

Name: Solutions

Problem	Total
Possible	100
Received	

DO NOT OPEN YOUR EXAM UNTIL TOLD TO DO SO.

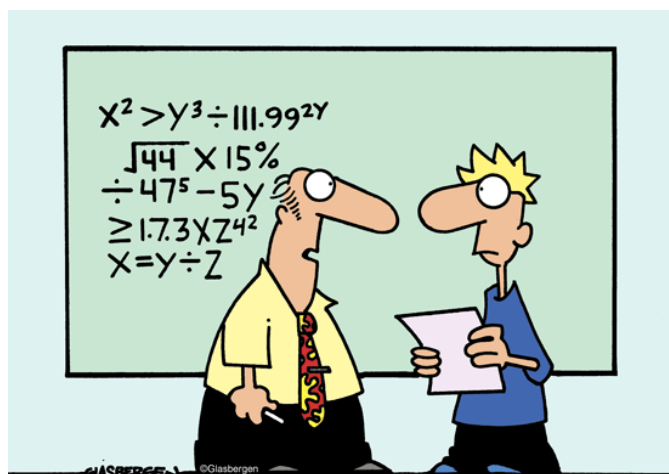
You may use a 3×5 (both sides) of handwritten notes.

You will NOT use a calculator.

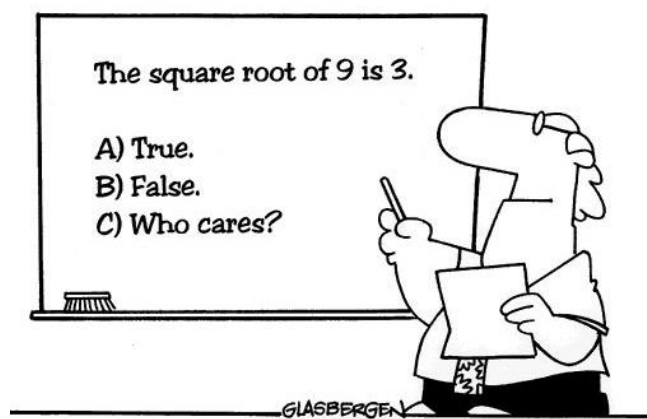
In giving your answers, except on page 4, do not simplify your answers.

For example, leave your answer in the form

$P(5,3)$ or $12!$ or $C(4,3) \cdot C(7,4)$ or $7 \cdot 6 \cdot 5$ or ...



"Can you keep a secret? I've been teaching this stuff for 15 years and I still don't understand it."



Many students actually look forward to Mr. Atwadder's math tests.

- /3 How many 8-letter passwords could be made using the 26 lower case and 26 upper case letters (so 52 letters total)?

$$52^8 \quad P(52, 8) \text{ if you cannot repeat}$$

- /3 In how many ways can a committee of 8 persons be chosen from 10 married couples if a husband and wife cannot both serve on the committee (i.e. the 8 persons must come from 8 different couples)?

$$C(10, 8) \cdot 2^8$$

- /3 In how many ways could we divide a basketball team of 14 players into the 5 starters and the 9 substitutes?

$$\binom{14}{5, 9}$$

- /3 In how many ways can I arrange 25 books on a shelf?

$$25!$$

- /3 In how many ways can I arrange 25 books on a shelf, if 5 of them are math books which must be placed together side-by-side?

$$21 \cdot 20! \cdot 5!$$

- /3 How many 4-digit numbers from 1000 to 9999 are there in which the digits are all different? Example: 3902.

$$9 \cdot 8 \cdot 7 \cdot 6$$

- /3 How many 3-digit numbers from 100 to 999 are there in which exactly two of the digits are the same? Example: 393 or 339 or

$$9 \cdot 8 \cdot 1 + 9 \cdot 1 \cdot 8 + 8 \cdot 9 \cdot 1$$

- /3 How many different meals can be chosen if there are 2 appetizers, 5 main dishes, and 4 desserts, assuming a meal consists of one item from each category?

$$2 \cdot 5 \cdot 4$$

- /3 For Halloween, if I have 10 pieces of candy, all of different types, and 10 kids show up at my door, in how many different ways could I hand out the 10 pieces of candy, one piece to each kid?

$$10!$$

/3 In how many ways can I arrange 3 items?

$$3!$$

/3 In how many ways can I choose 3 books from 8?

$$C(8, 3)$$

/3 In how many ways can I arrange 3 books of 8 on a shelf?

$$P(8, 3) = 8 \cdot 7 \cdot 6 = C(8, 3) \cdot 3!$$

/4 How many ways can I divide a group of 20 people into *two* groups of 5 and *one* group of 10?

$$\binom{20}{5, 5, 10} / 2!$$

/3 How many ways can you give a \$1, \$2, \$5 and \$10 bill to 4 of your 8 friends (one bill for each friend that you choose)?

$$P(8, 4) = 8 \cdot 7 \cdot 6 \cdot 5 = C(8, 4) \cdot 4!$$

/4 In how many ways can you choose 3 green balls and 4 red ones from a basket containing 6 green, 7 red and 8 yellow balls?

$$C(6, 3) \cdot C(7, 4)$$

/3 In the United States we have 10-digit phone numbers: 3-digit area code, 7-digit phone number. How many 10-digit phone numbers are there if the only restriction is that 0 cannot be the first digit of the area code or of the phone number?

$$9 \cdot 10 \cdot 10 \cdot 9 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10$$

/5 How many nine-letter words (including nonsense words) can you make in which:

No letter can be repeated

All five vowels must be used

None of the four consonants can be next to each other.

Example: **BEQIOTAXU**



$$C(6, 4) \cdot C(21, 4) \cdot 4! \cdot 5!$$

Where to put
consonants relative
to the vowels.

Choose the 4 consonants,
put them in some order.

For problems on this page, simplify your answers, i.e., give an actual number.

For the five questions on this page, suppose there are eight kids: four boys and four girls.

/4 In how many different ways could we choose two (different) girls?

$$C(4, 2) = \frac{4!}{2!2!} = \frac{4 \cdot 3}{2 \cdot 1} = 6$$

/4 In how many different ways could we choose any two of them (of either gender)?

$$C(8, 2) = \frac{8 \cdot 7}{2 \cdot 1} = 28$$

/5 In how many different ways could we divide the eight into four boy-girl couples?

$$4 \cdot 3 \cdot 2 \cdot 1$$

↑
which girl for boy 1, etc.

/5 In how many different ways could we divide the eight into three groups (of any gender) of sizes 2, 2 and 4?

$$\binom{8}{2, 2, 4} / 2! = \frac{8!}{2!2!4!2!} = \frac{\cancel{8} \cdot \cancel{7} \cdot \cancel{6} \cdot \cancel{5} \cdot \cancel{4}!}{\cancel{2} \cdot \cancel{2} \cdot \cancel{2} \cdot \cancel{4}!} = 7 \cdot 6 \cdot 5 = 7 \cdot 30 = 210$$

/5 In how many ways can they be seated in a row such that no person is seated next to someone of the same gender? (You do NOT need to simplify this answer.)

$$\begin{array}{l} \text{BGBGBGBG} \\ \text{GBGBGBGB} \end{array} \} 2 \cdot 4! \cdot 4! \quad \text{OR}$$

$$\underline{8} \cdot \underline{4} \cdot \underline{3} \cdot \underline{3} \cdot \underline{2} \cdot \underline{2} \cdot \underline{1} \cdot \underline{1}$$

**For the above problems (except the final problem),
simplify your answers, i.e., give an actual number.**

For the next two questions, note that in California a typical automobile license plate has a letter, then three numbers, then three letters, for example, U747ROC. Assume each number can be a digit from 0 through 9.

- /3 How different license plates could there be?

$$26 \cdot 10 \cdot 10 \cdot 10 \cdot 26 \cdot 26 \cdot 26$$

- /3 How many different license plates with no repetition of numbers or letters could there be?

$$26 \cdot 10 \cdot 9 \cdot 8 \cdot 25 \cdot 24 \cdot 23$$

For the next four questions, suppose there is a deck of 80 cards of 4 different colors of cards numbered 1 to 20. We will choose 7 cards.

- /5 In how many ways can you choose 7 cards and have exactly 3 of them be 5's (there could be duplication in the other 4 cards, but no other 5's), e.g. 5 5 5 17 3 1 17.

$$C(4, 3) \cdot C(76, 4) \leftarrow \begin{array}{l} \text{Choose the three 5's} \\ \text{Choose the four non-5's} \end{array}$$

- /3 In how many ways can you choose any 7 cards from the 80?

$$C(80, 7)$$

- /3 In how many ways can you choose the 7 cards and end up with a straight (e.g. 1-7 or 2-8 or ...), where color doesn't matter?

$$\underbrace{1-7 \text{ to } 14-20}_7 \cdot C(4, 1) \leftarrow \begin{array}{l} \text{Choose the color} \\ \text{for each number} \end{array}$$

- /5 In how many ways can you choose the 7 cards and have a really full house consisting of 3 of one number, 2 of another (different) number, and 2 more of another (different number) e.g. 5 5 5 11 11 8 8.

$$C(20, 3) \cdot 3 \cdot C(4, 3) \cdot C(4, 2) \cdot C(4, 2)$$

Extra credit: which of your three numbers to have three of

- /4 In how many ways can a housing director assign 8 students to 5 dorm rooms if 3 rooms are doubles and 2 rooms are singles, and if 2 of the students don't get along with each other and thus cannot be placed together?

$$\binom{8}{2, 2, 2, 1, 1} - 3 \binom{6}{2, 2, 1, 1} \leftarrow \begin{array}{l} \text{All possible room assignments} \\ \text{which double} \\ \text{Ways to put the two into the same room (a double)} \end{array}$$