y = 100$

How many solutions to this problem do you expect? Why?

None. # eqns > # unknowns.

20 points **3.** Find all of the solutions to the following systems of equations. If there is more than one solution, give the <u>general</u> solution plus <u>two</u> <u>specific</u> solutions. If there is no solution, show that this is the case.

/10

$$\begin{aligned} x - \ y + \ z &= 3 \\ -2x + 4y - 4z &= 1 \\ x + \ y - \ z &= 4 \end{aligned}$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 3 \\ -2 & 4 & -4 & 1 \\ 1 & 1 & -1 & 4 \end{array}\right] \xrightarrow[R3+(-1)R1]{R2+(2)R1} \left[\begin{array}{ccc|c} 1 & -1 & 1 & 3 \\ 0 & 2 & -2 & 7 \\ 0 & 2 & -2 & 1 \end{array}\right]$$

$$\xrightarrow{R3-R2} \left[\begin{array}{ccc|c} 1 & -1 & 1 & 3 \\ 0 & 2 & -2 & 7 \\ 0 & 0 & 0 & -6 \end{array}\right]$$

No solution.

/10

$$\begin{aligned} x + 5y + 3z &= 1 \\ 2x + 9y + 7z &= 4 \end{aligned}$$

$$\left[\begin{array}{ccc|c} 1 & 5 & 3 & 1 \\ 2 & 9 & 7 & 4 \end{array}\right]$$

$$\xrightarrow{R2+(-2)R1} \left[\begin{array}{ccc|c} 1 & 5 & 3 & 1 \\ 0 & -1 & 1 & 2 \end{array}\right] \xrightarrow{-R2} \left[\begin{array}{ccc|c} 1 & 5 & 3 & 1 \\ 0 & 1 & -1 & -2 \end{array}\right]$$

$$\xrightarrow{R1+(-5)R2} \left[\begin{array}{ccc|c} 1 & 0 & 8 & 11 \\ 0 & 1 & -1 & -2 \end{array}\right] \qquad \begin{aligned} x \quad + 8z &= 11 \\ y - z &= -2 \end{aligned}$$

General solution

$x = -8z + 11$

$y = z - 2$

$z =$ free

Specific solutions

$x = 11 \quad 3 \quad -5 \quad -13$

$y = -2, \ -1, \ 0, \ 1$, etc.

$z = 0 \quad 1 \quad 2 \quad 3$

Choose z first, then x & y follow.

10 points **4.** For what value(s) of k will this system of equations have a solution? What is the solution?

$$2x + 4y = 2$$
$$x + 7y = -4$$
$$kx + 8y = -2$$

$$\left[\begin{array}{cc|c} 2 & 4 & 2 \\ 1 & 7 & -4 \\ k & 8 & -2 \end{array}\right] \xrightarrow{\frac{1}{2}R1} \left[\begin{array}{cc|c} 1 & 2 & 1 \\ 1 & 7 & -4 \\ k & 8 & -2 \end{array}\right] \xrightarrow[R3+(-k)R1]{R2-R1} \left[\begin{array}{cc|c} 1 & 2 & 1 \\ 0 & 5 & -5 \\ 0 & 8-2k & -4 \end{array}\right]$$

$$\xrightarrow{\frac{1}{5}R2} \left[\begin{array}{cc|c} 1 & 2 & 1 \\ 0 & 1 & -1 \\ 0 & 8-2k & -4 \end{array}\right] \xrightarrow[R3+(2k-8)R2]{R1+(-2)R2} \left[\begin{array}{cc|c} 1 & 0 & 3 \\ 0 & 1 & -1 \\ 0 & 0 & -2k+4 \end{array}\right] \leftarrow -4+8-2k$$

$x = 3$
$y = -1$
$0 = -2k+4$

Need $-2k+4=0 \Rightarrow k=2$.

And $x = 3$
$y = -1$

10 points **5.** Use a matrix equation and a matrix inverse to solve the system of equations

$$.2x + .4y = 1$$
$$.8x + .6y = 2$$

$$\begin{bmatrix} .2 & .4 \\ .8 & .6 \end{bmatrix}^{-1} = \frac{1}{\underbrace{(.2)(.6)-(.4)(.8)}_{=-.20}} \begin{bmatrix} .6 & -.4 \\ -.8 & .2 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{.6}{-.2} & \frac{-.4}{-.2} \\ \frac{-.8}{-.2} & \frac{.2}{-.2} \end{bmatrix} = \begin{bmatrix} -3 & 2 \\ 4 & -1 \end{bmatrix}$$

Then $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -3 & 2 \\ 4 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}.$

3 points **6.** What is the inverse of $\begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$? $\begin{bmatrix} \frac{1}{a} & 0 & 0 \\ 0 & \frac{1}{b} & 0 \\ 0 & 0 & \frac{1}{c} \end{bmatrix}$

12 points **7.** Find the inverse of $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$ using the Gauss-Jordan Method $[\,A \mid I\,] \to [\,I \mid A^{-1}]$.

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right] \xrightarrow{R1+(-2)R2} \left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & -2 & 0 \\ 0 & 1 & 2 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$\xrightarrow[R2+(-2)R3]{R1+R3} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -2 & 1 \\ 0 & 1 & 0 & 0 & 1 & -2 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$$

The inverse.

15 points **8.** Suppose you have several nickels (5 cents) and dimes (10 cents). How many nickels and dimes would you need so that:

You have 15 coins total.

You have 100 cents (one dollar) total.

You have twice as many nickels as dimes (so $n = 2d$).

Don't just guess an answer—show your work. If there is no solution, show that this is the case.

$$\begin{aligned} n + d &= 15 \\ 5n + 10d &= 100 \\ n - 2d &= 0 \end{aligned} \qquad \left[\begin{array}{cc|c} 1 & 1 & 15 \\ 5 & 10 & 100 \\ 1 & -2 & 0 \end{array}\right] \xrightarrow[R3-R2]{R2+(-5)R1} \left[\begin{array}{cc|c} 1 & 1 & 15 \\ 0 & 5 & 25 \\ 0 & -3 & -15 \end{array}\right]$$

$$\longrightarrow \left[\begin{array}{cc|c} 1 & 1 & 15 \\ 0 & 1 & 5 \\ 0 & 1 & 5 \end{array}\right] \xrightarrow[R3-R2]{R1-R2} \left[\begin{array}{cc|c} 1 & 0 & 10 \\ 0 & 1 & 5 \\ 0 & 0 & 0 \end{array}\right]$$

$n = 5$

$d = 10$

14 points **9.** A company produces two items, but uses up some of each product in the production process, as described by the input-output (consumption) matrix

$$A = \begin{bmatrix} .1 & .5 \\ .3 & .5 \end{bmatrix}$$

/4 If you produced 10 units of each product, how much would remain of product <u>two</u>?

$$\begin{bmatrix} .1 & .5 \\ .3 & .5 \end{bmatrix}\begin{bmatrix} 10 \\ 10 \end{bmatrix} = \begin{bmatrix} 6 \\ 8 \end{bmatrix}, \text{ so } \begin{bmatrix} 10 \\ 10 \end{bmatrix} - \begin{bmatrix} 6 \\ 8 \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$$

/8 If you wanted to end up with 300 units of each product, how much would you need to produce of the two products?

$$(I-A)^{-1} = \begin{bmatrix} .9 & -.5 \\ -.3 & .5 \end{bmatrix}^{-1} = \frac{1}{\underbrace{(.9)(.5)-(-.3)(-.5)}_{.3}} \begin{bmatrix} .5 & .5 \\ .3 & .9 \end{bmatrix} = \begin{bmatrix} \frac{5}{3} & \frac{5}{3} \\ 1 & 3 \end{bmatrix}$$

$$\text{So produce } \begin{bmatrix} \frac{5}{3} & \frac{5}{3} \\ 1 & 3 \end{bmatrix}\begin{bmatrix} 300 \\ 300 \end{bmatrix} = \begin{bmatrix} 1000 \\ 1200 \end{bmatrix}$$

/2 If you wanted to end up with 1 more unit of product two (so in all, 300 units of product one and 301 units of product 2), how much <u>more</u> of the two products would you need to produce?

$$\text{Column } 2 \text{ of } (I-A)^{-1}: \begin{bmatrix} \frac{5}{3} \\ 3 \end{bmatrix}$$