

12.2

Final thought: Suppose all of your data were actually exactly on the same straight line

$$y = mx + b : \quad \begin{array}{c} X \\ \hline x_1 \\ \vdots \\ x_n \end{array} \quad \begin{array}{c} Y \\ \hline mx_1 + b \\ \vdots \\ mx_n + b \end{array}$$

$$\text{Then } \bar{x} = \frac{\sum_{i=1}^n x_i}{n}$$

$$\bar{y} = \frac{\sum_{i=1}^n (mx_i + b)}{n} = \frac{m \sum x_i + n \cdot b}{n}$$

$$= m\bar{x} + b$$

$$\text{Then } y_i - \bar{y} = (mx_i + b) - (m\bar{x} + b) \\ = m(x_i - \bar{x})$$

$$\text{So } r = \frac{\sum (x_i - \bar{x}) m (x_i - \bar{x})}{\sqrt{(\sum (x_i - \bar{x})^2) (\sum (m(x_i - \bar{x}))^2)}} \\ = \frac{m \sum (x_i - \bar{x})^2}{\sqrt{m^2} \sqrt{[\sum (x_i - \bar{x})^2]^2}} = \frac{m}{\sqrt{m^2}} = \begin{cases} 1 & \text{if } m > 0 \\ -1 & \text{if } m < 0 \end{cases}$$