Math 316

Section 7.5 One-Tailed t Tests

First some old ideas.

Recall that in general *the probability of something occurring* is the same as *what fraction of all the possible outcomes are that particular something*. Also, α is the maximum allowable probability that we (i.e. the scientific community) can live with of making a Type I error: when the population means are approximately the same, but we conclude they are different—that is, we reject H_0 and instead accept H_A.

Suppose that the two populations actually had the same mean. Then P is:

- The likelihood of getting two samples—one from each population—with a difference (as measured by t_s) in sample means as large as or larger than what we have in the given samples; that is, P is the *fraction* of all pairs of samples (one from each population) which would have a difference in sample means as large as or larger than what we have in the given samples; that is,
- P is the likelihood of making a Type I error: the population means are the same $\mu_1 = \mu_2$, but we reject H_0 (that $\mu_1 = \mu_2$) and accept H_A (that $\mu_1 \neq \mu_2$).

For example, suppose $P = .096$, and if (perhaps because the probability of Type I error that I can live with is $\alpha = .100$), I reject H_0 and accept H_A, and conclude that $\mu_1 \neq \mu_2$. Then there is a 9.6% chance I am making a Type I error in concluding that the population means are different, since there is a 9.6% chance that my samples actually came from two populations that actually have the same mean. See **Class Example 1a.**

The one new idea today: <u>one tailed</u> (i.e. directional) hypothesis testing.

Rather than test the hypothesis that the two sample means are simply different, we can test the hypothesis that one is greater than the other. See **Class Example 1b**. Assuming the same confidence level, it is easier to show that one mean is less than (rather than simply $\neq$) the other (the <u>one-tailed directional alternative</u>) than it is to show that two means are different (the <u>two-tailed nondirectional alterative</u>). This seems a little counterintuitive, since the directional alternative is more specific, so you would think that it would be more difficult to show. *In the one-tailed case we are depending on additional information ahead of time: that we have some reason ahead of time to believe that one population mean should be less than the other*. See Book Example 7.5.4 and the discussion in the "Why…" paragraph on page 262. Regarding how we write the null hypothesis in the directional case, see **Note on page 258** about null hypothesis H_0.

If your direction ($\mu_1 > \mu_2$ vs. $\mu_1 < \mu_2$) is wrong, then you will end up with a P value that is > 0.5. See **Class Example 1c.**

If time, we'll discuss how t_s and $t_{\alpha/2}$ are related. This is similar to the book's discussion on pages 241 – 242.