Before we get started, let's note that often when the values of data are larger, the mean is of course larger, and the values are often more spread out (so higher standard deviation). For example, the salaries of college students are usually lower and they are less spread out (less variability), while the salaries of adults further along in their careers are usually higher and more spread out (more variability). Another way to describe that data is more spread out is that their differences from the sample mean are greater. Finally, recall that <u>power</u> is how likely we will determine two populations are different (by rejecting H_0 and accepting H_A) if indeed they really are.

Section 11.5 Applicability of Methods

When is ANOVA appropriate? See **Standard Conditions** at the beginning of Section 11.5. Then see **Table 11.1.1**. Notice that $s_1 \approx s_2 \approx \cdots \approx s_5$. For normality, see **Figure 11.5.4**.

Next, let's look at **Figure 11.5.2**. In (a) the twelve values in each group are displayed relative to their group's mean. In (b), the five samples are in a different order, based on their group's sample mean (smallest to largest)—notice the symbol being used for each sample. In (b) we see the plots in (a), but the values in each sample are displayed relative to their respective means. The paragraph "While one could look…" describes why (b) might be more useful to us than (a) in visually determining how spread out each sample is, i.e. what the standard deviation is. Also notice in (b) that the standard deviation does not seem to increase as the mean increases. (As mentioned at the top of this handout, it is not uncommon for the standard deviation to increase as the sample mean increases.) An example of this is seen in **Figure 11.5.3**. This would violate the "equal standard deviations" condition mentioned at the beginning of Section 11.5.

Section 11.6 One-Way Randomized Blocks Design

In Sections 11.2 and 11.4 we looked at how varying organic treatments of insects affects corn weight. What if there is a second factor, such as how the five different types (treatments) of corn were located in the cornfield or in the lab? Let's discuss **Class Observations 1** for three possible placements. In the first two it might be easier to know how the placements would affect growth (e.g. as affected by sunlight), but the study is probably not very helpful if using these non-random placements. On the other hand, the third placement is probably better, but it might be more difficult to understand how the placements might have affected the corn growth. This section deals with how to account for and remove that second effect, in this case, the placement of the corn plants. By the way, One-Way refers to having one thing (e.g. treatment) that affects what we are measuring (e.g. corn growth). In Section 11.7 we will look at Two-Way ANOVA, in which we analyze how two different things (e.g. treatment, as well as types or amounts of fertilization) affect what we are measuring (e.g. corn growth).

We'll look at **Book Example 11.6.7** and **Table 11.6.3**. So we're interested in how the rain type affects alfalfa growth. However, a second factor—location of the plants ("Block")—might also be affecting the growth. **See Figure 11.6.1** for how the blocks were organized in this study. We want to understand and remove the effect of that second factor, the blocking, so that the effect of the rain type is clearer.

In **Figure 11.6.3**, you can see in the figure on the right that the blocks have a noticeable effect on the measurements (the five block means are different). So how do we account for that effect?

In **Figure 11.6.4** we see the effects of blocking in more detail. We see how the three values (from the three acidity levels) in the three blocks compare to the mean of their respective acidity level. For example, in Block 1 (using values from **Table 11.6.3**; Treatments are High acid, Low acid, and Control):

	High	Low	Ctrl
Block 1	1.30	1.78	2.67
Treatment mean	0.91	1.13	1.77
Difference	0.39	0.65	0.90

So it appears that Block 1 resulted in greater growth than any of the other blocks. Block 2 seems to cause the second most amount of growth. And so on. We also see this in the right part of **Figure 11.6.3**.

The grand mean of these 15 observations is 1.27. We see that Block 1 mean is $1.92 - 1.27 = 0.65$ larger than the grand mean, Block 2 mean is $1.55 - 1.27 = 0.28$ larger than the grand mean, and so on. In **Figure 11.6.5(a)** is a plot of the original data, and in **Figure 11.6.5 (b)** are those data but with the effects of blocking removed: we subtract from each value the effects of the block it came from, which is how much larger than the grand mean that block's mean is. For example, the High Acid value of 1.30 in Block 1 is modified to remove the effects of being in Block 1: we subtract 0.65 from 1.30 to get 0.65.

Notice that the means in each treatment group are the same (so the variability between groups remains the same), but the variability within each group is smaller, so the test statistic will be larger.

	High	Low	Ctrl	**Mean**	**Mean minus grand mean**
Block 1	1.30	1.78	2.67	**1.92**	**0.65**
Block 2	1.15	1.25	2.25	**1.55**	**0.28**
Block 3	0.50	1.27	1.46	**1.08**	**-0.19**
Block 4	0.30	.055	1.66	**0.84**	**-0.43**
Block 5	1.30	0.80	0.80	**0.97**	**-0.30**
Mean	**0.91**	**1.13**	**1.77**	$\boxed{1.27}$	0.00

Block 1	0.65	1.13	2.02	**1.27**
Block 2	0.87	0.97	1.97	**1.27**
Block 3	0.69	1.46	1.65	**1.27**
Block 4	0.73	0.98	2.09	**1.27**
Block 5	1.60	1.10	1.10	**1.27**
Mean	**0.91**	**1.13**	**1.77**	**1.27**

Subtract the effect of blocking from each block

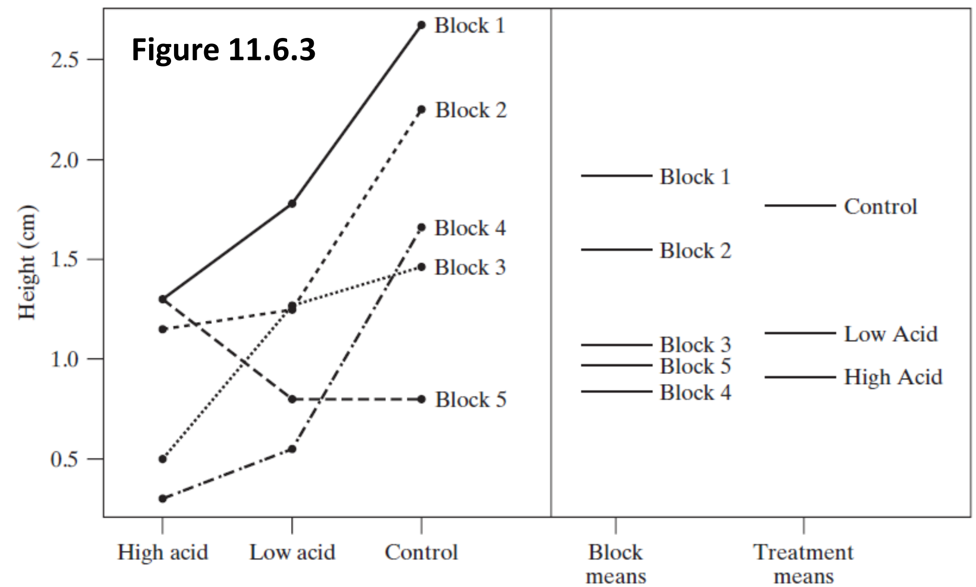

Figure 11.6.3

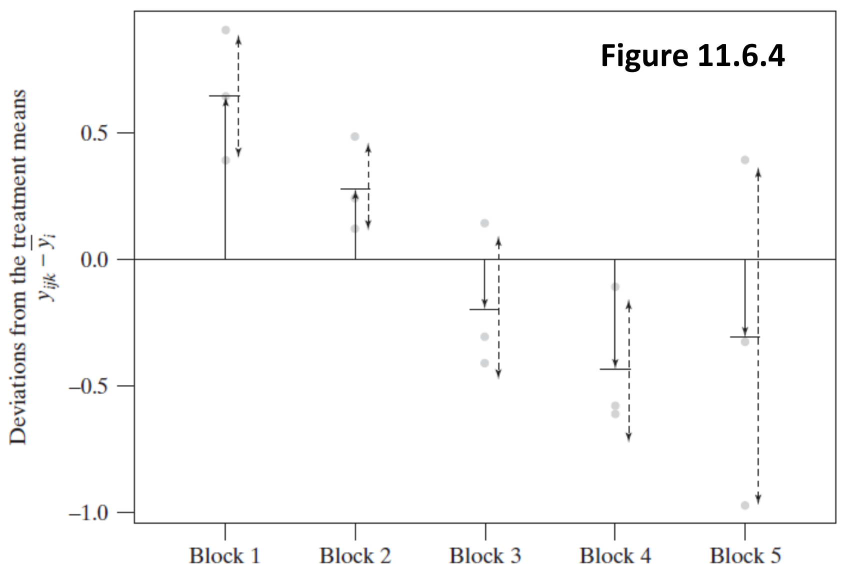

Figure 11.6.4

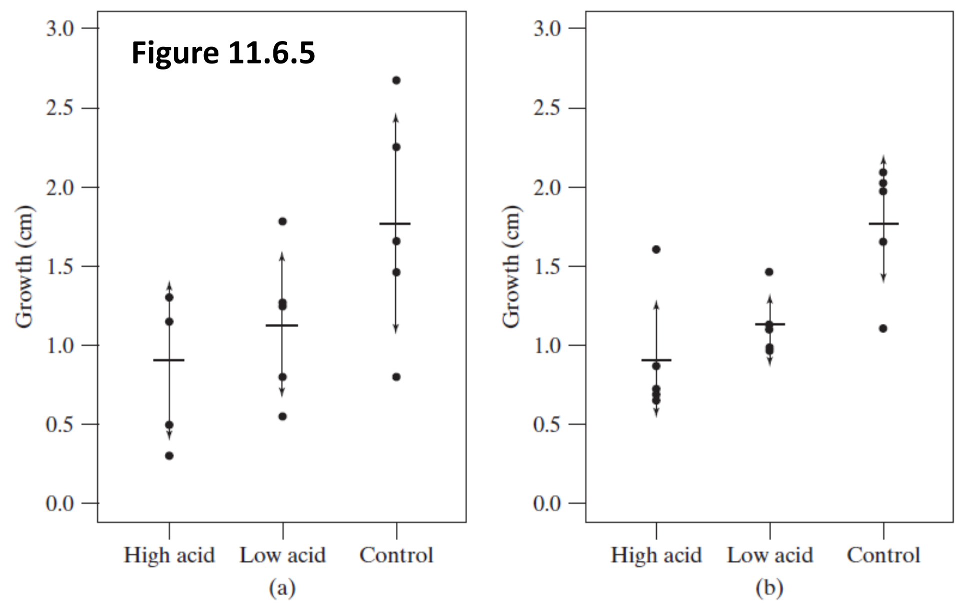

Figure 11.6.5

We've seen the effects of blocking. But how do we find our test statistic $\boldsymbol{F_s}$? Basically the same as we've already learned, but there is one more source of variance we have to consider, variance due to blocking. As before, $F_s = \frac{MS(between)}{MS(within)}$. We compute $MS(between)$ as before, but we now find $MS(within)$ differently. In One-way ANOVA,

$$SS(total) = SS(within) + SS(between\ treatments)$$

while in One-way ANOVA with blocking,

$$SS(total) = SS(within) + SS(between\ treatments) + SS(between\ blocks).$$

See page 473.

The $SS(within)$ is now comprised of two parts: the part due to the blocking (**see Figure 11.6.5(a) and (b)**) and the part that is there due to the treatments. So how do we find the part of $MS(within)$ that is not due to blocking?

As we saw for One-way ANOVA, the SS and MS for $between\ rain\ types$ is

Mean Square between Groups

$$\text{MS(between)} = \frac{\sum_{i=1}^{I} n_i(\bar{y}_i - \bar{\bar{y}})^2}{I - 1}$$

Sum of Squares and df between Groups

$$\text{SS(between)} = \sum_{i=1}^{I} n_i(\bar{y}_i - \bar{\bar{y}})^2$$

$$\text{df(between)} = I - 1$$

Similarly, the SS and MS for $between\ blocks$ is

Mean Squares between Blocks

$$\text{MS(blocks)} = \frac{\sum_{j=1}^{J} m_j(\bar{y}_{\bullet j} - \bar{\bar{y}})^2}{J - 1}$$

Sum of Squares and df between Blocks

$$\text{SS(blocks)} = \sum_{j=1}^{J} m_j(\bar{y}_{\bullet j} - \bar{\bar{y}})^2$$

$$\text{df(blocks)} = J - 1$$

So we can find

$$SS(within) = SS(total) - SS(between\ rain\ types) - SS(between\ blocks)$$

where

Definition of Total Sum of Squares

$$\text{SS(total)} = \sum_{i=1}^{I}\sum_{j=1}^{n_i}(y_{ij} - \bar{\bar{y}})^2$$

This is done in the book for the Rain and Alfalfa example, where $F_s = \frac{\frac{1.986}{2}}{\frac{1.452}{8}} = \frac{0.993}{0.182} \approx 5.47.$

	High acid	Low acid	Control	Block mean
Block 1	1.30	1.78	2.67	1.917
Block 2	1.15	1.25	2.25	1.550
Block 3	0.50	1.27	1.46	1.077
Block 4	0.30	0.55	1.66	0.837
Block 5	1.30	0.80	0.80	0.967
Treatment mean $= \bar{y}_i$	0.910	1.130	1.768	

Table 11.6.4 ANOVA table for alfalfa experiment

Source	df	SS	MS	F ratio
Between treatments	2	1.986	0.993	5.47
Between blocks	4	2.441	0.610	
Within groups	8	1.452	0.182	
Total	14			

Important: if we did *not* account for blocking, we would have $F_s = \frac{\frac{1.986}{2}}{\frac{2.441+1.452}{12}} \approx 3.06.$

Smaller F_s $\Rightarrow$ *larger P* $\Rightarrow$ *less likely* to reject H_0 and accept H_A (so a less powerful test).

Experiment a bit: Suppose we change the .80 in the Control to 1.80. How will that change our results? Look at **Figure 11.6.5** to visualize this. My guess, before doing any work: larger $MS(between)$, smaller $MS(within)$, and larger F_s. Let's see:

	High acid	Low acid	Control	Block mean
Block 1	1.30	1.78	2.67	1.917
Block 2	1.15	1.25	2.25	1.550
Block 3	0.50	1.27	1.46	1.077
Block 4	0.30	0.55	1.66	0.837
Block 5	1.30	0.80	$\boxed{1.80}$	1.300
Treatment mean $= \bar{y}_i$	0.910	1.130	1.97	

Table 11.6.4 ANOVA table for alfalfa experiment

Source	df	SS	MS	F ratio	
Between treatments	2	3.117	1.558	19.04	**Larger than before**
Between blocks	4	2.103	0.526		
Within groups	8	0.655	0.082		**Smaller than before**
Total	14				

To reiterate: removing blocking effects makes test more powerful: we are more likely to detect a difference (via larger test statistic) in the groups/treatments if there really is one.