

3.3.1

(a)  $\Pr\{S\} = \frac{1213}{6549} \approx .1852$       Also:  $\Pr\{HI\} = \frac{2115}{6549} \approx .3230$

(b)  $\Pr\{S|HI\} = \frac{247}{2115} \approx .1168$       Also:  $\Pr\{HI|S\} = \frac{247}{1213} \approx .2036$

(c)  $\Pr\{S|HI\} < \Pr\{S\}$       Also:  $\Pr\{HI|S\} < \Pr\{HI\}$

So  $S$  and  $HI$  are not independent: one being true makes the other less likely.

Also:  $\frac{\Pr\{S|HI\}}{\Pr\{S\}} \approx \frac{.1168}{.1852} \approx .6307$ . So  $\Pr\{S|HI\} \approx .6307 \Pr\{S\} \approx \frac{2}{3} \Pr\{S\}$ .

So if  $HI$  is true, then  $S$  is only about  $\frac{2}{3}$  as likely as it was (before knowing  $HI$ ).

Similarly,  $\frac{\Pr\{HI|S\}}{\Pr\{HI\}} \approx \frac{.2036}{.3230} \approx .6307$ . So  $\Pr\{HI|S\} \approx .6307 \Pr\{HI\} \approx \frac{2}{3} \Pr\{HI\}$ .

So if  $S$  is true, then  $HI$  is only about  $\frac{2}{3}$  as likely as it was (before knowing  $S$ ).

So we see that if either of  $S$  or  $HI$  is true, then the other is only about  $\frac{2}{3}$  as likely to be true.

So *Smokers* tend not to be *High Income* and *High Income* persons tend not to be *Smokers*.

3.3.5

Recall that two events  $A$  and  $B$  are independent if:

$$\Pr\{A|B\} = \Pr\{A\}$$

$$\Pr\{B|A\} = \Pr\{B\}$$

$$\Pr\{A \text{ and } B\} = \Pr\{A\} \cdot \Pr\{B\}$$

If any one of these is true, then the other two are also true.

So we can simply check to see if  $\Pr\{HS \text{ and } WS\} = \Pr\{HS\} \cdot \Pr\{WS\}$ .

Since  $\Pr\{HS \text{ and } WS\} = .08$  and  $\Pr\{HS\} \cdot \Pr\{WS\} = (.30)(.20) = .06 \neq .08$ ,

then  $HS$  and  $WS$  are not independent: one being true changes the likelihood of the other being true.

Additional observations:

$$\Pr\{HS|WS\} = \frac{\Pr\{HS \text{ and } WS\}}{\Pr\{WS\}} = \frac{.08}{.20} = .40, \text{ so } \frac{\Pr\{HS|WS\}}{\Pr\{HS\}} = \frac{.40}{.30} = \frac{4}{3}, \text{ so } \Pr\{HS|WS\} = \frac{4}{3} \Pr\{HS\}.$$

$$\Pr\{WS|HS\} = \frac{\Pr\{HS \text{ and } WS\}}{\Pr\{HS\}} = \frac{.08}{.30} = .2\bar{6}, \text{ so } \frac{\Pr\{WS|HS\}}{\Pr\{WS\}} = \frac{.2\bar{6}}{.20} = \frac{4}{3}, \text{ so } \Pr\{WS|HS\} = \frac{4}{3} \Pr\{WS\}.$$

So we see that if either the wife or husband smokes, then it is about 33% more likely (since  $\frac{4}{3} \approx 1.33$ ) that the other smokes. Perhaps smoking is kind of social thing: if one person smokes, then his/her spouse is more likely to smoke. Or perhaps smokers are simply attracted to other smokers.