Math 316

Background/reminders of ideas used today

Non-parametric: we don't assume anything about the population distribution. For example, a normal distribution is parametric, with parameters μ and σ. Non-parametric basically means we just have a bunch of data.

Discrete: specific values (e.g. number of siblings).

Continuous: a range of values (e.g. height, weight).

Recall the binomial distribution. Example: A 70% free throw shooter will shoot 10 shots. The probability of making 8 or more shots is

$$ {}_{10}C_8(.70)^8(.30)^2 + {}_{10}C_9(.70)^9(.30)^1 + {}_{10}C_{10}(.70)^{10}(.30)^0 \approx 0.2335 + 0.1211 + 0.0282 = 0.3825. $$

Similarly, suppose I flip a coin, say 19 times. Here are the outcomes and probabilities.

# heads	# tails	Probability
0	19	${}_{19}C_0(.50)^0(.50)^{19} \approx 0.0000$
1	18	${}_{19}C_1(.50)^1(.50)^{18} \approx 0.0000$
2	17	${}_{19}C_2(.50)^2(.50)^{17} \approx 0.0003$
3	16	${}_{19}C_3(.50)^3(.50)^{16} \approx 0.0018$
4	15	${}_{19}C_4(.50)^4(.50)^{15} \approx 0.0074$
5	14	${}_{19}C_5(.50)^5(.50)^{14} \approx 0.0222$
6	13	${}_{19}C_6(.50)^6(.50)^{13} \approx 0.0518$
7	12	${}_{19}C_7(.50)^7(.50)^{12} \approx 0.0961$
8	11	${}_{19}C_8(.50)^8(.50)^{11} \approx 0.1442$
9	10	${}_{19}C_9(.50)^9(.50)^{10} \approx 0.1762$
10	9	${}_{19}C_{10}(.50)^{10}(.50)^9 \approx 0.1762$
11	8	${}_{19}C_{11}(.50)^{11}(.50)^8 \approx 0.1442$
12	7	${}_{19}C_{12}(.50)^{12}(.50)^7 \approx 0.0961$
13	6	${}_{19}C_{13}(.50)^{13}(.50)^6 \approx 0.0518$
14	5	${}_{19}C_{14}(.50)^{14}(.50)^5 \approx 0.0222$
15	4	${}_{19}C_{15}(.50)^{15}(.50)^4 \approx 0.0074$
16	3	${}_{19}C_{16}(.50)^{16}(.50)^3 \approx 0.0018$
17	2	${}_{19}C_{17}(.50)^{17}(.50)^2 \approx 0.0003$
18	1	${}_{19}C_{18}(.50)^{18}(.50)^1 \approx 0.0000$
19	0	${}_{19}C_{19}(.50)^{19}(.50)^0 \approx 0.0000$

So the probability of getting ≥ 15 heads or ≥ 15 tails (the values boxed in above) is

$$ \approx 2(.0074 + .0018 + .0003 + .0000 + .0000) \approx 0.019. $$

Similarly, the probability of getting ≥ 16 heads or ≥ 16 tails is

$$ \approx 2(.0018 + .0003 + .0000 + .0000) \approx 0.004. $$

Section 8.4 The Sign Test

Four possibilities on comparing two samples and where to find these in the book:

	Not paired	Paired
Normal distribution (e.g. due to large samples)	6.6 – 7.5	8.2
Not normal (i.e. don't know)	7.10	8.4, 8.5

Consider the following example (from a previous edition of our textbook):

Growth of Viruses In a series of experiments on a certain virus (mengovirus), a microbiologist measured the growth of two strains of the virus—a mutant strain and a nonmutant strain—on mouse cells in petri dishes. Replicate experiments were run on 19 different days. The data are shown in Table 8.3.2. Each number represents the total growth in 24 hours of the viruses in a single dish.[12]

Note that there is considerable variation from one run to another. For instance, run 1 gave relatively large values (160 and 97), whereas run 2 gave relatively small values (36 and 55). This variation between runs arises from unavoidable small variations in the experimental conditions. For instance, both the growth of the viruses and the measurement technique are highly sensitive to environmental conditions such as the temperature and CO_2 concentration in the incubator. Slight fluctuations in the environmental conditions cannot be prevented, and these fluctuations cause the variation that is reflected in the data. In this kind of situation the advantage of running the two strains concurrently (that is, in pairs) is particularly striking. ■

Table 8.3.2 Virus growth at twenty-four hours

Run	Nonmutant strain	Mutant strain	Run	Nonmutant strain	Mutant strain
1	160	97	11	61	15
2	36	55	12	14	10
3	82	31	13	140	150
4	100	95	14	68	44
5	140	80	15	110	31
6	73	110	16	37	14
7	110	100	17	95	57
8	180	100	18	64	70
9	62	6	19	58	45
10	43	7			

Table 8.4.2 Virus growth at twenty-four hours

Run	Nonmutant strain y_1	Mutant strain y_2	Sign of $d = y_1 - y_2$	Run	Nonmutant strain y_1	Mutant strain y_2	Sign of $d = y_1 - y_2$
1	160	97	+	11	61	15	+
2	36	55	–	12	14	10	+
3	82	31	+	13	140	150	–
4	100	95	+	14	68	44	+
5	140	80	+	15	110	31	+
6	73	110	–	16	37	14	+
7	110	100	+	17	95	57	+
8	180	100	+	18	64	70	–
9	62	6	+	19	58	45	+
10	43	7	+				

For the above example:

N_+ = number of $+$ signs $= 15$

N_- = number of – signs $= 4$

B_s = max of $N_+, N_- = 15$

Table 7 for the Sign Test:

$n_d = 19$

$B_s = 15 \Rightarrow P = 0.019$ (this is for a two-tailed test) from Table 7.

As usual, we would compare P to α, etc.

As the book says, the Sign Test for comparing two paired samples is a "blunt but handy tool." **Class Observation 1** gives some intuition regarding Table 7. Similar to Table 6, the table header

$$n_d \quad .20 \quad .10 \quad \cdots$$

tells us what test statistic B_s is needed to get a P-value less than $.20 \quad .10 \quad \cdots$. For example, for $n_d = 19$, to get $P \leq 0.05$ we would need $B_s \geq 15$. (Notice that $B_s = 14 \Rightarrow P = 0.064$ and $B_s = 15 \Rightarrow P = 0.019$.) Also, like in Table 6, values in Table 7 are given for *non-directional* (two-tailed) test. So to get the directional (one-tailed) P value, divide the given P value by 2. Basic intuition, as seen before:

More difference between samples (in this case, more $+$'s than –'s, or conversely)

$\Rightarrow$ Larger test statistic B_s

$\Rightarrow$ Smaller P

$\Rightarrow$ More likely to conclude there is a difference in the two populations, i.e. more likely to reject $H_0: \mu_1 = \mu_2$ and accept $H_A: \mu_1 \neq \mu_2$.

So where do the Table 7 values come from? The null hypothesis H_0 in this example is that there is no difference between the Nonmutant and Mutant strains. This means that for each pair of values, there is a 50% chance that the Nonmutant strain growth $>$ Mutant strain growth and a 50% chance that the Nonmutant strain growth $<$ Mutant strain growth. (**Why?**) This leads to a probability of ≈ 0.019 that in the 19 pairs (the 19 differences) there will be ≥ 15 $+$'s or ≥ 15 $-$'s. (Details for finding 0.019 can be found in the coins example and table on the **front side of this sheet**.) All values in Table 7 can be computed in this way. Luckily the book has computed all of these values for us.

Note: if you swap Sample 1 with Sample 2, you change the sign of $d = y_1 - y_2$, so for example, instead of 15 $+$'s and 4 $-$'s we would instead have 15 $-$'s and 4 $+$'s, and the test statistic B_s and P would be the same. Also: see **Treatment of Zeros on page 328**. Personally, I think dropping the pairs whose difference is 0 is a bad idea: you are losing information.

Section 8.5 The Wilcoxon Signed-Rank Test

The Wilcoxon Signed-Rank Test looks not only at the *sign* of the difference with a pair of values but also the *size* of the difference. (Hmmm, that seems like a better idea.) Because it uses more information than the Sign Test (which makes it slightly more complicated), it is more powerful (we are more likely to detect a difference in the two populations if there really is one). See **Book Example 8.5.1**. See the "**From Table 8.5.4…" paragraph** on page 333. See **Class Observation 2**. Let's do a **Variation of Example 8.5.1**. On your own, see Treatment of Zeros and Treatment of Ties on page 334.

8.6 Perspective

If time, let's look at **Example 8.6.1**. Important info: "All volunteers received health education literature and a brief lecture."