Math 141 Spring 2024 **Midterm Exam 2** **February 20, 2024**

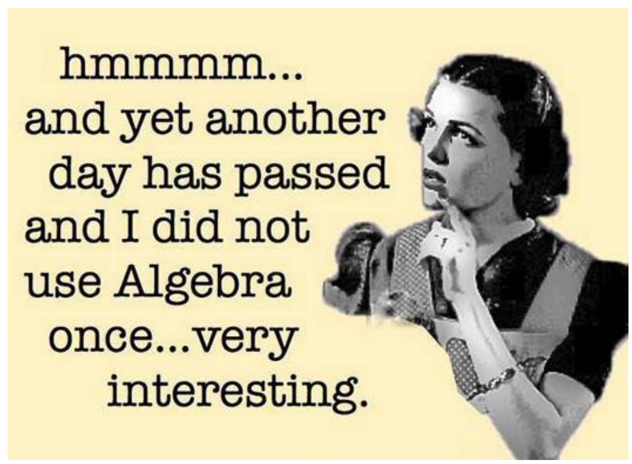
Name: ____

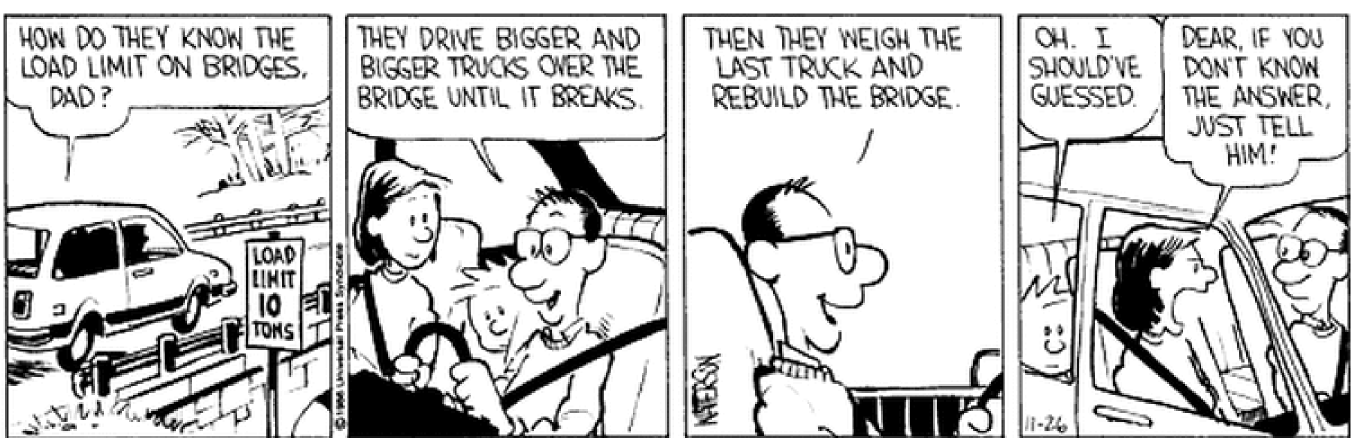
Problem	1 / 2 / 3	4 / 5	6 / 7	8	9	Total
Possible	27	19	26	12	16	100
Received						

DO NOT OPEN YOUR EXAM UNTIL TOLD TO DO SO.

You may use a 3 x 5 card (both sides) of handwritten notes and a calculator.

FOR FULL CREDIT, SHOW ALL WORK RELATED TO FINDING EACH SOLUTION.





4 points **1.** In a New York City people often take the subway or they take a taxi (taxis and subways are in competition with each other). Let $f(p_1, p_2)$ be the number of people who take the subway, where p_1 is the price of a subway ride and p_2 is the price of a taxi ride. What would be the sign of each of the following? (Just circle one for each derivative.)

$\frac{\partial f}{\partial p_1}$ would be: > 0 $= 0$ < 0

$\frac{\partial f}{\partial p_2}$ would be: > 0 $= 0$ < 0

14 points **2.** For $f(x, y) = e^{y/x^2}$, find the following derivatives:

/3 $\frac{\partial f}{\partial x} =$

/3 $\frac{\partial f}{\partial y} =$

/4 $\frac{\partial^2 f}{\partial y^2} =$

/4 $\frac{\partial^2 f}{\partial x \partial y} =$

9 points **3.** Consider the production function $f(x, y) = 10x^{1/2}y^{1/2}$ where x is units of labor and y is units of capital.

/3 Find and interpret $f(25, 49)$.

/4 Find and interpret $\frac{\partial f}{\partial x}(25, 49)$.

/2 Find the marginal productivity of labor at $(x, y) = (25, 49)$.

9 points **4.** Suppose for $f(x, y)$ that: $f(1,2) = 20$, $\frac{\partial f}{\partial x}(1,2) = 10$, and $\frac{\partial f}{\partial y}(1,2) = 5$.
Using this info, estimate each of the following:

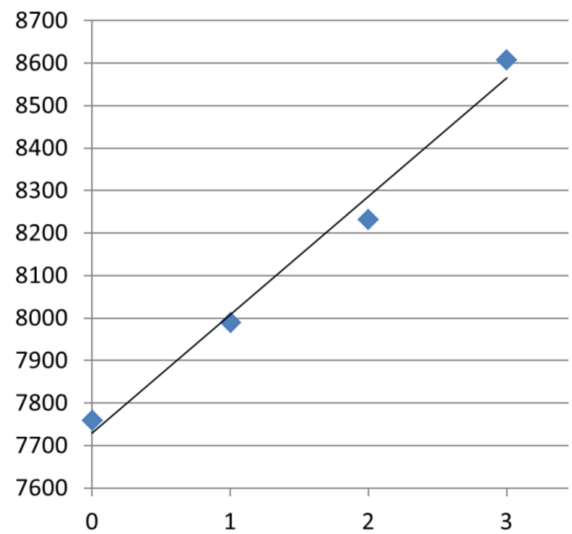
/4 $f(\mathbf{4}, 2) \approx$

/4 $f(1, \mathbf{4}) \approx$

/1 $f(\mathbf{4}, \mathbf{4}) \approx$

10 points **5.** Consider the following data which lists the per capita spending on health care in the United States from 2020 to 2023.

Years t after 2020	Spending S
0	\$7760
1	\$8010
2	\$8230
3	\$8600



The line that best fits these data is

$$S = 275t + 7750,$$

where t is the number of years after 2020.

/2 What does the value 275 represent? (Don't just say slope—explain what it tells us.)

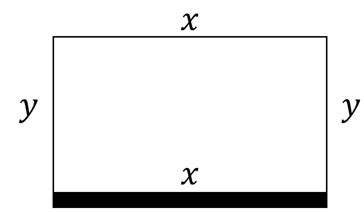
/4 Based on these data (i.e. this line), what do you estimate the per capita spending will be in the year 2030 (10 years after 2020)?

/4 Based on these data, by approximately **what year** do you expect per capita spending to reach \$15,000?

13 points **6.** Suppose we are trying to enclose a rectangular garden, where the lower wall is made of stone and costs \$90 per foot, and the upper and two side walls are made of wood at \$30 per foot. So the cost of the four walls would be

$$90x + 30x + 30y + 30y = 120x + 60y.$$

Find the values of x and y which <u>maximize the area xy</u> given the <u>constraint that we have \$1200 to spend on the wall</u>. <u>Use the Lagrange Multiplier Method</u>.



Extra credit: by approximately how much would the area increase if we could spend \$1 more on building the wall? Use λ to answer this question.

13 points **7.** Same information as in the previous problem, but now we want to find the values of x and y which <u>minimize cost</u> with the <u>constraint that the area be $400\ \text{ft}^2$</u>. Use any Calculus-based method you like—you do not necessarily have to use the Lagrange Multiplier Method (of course you can), but you do need to use Calculus (derivatives) in solving this.

12 points **8.** Suppose a monopolist produces and sells a certain product in two different countries.

Country	Number sold	Price
1	x	$90 - \frac{x}{20}$
2	y	$100 - \frac{y}{10}$

Then the total revenue is

$$R(x,y) = x\left(90 - \frac{x}{20}\right) + y\left(100 - \frac{y}{10}\right).$$

If there is a fixed cost of \$5000 and a cost-per-item of \$10, then the total cost is

$$C(x,y) = 5000 + 10(x + y).$$

Then *Profit* = *Revenue* – *Cost*:

$$P(x,y) = x\left(90 - \frac{x}{20}\right) + y\left(100 - \frac{y}{10}\right) - [5000 + 10(x+y)]$$
$$= -\frac{1}{20}x^2 + 80x - \frac{1}{10}y^2 + 90y - 5000$$

<u>So here is the problem</u>: find the values of x and y which maximize profit

$$P(x,y) = -\frac{1}{20}x^2 + 80x - \frac{1}{10}y^2 + 90y - 5000.$$

Use <u>first derivatives</u> to find the values of x and y, and use <u>second derivatives</u> to make sure that you have found where $P(x,y)$ is *maximized* (rather than *minimized*).

16 points **9.** Suppose now that the monopolist in Problem 8 must charge the same price in both countries. That is, it must be that $90 - \frac{x}{20} = 100 - \frac{y}{10}$.

Find the values of x and y which maximize profit

$$P(x, y) = -\frac{1}{20}x^2 + 80x - \frac{1}{10}y^2 + 90y - 5000.$$

subject to the equal price constraint that $90 - \frac{x}{20} = 100 - \frac{y}{10}$.