Math 316

Section 11.7 Two-way ANOVA

We've seen One-way ANOVA and One-way ANOVA with blocks. Recall that blocking was a second factor that affects what we were measuring (e.g. alfalfa plant growth). In this section we look at Two-way ANOVA, in which there are two variables/<u>factors</u>/<u>sources</u> that affect what we are measuring, and now we are interested in the effect of both factors.

See **Table 11.7.1**. There are two variables that affect growth of soybeans: stress level and light level. **Figure 11.7.1** displays the data, and **Figures 11.7.2 and 11.7.3** give us a summary of the effect of light level and stress level.

If we didn't know better, we might try comparing the data two pairs at a time, e.g. Low vs. Moderate Light within the Control group or Control vs. Stress within the Low Light group, and so on. It turns out we can (and should) analyze all of the data simultaneously. In the end, for this example we will end up with the **Table 11.7.5**. There are the $MS(between\ factor\ 1)$ and $MS(between\ factor\ 2)$ values, like in One-way ANOVA with blocking, and there is now also a new $MS(Interaction)$ value as well. Since these values (in particular, $MS(Interaction)$) are becoming more complicated to compute, we don't have to find them: the book just gives the summary table to us. But it's still good to have some intuition about what affects each of these values.

So there are three different F_s test statistics. One for each of the two possible influences on the soybean growth (stress level and light level), plus a third one for the interaction between those two influences. If any of those three values is significant (i.e. gives us a P value less than α) then we reject the null hypothesis and accept the alternative.

So what's this "Interaction" thing? It is possible that two different factors interact and that this interaction is sort of a third factor. For example, if we look at how smoking and drinking each individually affect life span, it might be that the combination (interaction) of the two have their own effect, beyond the individual effects of the smoking or drinking.

See the **paragraph just above Example 11.7.5** We usually test for the presence of interactions first. If there is no significant effect of interactions, then we test for the significance of the effects of each effect individually. The possible outcomes/conclusions:

1. There is significant effect due to interactions; else
2. There is significant effect due to factor 1 and/or factor 2; else
3. There is not significant effect due to any of the three sources: neither of the two factors individually, nor their interaction.

We see in **Table 11.7.5**, based on the F values, that there is some effect on soybean growth due to the variation in stress levels as well as the variation of light levels. This is seen in **Figures 11.7.1 – 3**. We also see, based on the F value, that there is little effect on soybean growth due to the interaction between light and stress levels. How to visualize this? **See Figure 11.7.3**. Perfectly parallel lines mean no interaction.

Now compare this to **Example 11.7.3** and **Figure 11.7.4**. The non-parallel lines indicate that the interaction between the two factors (Zn level and Fe level) is itself another factor. This is seen in the ANOVA **Table 11.7.4**. From Figure 11.7.4:

- There seems to be a big difference between Low Fe and High Fe, which leads to a very large F_s value.
- There seems to be contradictory (and on average, almost no) differences between Low Zn and High Zn, which leads to a very small F_s value.
- The very non-parallel lines seen are an indication of strong interaction between Fe level and Zn level, which leads to a very large F_s value.

Using the online ANOVA tool (at the class homepage), let's experiment with two extreme cases to get more of a feel for MS and F_s for Between and Interaction. We can have more than two levels within each of the two factors/treatments/variables. See **Example 11.7.6** and **Figure 11.7.5** and **Table 11.7.7**.

Recall two things that make $F_s = \frac{MS(between)}{MS(within)}$ larger and thus P smaller, thus making it more likely that we will reject H_0 (that all treatments/groups are the same) and accept H_A (that at least one of the treatments/groups is different from the others):

1. Larger differences between $\bar{y}_1$ and/or $\bar{y}_2$ and/or … $\Rightarrow$ larger $MS(between)$.
2. Smaller variation within groups (smaller standard deviations within each group) $\Rightarrow$ smaller $MS(within)$.

Both of these result in less overlap between groups (for example, see **Figure 11.7.2**).

If the results remain same, but we have more data (larger samples), we end up with a smaller P value. Let's experiment with this a bit in class with the **online ANOVA tool**.

Finally, in the model $y_{ijk} = \mu + \tau_i + \beta_j + \gamma_{ij} + \varepsilon_{ijk}$ we have

y_{ijk} is the kth value in treatment/group i and treatment/group j

μ is the overall average, that is, what each y_{ijk} "should" be

τ_i is the amount of y_{ijk} (the change from μ) due to the first factor, level i

β_i is the amount of y_{ijk} (the change from μ) due to the second factor, level j

γ_{ij} is the amount of y_{ijk} (the change from μ) due to the interaction between the first factor, level i and the second factor, level j

ε_{ijk} is the random error present in each observation y_{ijk}