Recall the word mean is the sum of a bunch of values divided by some number. Also recall weighted averages: the average of the numbers 1 1 1 4 4 is $\frac{3\cdot 1+2\cdot 4}{3+2} = \frac{11}{5}$, where 3 and 2 are the weightings of the values of 1 and 4.

Section 11.1 Introduction

To get an idea about this chapter, let's look at **Example 11.1.1** (Table 11.1.1 and **Figure 11.1.1**). First, suppose that we just had samples from the first two of those five groups. Would we decide that the two samples are different enough to decide that their two respective populations are different, i.e., that those two treatments have different effects on growing corn? Recall the test statistic

$$t_s = \frac{\bar{y}_1 - \bar{y}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

from which we find a P value, etc. So we are comparing the *variation between the samples* to (i.e. dividing by) a sort of average of the *variation within the samples* (as measured by standard deviations, and of course sample sizes always plays a role). Comparing *variance between* to *variance within* is called Analysis of Variance, i.e. ANOVA.

Section 11.2 Basic One-Way Analysis of Variance
Section 11.4 The Global F Test

For more than two samples, as in **Example 11.1.1**, how do we compare the variation *between* the samples to to the variability *within* those samples? For this there is a new test and corresponding test statistic F_s, from which we'll get a P value from our final new table, Table 10. And we'll do this for all five groups simultaneously, rather than two groups at a time. On your own, see "Why Not Repeated t Tests" between two samples at a time, on page 444, for some thoughts on that issue.

We'll find $F_s = \frac{MS(between)}{MS(within)}$ where $MS(between)$ measures the variance between all of the samples, and $MS(within)$ measures the variance within all of the samples. (MS means Mean Square.) So how to define/find $MS(between)$ and $MS(within)$? First, some notation:

$$\begin{aligned}
I &= number\ of\ groups \\
i &= group\ number \\
n_i &= size\ of\ group\ i \\
n_{.} &= total\ \#\ of\ observations = n_1 + n_2 + \cdots + n_I \\
s_i^2 &= variance\ (standard\ deviation\ squared)\ of\ group\ i \\
\bar{y}_i &= mean\ of\ group\ i\ sample \\
y_{ij} &= group\ i, observation/value\ j \\
\bar{\bar{y}} &= grand\ mean, i.e. mean\ of\ all\ observations\ in\ all\ groups
\end{aligned}$$

The grand mean $\bar{\bar{y}}$ is easy to find: it's just the average of all of the observations, i.e. all of the values in all of the samples. From the book:

The total number of observations is

$$n. = \sum_{i=1}^{I} n_i$$

Finally, the **grand mean**—the mean of all the observations—is

$$\bar{\bar{y}} = \frac{\sum_{i=1}^{I}\sum_{j=1}^{n_i} y_{ij}}{n.}$$

Equivalently we can express $\bar{\bar{y}}$ as a weighted average of the group means

$$\bar{\bar{y}} = \frac{\sum_{i=1}^{I} n_i \bar{y}_i}{\sum_{i}^{I} n_i} = \frac{\sum_{i=1}^{I} n_i \bar{y}_i}{n.}$$

Let's find this for Table 11.1.1, which I've **posted online** at the class homepage**.**

Next, the way we find $MS(between)$ is simply a weighted average of how different each sample mean is from the grand mean.

Mean Square between Groups

$$\text{MS(between)} = \frac{\sum_{i=1}^{I} n_i(\bar{y}_i - \bar{\bar{y}})^2}{I - 1}$$

Let's find this for Table 11.1.1.

Next, how do we find a single number which measures the variance within all of the samples? We simply find a weighted average of the variances of the samples.

$$s_{\text{pooled}}^2 = s^2 = \frac{\sum_{i=1}^{I} (n_i - 1)s_i^2}{\sum_{i=1}^{I} (n_i - 1)}$$

Let's do this for Table 11.1.1.

(You might recall that s_{pooled}^2 is an alternate way of combining, i.e. "pooling," variances for two or more samples, as we saw in Section 6.6 on **pages 207 and 209**.) s_{pooled}^2 is also sometimes referred to as "Mean Square Within," that is, $s_{pooled}^2 = MS(within)$.

All of these values can be summarized as on page 454.

ANOVA Quantities with Formulas

Source	df	SS (Sum of Squares)	MS (Mean Square)
Between groups	$I - 1$	$\sum_{i=1}^{I} n_i(\bar{y}_i - \bar{\bar{y}})^2$	SS/df
Within groups	$n. - I$	$\sum_{i=1}^{I} (n_i - 1)s_i^2$	SS/df
Total	$n. - 1$	$\sum_{i=1}^{I}\sum_{j=1}^{n_i} (y_{ij} - \bar{\bar{y}})^2$	

Let's do this for Table 11.1.1.

The Mean Square is simply the Sum of Squares divided by degrees of freedom. Notice that the two Sums of Squares add up to the Total Sum of Squares, which can be computed by the formula listed above. That formula is simply the sum of the squares of the differences between all of the data from the grand mean. The sum of squares fact that $SS(between) + SS(within) = SS(total)$ will be important to us in later sections. For now, let's simply **notice this** in the spreadsheet.

So

$$F_s = \frac{MS(between)}{MS(within)}$$

and the corresponding P value is found in Table 10. **Let's do this for Table 11.1.1.**

This test cannot be directional. What would the direction(s) even be in H_A? Remember:

- The null hypothesis states that all of the populations have the same mean, thus the samples should have approximately the same.
- The test statistic measures how different the samples are from each other: larger test statistic means the samples are more different from each other.
- The P value is the likelihood of getting samples that are this different from each other (or more so) if the null hypothesis is true (if populations have same mean).

Let's change the samples a bit and see what values change. The spreadsheet we have been looking at is online, if you want to experiment with it a bit.

A note on the model $y_{ij} = \mu + \tau_i + \varepsilon_{ij}$ discussed at the beginning of Section 11.3, and summarized here:

y_{ij} is the jth value in group/treatment i

μ is the overall average, that is, what each y_{ij} "should" be

τ_i is the amount of y_{ij} (the change from μ) due to treatment i

ε_{ij} is the random error present in each recorded measurement y_{ij}

We'll look at these ideas more later as the ideas come up in subsequent sections.