

	Ancrod	Placebo	Total
Yes	13 (8.928)	5 (9.072)	18
No	235 (239.072)	247 (242.928)	482
Total	248	252	500

$\chi^2 = 3.82$. With $df = 1$, Table 9 gives $\chi^2_{0.10} = 2.71$ and $\chi^2_{0.05} = 3.84$, so $0.05 < P < 0.10$. We do not reject H_0 ; there is insufficient evidence ($0.05 < P < 0.10$) to conclude that hemorrhaging is more likely under one treatment than under the other.

- 10.2.14 (a) H_0 : There is no relationship between a woman's response to a man and her menstrual phase;
 H_A : Women are more likely to respond to a man during their fertile phase.

(b) The sample proportions for Yes are $\hat{p}_1 = 13/60 \approx 0.22 = 0.22$ and $\hat{p}_2 = 11/140 \approx 0.08$. The expected cell counts under the null hypothesis are $e_{11} = 7.2$; $e_{12} = 16.8$; $e_{21} = 52.8$; $e_{22} = 123.2$.

(c) The data point in the direction predicted by H_A , so the P-value is $0.0059/2 = 0.00295$. We reject H_0 because the P-value is smaller than 0.02. We have strong evidence ($P=0.00295$) to conclude that women are more receptive to men during their fertile phase of their menstrual cycle.

- 10.2.15 (a) Six-banded armadillos make up the same proportion of roadkill in both locations.

(b) $\Pr\{\text{Armadillo killed} \mid \text{Atlantic Forest roadkill}\} = \Pr\{\text{Armadillo killed} \mid \text{Cerrado roadkill}\}$

(c)

	Atlantic Forest	Cerrado
Total Animals Killed (n)	178	318
Armadillos	51	66
Percent	28.7%	20.8%

(d)

Count (Expected)	Armadillo	Other	Total
Atlantic Forest	51 (41.9879)	127 (136.012)	178
Cerrado	66 (75.0121)	252 (242.988)	318
Total	117	379	496

(e) There is statistically significant evidence ($P\text{-value} = 0.047 < \alpha = 0.05$) that the proportion of roadkill that is Armadillo is not the same for both locations.

(f) While it is reasonable to believe that the armadillos make up different proportions of roadkill at the two locations, that doesn't necessarily mean that there is more danger in one location than the other.

For example, if armadillos make up a larger proportion of fauna in one habitat than the other, one would naturally expect that they would make up a larger proportion of roadkill.

- 10.3.1 (a) Among black mice, the fraction having smooth coats is $\Pr\{S|B\}$. Among gray mice, the fraction having smooth coats is $\Pr\{S|G\}$. Thus, the statement "Smooth coats are more common among black mice than among gray mice" asserts that $\Pr\{S|B\} > \Pr\{S|G\}$.

(b) $\Pr\{S|B\} > \Pr\{W|B\}$ (which means that $\Pr\{S|B\} > 0.5$).

(c) $\Pr\{B|S\} > \Pr\{B|W\}$.

(d) $\Pr\{B|S\} > \Pr\{G|S\}$ (which means that $\Pr\{B|S\} > 0.5$).

(e) $\Pr\{S\} > \Pr\{W\}$ (which means that $\Pr\{S\} > 0.5$).

- 10.3.2 (a) (i) There is no single correct answer. One typical answer is

	B	G	Total
W	30	50	80
S	30	100	130
Total	60	150	210

$$\hat{p}_T\{W|B\} = 30/60 = 0.5; \hat{p}_T\{W|G\} = 50/150 = 1/3; 0.5 > 1/3.$$

(ii)

	B	G	Total
W	20	50	70
S	40	100	140
Total	60	150	210

$$\hat{p}_T\{W|B\} = 20/60 = 1/3; \hat{p}_T\{W|G\} = 50/150 = 1/3; 1/3 = 1/3.$$

(b) (i) Using the typical answer from part (a) (i),

$$\hat{p}_T\{B|W\} = 30/80 = 0.375; \hat{p}_T\{B|S\} = 30/130 = 0.231.$$

(ii) $\hat{p}_T\{B|W\} = 20/70 = 2/7$; $\hat{p}_T\{B|S\} = 40/140 = 2/7$.

(c) The first data set, with $\hat{p}_T\{W|B\} > \hat{p}_T\{W|G\}$, has $\hat{p}_T\{B|W\} > \hat{p}_T\{B|S\}$. Because of Fact 10.3.2, it is not possible to have a data set for which $\hat{p}_T\{W|B\} > \hat{p}_T\{W|G\}$ but $\hat{p}_T\{B|W\} < \hat{p}_T\{B|S\}$.

- 10.3.3 (a) The estimated conditional probability that a patient dies, given that he had surgery, is

$$\hat{p}_T\{D|S\} = \frac{83}{347} = 0.239. \text{ Likewise, } \hat{p}_T\{D|WW\} = \frac{106}{348} = 0.3046.$$

The estimated conditional probability that a patient had surgery, given that the patient died, is

$$\hat{p}_T\{S|D\} = \frac{83}{189} = 0.7615. \text{ Likewise, } \hat{p}_T\{S|A\} = \frac{264}{506} = 0.5217.$$

- (b) H_0 : There is no relationship between treatment and survival.
 H_A : There is a relationship between treatment and survival.

$\chi^2 = 3.75$. With $df = 1$, Table 9 gives $\chi^2_{0.10} = 2.71$ and $\chi^2_{0.05} = 3.84$, so $0.05 < P < 0.10$.

We do not reject H_0 ; there is no significant evidence ($0.05 < P < 0.10$) that treatment and survival are related.

- 10.3.4 The following table shows the data arranged as a contingency table.

		Preferred Hand		Total
		Right	Left	
Preferred Foot	Right	2012	121	2133
	Left	142	116	258
Total		2154	237	2391

Let RH, LH, RF, and LF denote right-handed, left-handed, right-footed, and left-footed.

- (a) The estimated conditional probability that a woman is right-footed, given that she is right-handed, is $\hat{p}_r\{RF|RH\} = \frac{2012}{2154} = 0.934$.
- (b) The estimated conditional probability that a woman is right-footed, given that she is left-handed, is $\hat{p}_r\{RF|LH\} = \frac{121}{237} = 0.511$.
- (c) We test independence of hand preference and foot preference using a χ^2 test. We calculated expected frequencies from the formula

$$E = \frac{(\text{Row total})\pi(\text{Column total})}{\text{Grand total}}$$

The following table shows the expected frequencies:

		Preferred Hand		Total
		Right	Left	
Preferred Foot	Right	1921.57	211.43	2133
	Left	232.43	25.57	258
Total		2154	237	2391

The χ^2 test statistic is

$$\chi^2 = \frac{(2012 - 1921.57)^2}{1921.57} + \frac{(121 - 211.43)^2}{211.43} + \frac{(142 - 232.43)^2}{232.43} + \frac{(116 - 25.57)^2}{25.57} = 398.$$

- (d) The null hypothesis can be expressed as

$$H_0: \Pr\{RF|RH\} = 0.5 = \Pr\{LF|RH\}$$

This requires a goodness-of-fit test. The expected frequencies are calculated from H_0 as

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Right-footed: $E = (0.5)(2154) = 1077$

Left-footed: $E = (0.5)(2154) = 1077$

The observed frequencies are

Right-footed: 2012

Left-footed: 142

The χ^2 test statistic is

$$\chi^2 = \frac{(2012 - 1077)^2}{1077} + \frac{(142 - 1077)^2}{1077} = 1,623.$$

- 10.3.5 (a) $\Pr\{DY|EY\} > \Pr\{DY|EN\}$

(b) $\Pr\{EY|DY\} > \Pr\{EY|DN\}$

(c) $\Pr\{EY|DY\} > \Pr\{EN|DY\}$

(d) $\Pr\{EY|DY\} > 0.5$ (or $\Pr\{EY|DY\} > \Pr\{EN|DY\}$)

(e) $\Pr\{DY|EY\} > 0.5$ (or $\Pr\{DY|EY\} > \Pr\{DN|EY\}$)

(f) $\Pr\{DY|EY\} > \Pr\{DY|EN\}$

(g) $\Pr\{DY|EY\} > \Pr\{DN|EY\}$

- 10.3.6 Statements (a), (b), and (f) are all equivalent and all express the assertion that occurrence of the disease is associated with exposure to the agent.

- 10.3.7 (a) There is no single correct answer. One typical answer is

		EY	EN	Total
		DY	30	50
DN	70	450	520	
Total		100	500	600

$$\hat{p}_r\{DY|EY\} = 30/100 = 0.3; \hat{p}_r\{DY|EN\} = 50/500 = 0.1; 0.3 > 0.1.$$

$$\hat{p}_r\{EY|DY\} = 30/80 = 0.375; \hat{p}_r\{EN|DY\} = 50/80 = 0.625; 0.375 < 0.625.$$

- (b) There is no single correct answer. One answer would be the example given in part (a).
- (c) This is not possible, due to Fact 10.3.2.

10.3.8 Let MP, MA, HP, and HA denote maples present, maples absent, hickories present, and hickories absent.

	MP	MA	Total
HP	26	63	89
HA	29	26	55
Total	55	89	144

H_0 : Maples and hickories are distributed independently ($\Pr\{HP|MP\} = \Pr\{HP|MA\}$)

H_A : Maples and hickories are not distributed independently ($\Pr\{HP|MP\} \neq \Pr\{HP|MA\}$)

$\hat{p}_T\{HP|MP\} = 26/55 \approx 0.47$; $\hat{p}_T\{HP|MA\} = 63/89 \approx 0.71$. With $df = 1$, Table 9 gives $\chi^2_{0.01} = 6.63$ and $\chi^2_{0.001} = 10.83$, so $0.001 < P < 0.01$ and we reject H_0 . There is sufficient evidence ($0.001 < P < 0.01$) to conclude that hickories are more likely to be present when maples are absent than if maples are present. Thus, there is repulsion between the species. (This repulsion need not be an actual biological interaction; it could be that the two species simply prefer different local habitats.)

10.3.9 Let AP, AA, BP, and BA denote species A present, species A absent, species B present, and species B absent.

	AP	AA	Total
BP	30	10	40
BA	49	55	104
Total	79	65	144

H_0 : Species A and species B are distributed independently ($\Pr\{BP|AP\} = \Pr\{BP|AA\}$)

H_A : Species A and species B are not distributed independently ($\Pr\{BP|AP\} \neq \Pr\{BP|AA\}$)

$\hat{p}_T\{BP|AP\} = 30/79 \approx 0.38$; $\hat{p}_T\{BP|AA\} = 10/65 \approx 0.15$. With $df = 1$, Table 9 gives $\chi^2_{0.01} = 6.63$ and $\chi^2_{0.001} = 10.83$, so $0.001 < P < 0.01$ and we reject H_0 . There is sufficient evidence ($0.001 < P < 0.01$) to conclude that species B is more likely to be present when species A is present than if species A is absent. Thus, there is attraction between the species. (This attraction need not be an actual biological interaction; it could be that the two species simply prefer similar local habitats.)

10.3.10 (a) $\hat{p}_T\{\text{Yes}|A\} = 111/513 \approx 0.22$; $\hat{p}_T\{\text{Yes}|B\} = 74/515 \approx 0.14$.

(b) $\hat{p}_T\{A|\text{Yes}\} = 111/185 = 0.6$; $\hat{p}_T\{A|\text{No}\} = 402/843 \approx 0.48$.

10.3.11 There is no single correct answer. One typical answer is

		Treatment		Total
		Angioplasty	Bypass	
Angina?	Yes	20	80	100
	No	100	800	900
Total		120	880	1000

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$\hat{p}_T\{\text{Yes}|A\} = 20/100 = 0.2$; $\hat{p}_T\{\text{Yes}|B\} = 80/800 = 0.1$. Nevertheless, the majority of patients with angina (80 of them) had bypass surgery.

10.3.12 (a) True. (780 of the 1000 pairs have the same handedness.)

(b) True. (780 of the 1000 pairs have the same handedness.)

(c) True. (780 of the 1000 pairs have the same handedness.)

(d) True. ($\hat{p}_T\{\text{RH sister}|\text{RH brother}\} = 765/900 = 0.85 = 85/100 = \hat{p}_T\{\text{RH sister}|\text{LH brother}\}$.)

(e) True. ($\hat{p}_T\{\text{RH brother}|\text{LH sister}\} = 135/150 = 0.9$.)

• 10.4.1 Tables that more strongly support H_A are those with fewer than 2 deaths on treatment B. There are two such tables:

5	1
9	15

6	0
8	16

10.4.2 There are three tables that more strongly favor treatment A:

6	2
11	14

7	1
10	15

8	0
9	16

10.4.3 The P-value is quite small. The data provide strong evidence that the probability of insomnia is higher on the antidepressant than on placebo.

10.4.4 (a) There is no significant evidence for any reasonable α that the meal-replacement increases the risk of preterm birth compared to the control diet. The P-value is very large.

(b) No. While there is no evidence the meal-replacements are doing any harm, this is not compelling evidence for the safety of the meal-replacement diet.

(c) In favor of a directional test: If the goal is to detect any safety concerns (i.e., that the meal replacement is harmful), a directional test is more powerful than a non-directional one and is preferred.

(d) In favor of a non-directional test: It is quite possible (and maybe even suspected) that women with a healthy weight will have better delivery outcomes. Thus, the meal-replacement may offer a benefit and reduce the risk of preterm birth. A directional test (for safety) would not be able to test for this possible beneficial effect, while a non-directional test would.

(b)

	Band Color			Total
	Clear	Dark	Unreadable	
February	20.45	59.09	20.45	100
March	17.65	73.53	8.82	100

(c) A variety of software packages can be used, or the problem can be solved by hand. R gives the following output.

```
Pearson's Chi-squared test
```

```
data: Month and Color
X-squared = 2.3766, df = 2, p-value = 0.3047
```

Since the P-value is greater than $\alpha = 0.10$, there is no statistically significant evidence that distribution of band colors is different across the two months.

10.5.7 H_0 : There is no association between treatment group and condition

H_A : Treatment group and condition are related

	No response	Moderate response	Marked response	Remission	Total
Fluvoxamine	15 (17.831)	7 (6.747)	3 (2.892)	15 (12.530)	40
Placebo	22 (19.169)	7 (7.253)	3 (3.108)	11 (13.470)	43
Total	37	14	6	26	83

The test statistic is $\chi^2_1 = 1.83$. The degrees of freedom are $df = (2 - 1)(4 - 1) = 3$. From Table 9 we find that $\chi^2_{3,0.20} = 4.64$. Thus, $P > 0.20$, so we do not reject H_0 . There is insufficient evidence ($P > 0.20$) to conclude that treatment group and condition are related.

10.5.8 (a) H_0 : There is no association between treatment group and condition

H_A : Treatment group and condition are related

(b) The degrees of freedom are $(2 - 1)(4 - 1) = 3$.

(c) We do not reject H_0 . There is little or no evidence ($P = 0.87$) to conclude that treatment group and condition are related.

10.5.9 (a) H_0 : The chance of needing penicillin is the same for all four treatment groups; H_A : The chance of needing penicillin depends on which group a patient is in.

(b) $e_{11} = 12 \cdot 55 / 210 = 3.14$.

(c) We retain H_0 because the P-value is larger than 0.05. There is no evidence ($P = 0.90$) that the chance of needing penicillin depends on group membership.

10.5.10 (a) H_0 : The probability of improvement is the same for all four treatment groups; H_A : The probability of improvement depends on which group a patient is in.

(b) The four percentages are, in order, 65%, 65%, 59%, and 50%.

(c) We reject H_0 because the P-value is smaller than 0.05. There is sufficient evidence ($P = 0.03$) to conclude that the chance of improvement depends on group membership.

10.5.11 (a) We retain H_0 because the P-value is larger than 0.05. There is no evidence ($P = 0.4551$) that the chance of needing penicillin depends on group membership.

(b) There is evidence that patients respond better to acupuncture than to usual care for chronic back pain. However, there is no evidence that real acupuncture works better than simulated acupuncture.

10.6.1 Yes, because the expected frequencies (8.3, 5.7, 7.7, and 5.3) all exceed 5.

• 10.6.2 This analysis is not appropriate because the observational units (mice) are nested within the units (litters) that were randomly allocated to treatments. This hierarchical structure casts doubt on the condition that the observations on the 224 mice are independent, especially in light of the investigator's comment that the response varied considerably from litter to litter.

10.6.3 Flaw 1: The sugar observations are not independent of the starch observations, because they were measured on the same people; thus, the two samples are paired. Flaw 2: The 110 sugar observations are not independent of each other because there were 11 observations on each person (hierarchical structure); similarly, the 110 starch observations are not independent of each other.