Math 316

Read Section 10.1 on your own. It's a nice intro to what this chapter is about.

Section 10.2 The 2×2 Contigency Table

Section 10.5 The $r \times k$ Contigency Table

In Section 9.4 we first worked with χ_s^2. That same formula can be used when there are multiple populations. In Section 10.5 is the general case, and Section 10.2 deals with a specific case. I think it is easier to simply look at the general case from the beginning. Let's work **Class Example 1**. For problems with more than one population, there is not an expected fraction/percentage/proportion within each population. **On the backside of this sheet** are more details about how to compute the expected value in each case.

In general, as seen in the past for other test statistics, if we have the same results, but with larger samples sizes, we end up with a larger test statistic and smaller P value. Suppose in **Class Example 1 we had the same proportions in each of the six values, but with double the sample size**. We see that we end up with a test statistic that is twice as large as before.

In Section 10.2 we deal with the 2×2 case. I guess that the book figures it's easier to deal with this special case (in Section 10.2) first before the general case (in Section 10.5). I don't think so. There are a couple of things unique to the 2×2 case: this is the only case in which the alternative hypothesis H_A can be directional, and (as we'll see in Section 10.8) for the 2×2 case we can have paired data. Let's work **Class Example 2**.

When can you swap "population" and "categories." A few examples:

Class Example 1 (hair color vs. eye color)?

HW 10.2.5?

HW 10.2.6?

HW 10.5.3?

By the way, here is another way to describe/write the hair color vs. eye color problem. Recall notation $\Pr\{A|B\}$ is "the probability that A is true or will occur, given that B is true or has occurred." Also recall that the hat $\widehat{\ }$ means we are dealing with the sample, rather than the population. So in the hair color vs. eye color problem, where BE means blue eyes and LH means light hair, we have $\widehat{Pr}\{BE\} = \frac{60}{250} = .24$ and $\widehat{Pr}\{BE|LH\} = \frac{30}{150} = .20$. So the question is whether these two *sample* probabilities of having blue eyes (one when we don't know hair color, and the other when we do) are different enough to conclude that there is also a difference in the corresponding *populations*, that is, we're trying to determine if hair color influences eye color. We might have a null hypothesis $H_0: Pr\{BE|LH\} = \Pr\{BE\}$ and alternative $H_A: \Pr\{BE|LH\} \neq \Pr\{BE\}$.

More details on how we compute the expected value in contingency tables

Here are the details on how the expected values are found in Class Example 1. Since $\frac{60}{250} = 0.24 = 24\%$ of everyone has brown eyes, then (assuming that hair color does not affect eye color) we would expect that:

24% of the light haired people would have brown eyes

24% of the dark haired people would also have brown eyes.

So

$0.24 \cdot 150 = 36$ of the light haired people would have brown eyes

$0.24 \cdot 100 = 24$ of the dark haired people would have brown eyes.

That is

The number of brown eyed, light haired people would be $\frac{60}{250} \cdot 150 = \frac{60 \cdot 150}{250}$

The number of brown eyed, dark haired people would be $\frac{60}{250} \cdot 100 = \frac{60 \cdot 1000}{250}$.

Here is a second way to look at this.

Since $\frac{150}{250} = 0.60 = 60\%$ of everyone has light hair, then (assuming that eye color does not affect hair color) we would expect that:

60% of the brown eyed people would have light hair

60% of the grey/green eyed people would have light hair

60% of the blue eyed people would have light hair

So

$0.60 \cdot 60 = 36$ of the brown eye people would have light hair

$0.60 \cdot 110 = 66$ of the green/grey eyed people would have light hair

$0.60 \cdot 30 = 18$ of the blue eyed people would have light hair

So

The number of light haired, brown eyed people would be $\frac{150}{250} \cdot 60 = \frac{150 \cdot 60}{250}$

The number of light haired, grey/green eyed people would be $\frac{150}{250} \cdot 66 = \frac{150 \cdot 66}{250}$

The number of light haired, blue eyed people would be $\frac{150}{250} \cdot 18 = \frac{150 \cdot 18}{250}$

I'm guessing by now you're seeing the pattern of to compute the expected values.

On another note. Why the word “contingency”? “Contingent” means “dependent,” as in “Is hair color contingent or not upon eye color?”