Math 141 Spring 2025 **Midterm Exam 2** **February 14, 2025**

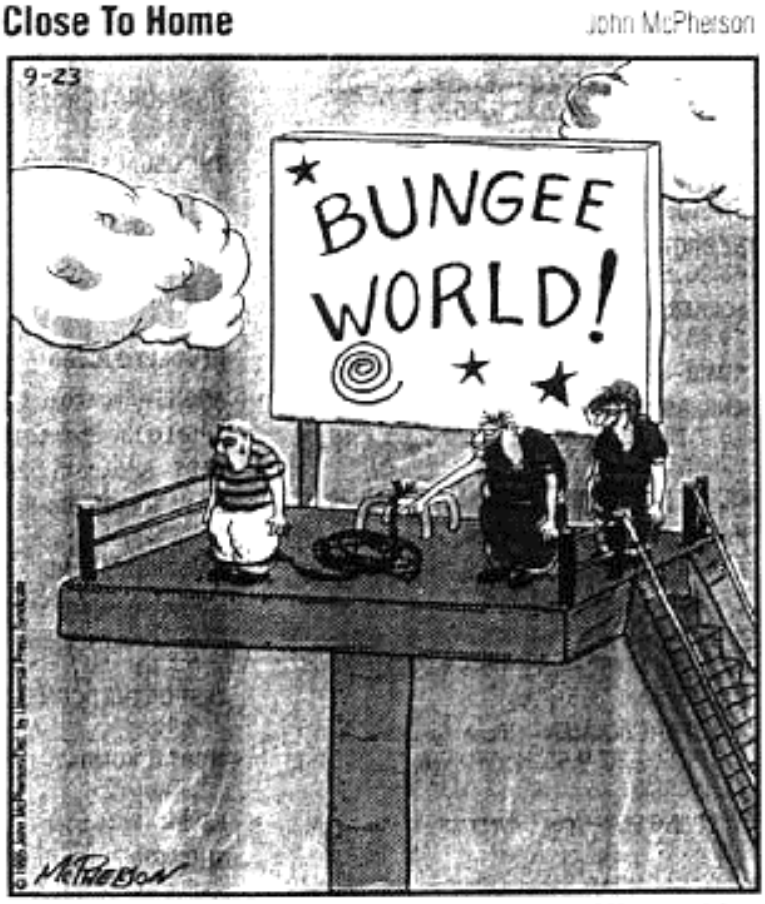
Name: Solutions

Problem	1	2	3 / 4	5 / 6	7	Total
Possible	15	14	25	24	22	100
Received						

DO NOT OPEN YOUR EXAM UNTIL TOLD TO DO SO.

You may use a 3 x 5 card (both sides) of notes and a calculator.

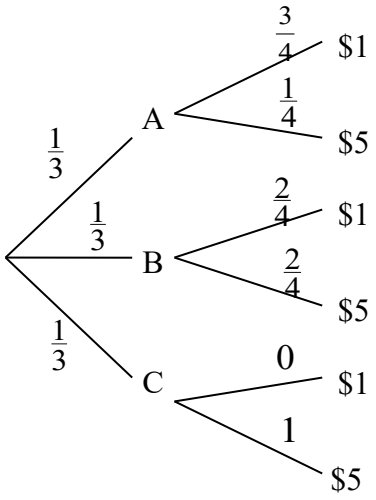
FOR FULL CREDIT, SHOW ALL WORK RELATED TO FINDING EACH SOLUTION.



"Okee-doke! Let's just double-check. We're 130 feet up and we've got 45 yards of bungee cord, that's uh ... 90 feet. Allow for 30 feet of stretching, that gives us a total of ...120 feet. Perfect!"

15 points 1. Each of three envelopes contains four bills:

- Envelope A contains:
  - Three \$1 bills.
  - One \$5 bill
- Envelope B contains:
  - Two \$1 bills
  - Two \$5 bills
- Envelope C contains:
  - No \$1 bill
  - Four \$5 bills



An envelope is randomly selected and a bill is randomly selected from that envelope.

/2 What is the probability the bill is \$1 from Envelope B?

$$\frac{1}{3} \cdot \frac{2}{4} = \frac{2}{12}$$

/4 What is the probability the bill is \$1 (from any envelope)?

$$\frac{1}{3} \cdot \frac{3}{4} + \frac{1}{3} \cdot \frac{2}{4} + \frac{1}{3} \cdot 0 = \frac{5}{12}$$

/4 If the bill is \$1, what is the probability the envelope it came from was Envelope B?

$$\Pr(B \mid \$1) = \frac{\Pr(B \text{ and } \$1)}{\Pr(\$1)} = \frac{\frac{2}{12}}{\frac{5}{12}} = \frac{2}{5}$$

/5 If the bill is \$1, and we then choose *another* bill from the *same* envelope, what is the probability that the other bill we choose is also \$1?

$$\Pr(\$1 \text{ again} \mid \$1 \text{ first time}) = \frac{\Pr(\$1 \text{ twice from same envelope})}{\Pr(\$1 \text{ first time})}$$

$$= \frac{\frac{1}{3} \cdot \frac{3}{4} \cdot \frac{2}{3} + \frac{1}{3} \cdot \frac{2}{4} \cdot \frac{1}{3} + \frac{1}{3} \cdot 0}{\frac{5}{12}} = \frac{\frac{8}{36}}{\frac{5}{12}}$$

14 points 2. Suppose you roll two dice, and you are interested in their sum. The possible outcomes are listed at right.

Sum	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

/2 What is the probability of rolling a sum of 5 or a 9?

$\frac{8}{36}$

/3 Suppose someone can see the dice (and you cannot), and tells you the sum is odd. What is the probability you rolled a sum of 5 or 9?

$\frac{8}{18}$

/3 Suppose someone can see the dice (and you cannot), and says you rolled a sum of either 5 or a 9. What is the probability one of the dice is a 4?

$\frac{4}{8}$

/3 Suppose you can see that one die is a 4, but you cannot see the other die. What is the probability you rolled a sum of 5 or a 9?

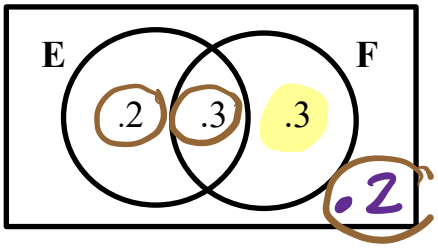
$\frac{2}{6}$

 and  — 1, 2, 3, 4, 5 or 6

/3 What is the probability of rolling a sum that is odd or ≥ 9?

$$\frac{18 + 10 - 6}{36} = \frac{22}{36}$$

17 points 3. Suppose $\Pr(E) = .5$, $\Pr(F) = .6$ and $\Pr(E \cap F) = .3$. Find each of the following.



/1 $\Pr(E') =$ $1 - .5 = .5$

/1 $\Pr(F') =$ $1 - .6 = .4$

/2 $\Pr(E \cup F') =$ $.2 + .3 + .2 = .7$

/2 $\Pr(E' \cap F) =$ $.3$

/3 $\Pr(E|F') =$ $\dfrac{\Pr(E \text{ and } F')}{\Pr(F')} = \dfrac{.2}{.4} = .5$

/3 $\Pr(E'|F') =$ $\dfrac{\Pr(E' \cap F')}{\Pr(F')} = \dfrac{.2}{.4} = .5$

/3 $\Pr(F|E) =$ $\dfrac{\Pr(F \cap E)}{\Pr(E)} = \dfrac{.3}{.5} = .6$

/2 Are events E and F independent? Why or why not?

Yes. $\Pr(F|E) = \Pr(F)$, etc.

8 points 4. We are interested in what proportion of voters in each political party actually vote.

Political party	Proportion registered	Proportion voter turnout (call this "V")
Democrat (D)	.40	.70
Republican (R)	.50	.50
Independent (I)	.10	.80

/2 First, without doing any work, give a bound on $\Pr(V)$, the proportion of people who voted, that is, the probability that any given person voted.

$.50 < \Pr(V) < .80$

/6 If a person tells us that he/she voted, how likely is it that he/she is Republican? That is, what is $\Pr(R \mid V)$?

$$\Pr(R|V) = \frac{\Pr(R \text{ and } V)}{\Pr(V)} = \frac{(.50)(.50)}{(.40)(.70)+(.50)(.50)+(.10)(.80)}$$

$$= \frac{.25}{.61} \approx .41.$$

YOU DO NOT NEED TO SIMPLY THE ANSWERS FOR THE PROBLEMS ON THIS PAGE.

19 points 5. Four balls are chosen at random without replacement (that is, without putting the ball back into the basket once it has been chosen). There are thirteen balls total: 6 green and 7 blue.

/4 If you select four balls, what is the probability that all four balls are blue?

$$\left(\frac{7}{13}\right)\left(\frac{6}{12}\right)\left(\frac{5}{11}\right)\left(\frac{4}{10}\right) = \frac{C(7,4)}{C(13,4)}$$

/2 If you select four balls, what is the probability that at least one of them is green?

$$1 - \downarrow$$

/4 If you select four balls, all at once, what is the probability that two of the balls are green and two are blue?

$$\frac{C(6,2)\cdot C(7,2)}{C(13,4)}$$

/4 If you select four balls, one at a time, what is the probability that the balls are (in this order) green, blue, blue and green?

$$\left(\frac{6}{13}\right)\left(\frac{7}{12}\right)\left(\frac{6}{11}\right)\left(\frac{5}{10}\right)$$

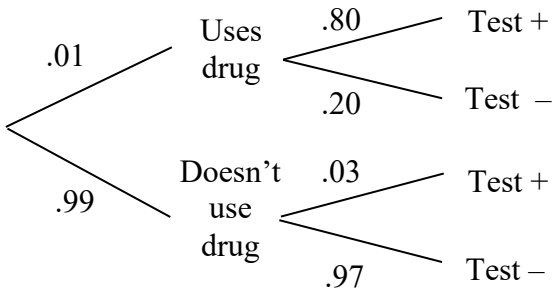
/5 If you select one ball, but you do not look at it (so you do not know its color), what is the probability that the second ball you select would be blue? **Show your work—don't just give an answer**.

$$Pr(B2) = Pr(B1 \text{ and } B2) + Pr(G1 \text{ and } B2)$$
$$= \left(\frac{7}{13}\right)\left(\frac{6}{12}\right) + \left(\frac{6}{13}\right)\left(\frac{7}{12}\right) = \cdots = \frac{7}{13}$$

5 points 6. Suppose that each of 4 persons randomly chooses a number between 1 and 50. What is the probability that at least two persons choose the same number?

$$1 - Pr(\text{all choose different})$$
$$= 1 - \frac{50\cdot 49\cdot 48\cdot 47}{50\cdot 50\cdot 50\cdot 50}$$

22 points 7. Suppose that 1% of a certain group of people use a certain drug, and suppose that a particular test which is used to determine whether a person uses the drug gives false negatives 20% of the time and false positives 3% of the time.



/16 Fill in the following table. **Be sure to <u>show all pertinent work</u> below the table**.

	No Test	Results of test: Positive	Results of test: Negative
Pr(Uses drug)	.01	.212	.002
Pr(Doesn't use drug)	.99	.788	.998

$Pr(D|+) = \frac{Pr(D \text{ and } +)}{Pr(+)} = \frac{(.01)(.80)}{(.01)(.80)+(.99)(.03)} = \frac{.008}{.0377} = .212$

$Pr(D'|+) = \frac{Pr(D' \text{ and } +)}{Pr(+)} = \frac{(.99)(.03)}{.0377} = \frac{.0297}{.0377} = .788$

$Pr(D|-) = \frac{Pr(D \text{ and } -)}{Pr(-)} = \frac{(.01)(.20)}{(.01)(.20)+(.99)(.97)} = \frac{.002}{.9623} = .002$

$Pr(D'|-) = Pr(D' \text{ and } -) = (.99)(.97) = \frac{.9603}{.9623} = .998$

/6 What is the probability that a randomly selected person who has tested <u>negative</u> once would test <u>positive</u> if he/she were tested again?

$Pr(+|-) = \frac{Pr(- \text{ then } +)}{Pr(-)}$

$= \frac{(.01)(.20)(.80)+(.99)(.97)(.03)}{(.01)(.20)+(.99)(.97)}$

$= .0316.$