Math 141 Spring 2025 **Midterm Exam 3** **March 13, 2025**

Name: Solutions

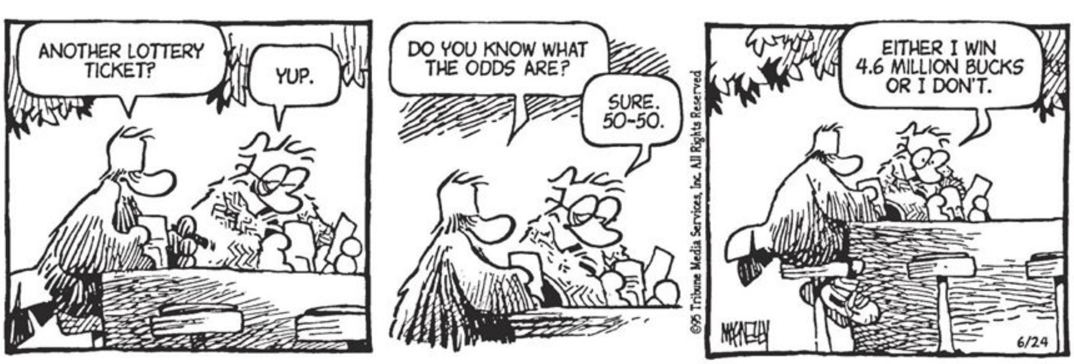
Problem	1 / 2	3	4 / 5	6 / 7	Total
Possible	32	21	20	27	100
Received					

DO NOT OPEN YOUR EXAM UNTIL TOLD TO DO SO.

You may use a 3 x 5 card of notes, both sides, and a calculator.

FOR FULL CREDIT, SHOW ALL WORK FOR FINDING EACH SOLUTION.

FIND THE FINAL ACTUAL VALUE FOR ALL PROBLEMS.



17 points 1. An archer (a person who shoots arrows) has a 0.45 probability of hitting a target. He will shoot 10 arrows at the target.

/5 What is the probability of hitting the target **exactly** 2 times out of 10?

$C(10,2)(.45)^2(.55)^8 = .0763$

/10 What is the probability of hitting the target **at least** 2 of the 10 times?

$= 1 - Pr(\text{not} \geq 2) = 1 - [Pr(0) + Pr(1)]$

$= 1 - [C(10,0)(.45)^0(.55)^{10} + C(10,1)(.45)^1(.55)^9]$

$= 1 - [.0025 + .0207] = .9768$

/2 What is the *expected number* of times he would hit the target (out of 10)? That is, if he were to shoot 10 shots, and then another 10 shots, and then another 10 shots, over and over, then on average how many shots (out of 10) would he make?

$10(.45) = 4.5$

15 points 2. Consider the game at right.

Winnings	Probability
\$1	.6
\$2	.2
\$10	.2

/5 Find the expected value of winnings:

$\$1(.6) + \$2(.2) + \$10(.2)$
$= \$3$

/10 Find the variance and standard deviation of winnings:

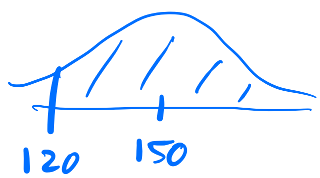
$\sigma^2 = (1-3)^2(.6) + (2-3)^2(.2) + (10-3)^2(.2)$

$= \cdots = 12.4$

$\sigma = \sqrt{12.4}$

21 points 3. The weights of a certain type of apple follow a normal distribution with mean 150 grams and standard deviation 20 [redacted] grams.

/5 What percentage of the apples weigh [redacted] 120 [redacted] grams or more?



$z = \frac{120-150}{20} = -1.5$

$A(-1.5) = .0668$

$1 - .0668 = .9332$

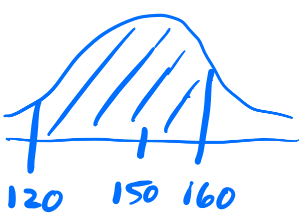
/2 What percentage of the apples weigh 120 [redacted] grams or less?

$.0668$

/1 What percentage of the apples weigh **exactly** 120 [redacted] grams?

0

/8 What percentage of the apples weigh between 120 [redacted] grams and 160 [redacted] grams?



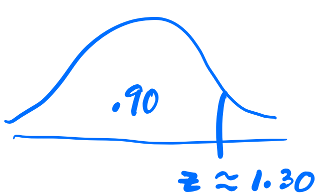
$z=-1.5$ $\quad z = \frac{160-150}{20} = -.5$

$A(.5) - A(-1.5)$

$= .6915 - .0668$

$= .6247$

/5 What [redacted] apple weight is at the 90th percentile?



So weight is

$150 + 1.30(20)$

$= 1.76$

z	$A(z)$	z	$A(z)$
−3.50	.0002	−2.00	.0228
−3.45	.0003	−1.95	.0256
−3.40	.0003	−1.90	.0287
−3.35	.0004	−1.85	.0322
−3.30	.0005	−1.80	.0359
−3.25	.0006	−1.75	.0401
−3.20	.0007	−1.70	.0446
−3.15	.0008	−1.65	.0495
−3.10	.0010	−1.60	.0548
−3.05	.0011	−1.55	.0606
−3.00	.0013	−1.50	.0668
−2.95	.0016	−1.45	.0735
−2.90	.0019	−1.40	.0808
−2.85	.0022	−1.35	.0885
−2.80	.0026	−1.30	.0968
−2.75	.0030	−1.25	.1056
−2.70	.0035	−1.20	.1151
−2.65	.0040	−1.15	.1251
−2.60	.0047	−1.10	.1357
−2.55	.0054	−1.05	.1469
−2.50	.0062	−1.00	.1587
−2.45	.0071	−.95	.1711
−2.40	.0082	−.90	.1841
−2.35	.0094	−.85	.1977
−2.30	.0107	−.80	.2119
−2.25	.0122	−.75	.2266
−2.20	.0139	−.70	.2420
−2.15	.0158	−.65	.2578
−2.10	.0179	−.60	.2743
−2.05	.0202	−.55	.2912

z	$A(z)$	z	$A(z)$
−.50	.3085	1.00	.8413
−.45	.3264	1.05	.8531
−.40	.3446	1.10	.8643
−.35	.3632	1.15	.8749
−.30	.3821	1.20	.8849
−.25	.4013	1.25	.8944
−.20	.4207	1.30	.9032
−.15	.4404	1.35	.9115
−.10	.4602	1.40	.9192
−.05	.4801	1.45	.9265
.00	.5000	1.50	.9332
.05	.5199	1.55	.9394
.10	.5398	1.60	.9452
.15	.5596	1.65	.9505
.20	.5793	1.70	.9554
.25	.5987	1.75	.9599
.30	.6179	1.80	.9641
.35	.6368	1.85	.9678
.40	.6554	1.90	.9713
.45	.6736	1.95	.9744
.50	.6915	2.00	.9772
.55	.7088	2.05	.9798
.60	.7257	2.10	.9821
.65	.7422	2.15	.9842
.70	.7580	2.20	.9861
.75	.7734	2.25	.9878
.80	.7881	2.30	.9893
.85	.8023	2.35	.9906
.90	.8159	2.40	.9918
.95	.8289	2.45	.9929

10 points 4. Suppose you select three balls from 3 orange and 5 blue balls. We are interested in the number of **orange** balls, which we call X.

/8 Complete the probability distribution for the number of **orange** balls:

k	$\Pr(X = k)$
0	$10/56$
1	$30/56$
2	$C(3,2) \cdot C(5,1) / C(8,3) = 3 \cdot 5/56 = 15/56$
3	$C(3,3) \cdot C(5,0) / C(8,3) = 1 \cdot 1/56 = 1/56$

/2 What is the expected number of orange balls?

$$0\left(\frac{10}{56}\right) + 1\left(\frac{30}{56}\right) + 2\left(\frac{15}{56}\right) + 3\left(\frac{1}{56}\right)$$

$$= \frac{63}{56} = \frac{9}{8}$$

10 points 5. Find (and **show appropriate work**) the mean, variance and standard deviation of the following ten values:

1 1 1 1 1 5 5 5 10 10

$$\mu = \frac{1 \cdot 5 + 5 \cdot 3 + 10 \cdot 2}{10} = 4$$

$$\sigma^2 = \frac{(1-4)^2 \cdot 5 + (5-4)^2 \cdot 3 + (10-4)^2 \cdot 2}{10} = 12$$

$$\sigma = \sqrt{12}$$

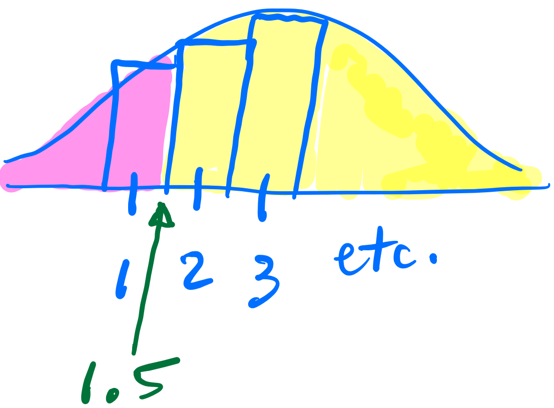
22 points 6. Suppose we roll a die 20 times. We are interested in getting fives. ⚄

/5 What is the probability of getting **exactly** 2 fives?

$C(20,2)\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^{18} = .1982$

Use the normal curve to approximate the next two probabilities.

/10 What is the probability of getting **at least** 2 fives?



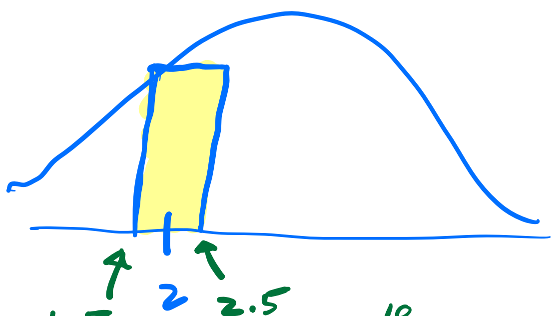
$z = \dfrac{1.5 - 20\left(\frac{1}{6}\right)}{\sqrt{20\left(\frac{1}{6}\right)\left(\frac{5}{6}\right)}} = -1.1$

$20\left(\frac{1}{6}\right) = \frac{10}{3}$, $20\left(\frac{1}{6}\right)\left(\frac{5}{6}\right) = \frac{5}{3}$

$A(-1.1) = .1357$

$1 - .1357 = .8643$

/7 What is the probability of getting **exactly** 2 fives? (Remember to use the **normal curve** to **approximate** this value.)



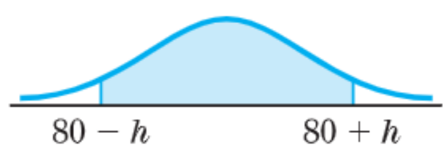
$z = -1.1$ $z = \dfrac{2.5 - \frac{10}{3}}{\frac{5}{3}} = -.5$

$A(-.5) - A(-1.1)$

$= .3085 - .1357$

$= .1728$

5 points 7. In the normal curve at right, $\mu = 80$ and $\sigma = 10$. Find the value of h for which the area of the shaded region is 0.9544.



Area in tails is $1 - .9544 = .0456$.

Area in one tail is $.0456/2 = .0228$.

So $A(z) = .0228 \Rightarrow z = -2$.

So $h = 2(10) = 20$