

or an analysis of the power of the test.)

7.6.3 Let 1 denote male and 2 denote female.

$$SE_{(\bar{x}_1 - \bar{x}_2)} = \sqrt{0.62^2 + 0.53^2} = 0.8157.$$

$$(137.21 - 137.18) \pm (1.977)(0.8157) \quad (\text{using } df = 140)$$

$$(-1.6, 1.6) \text{ or } -1.6 < \mu_1 - \mu_2 < 1.6 \text{ beats per minute.}$$

We can be 95% confident that the mean difference does not exceed 1.6 beats per minute, which is small and unimportant (in comparison with, for example, ordinary fluctuations in heart rate from one minute to the next.)

• 7.6.4 The mean difference in concentration of coumaric acid is  $\mu_1 - \mu_2$ , where 1 denotes dark and 2 denotes photoperiod. We construct a 95% confidence interval for  $\mu_1 - \mu_2$ .

$$SE_{(\bar{x}_1 - \bar{x}_2)} = \sqrt{\frac{21^2}{4} + \frac{27^2}{4}} = 17.103.$$

The critical value  $t_{0.025}$  is found from Student's t distribution with  $df = n_1 + n_2 - 2 = 6$ . (Formula (6.7.1) gives  $df = 5.7$ .) From Table 4, we find  $t_{6,0.025} = 2.447$ . The 95% confidence interval is

$$\bar{y}_1 - \bar{y}_2 \pm t_{0.025} SE_{(\bar{x}_1 - \bar{x}_2)}$$

$$(106 - 102) \pm (2.447)(17.103)$$

$$(-37.9, 45.9) \text{ or } -37.9 < \mu_1 - \mu_2 < 45.9 \text{ nmol/gm.}$$

The difference could be larger than 20 nmol/gm or much smaller, so the data do not indicate whether the difference is "important."

$$7.6.5 \quad SE_{(\bar{x}_1 - \bar{x}_2)} = \sqrt{\frac{21^2}{40} + \frac{27^2}{40}} = 5.408.$$

$$(106 - 102) \pm (1.994)(5.408)$$

$$(-6.8, 14.8) \text{ or } -6.8 < \mu_1 - \mu_2 < 14.8 \text{ nmol/gm.}$$

The difference is less than 20 nmol/gm, so the data indicate that the difference is not "important."

• 7.6.6 Using the larger SD the sample effect size is computed as

$$\frac{55.3 - 53.3}{6.1} = 0.3279.$$

7.6.7 The mean difference in serum concentration of uric acid is  $\mu_1 - \mu_2$ , where 1 denotes men and 2 denotes women. We construct a 95% confidence interval for  $\mu_1 - \mu_2$ .

$$SE_{(\bar{x}_1 - \bar{x}_2)} = \sqrt{\frac{0.058^2}{530} + \frac{0.051^2}{420}} = 0.00354.$$

The critical value  $t_{0.025}$  is found from Student's t distribution with  $df = n_1 + n_2 - 2 = 948 \approx 1000$ . (Formula (6.7.1) gives  $df = 937.8$ .) From Table 4, we find  $t_{1000,0.025} = 1.962$ . The 95% confidence interval is

$$\bar{y}_1 - \bar{y}_2 \pm t_{0.025} SE_{(\bar{x}_1 - \bar{x}_2)}$$

$$(0.354 - 0.263) \pm (1.962)(0.00354)$$

$$(0.0841, 0.0979) \text{ or } 0.0841 < \mu_1 - \mu_2 < 0.0979 \text{ mmol/l.}$$

All values in the confidence interval are greater than 0.08 mmol/l. Therefore, according to the confidence interval the data indicate that the difference is "clinically important."

$$7.6.8 \quad SE_{(\bar{x}_1 - \bar{x}_2)} = \sqrt{\frac{0.058^2}{53} + \frac{0.051^2}{42}} = 0.0112.$$

$$(0.354 - 0.263) \pm (1.984)(0.0112) \quad (\text{using } df = 100)$$

$$(0.069, 0.113) \text{ or } 0.069 < \mu_1 - \mu_2 < 0.113 \text{ mmol/l.}$$

The difference could be greater than or less than 0.08 mmol/l, so the data do not indicate whether the difference is "clinically important."

$$7.6.9 \quad (92.6 - 86.2)/18.3 = 0.35$$

7.6.10 No. The difference could be larger than 5 gm or much smaller, so the data do not indicate whether the difference is "important." (The small P-value indicates statistical significance but not importance.)

$$7.6.11 \text{ (a) } (16.8 - 11.9)/18.7 = 0.26$$

(b) No, they would not be ill-advised to conduct a larger study. The confidence interval is quite wide because of the limited sample size and contains values that are greater than 5 in magnitude. In particular, the confidence interval indicates that population mean Oswerty Index under the experimental treatment could be as many as 18 percentage points greater than the control population mean. A larger study could find compelling evidence that the experimental conditions are clinically useful (i.e., the mean difference is greater than 5 percentage points).

(c) The researchers should probably not conduct any further study. The confidence interval indicates (with 95% confidence) that the magnitude of the difference is no more than 3.2 percentage points. Moreover, there may be no difference at all since zero is in the interval. In particular, we are 95%

confident that the population mean Oswerty Index for the experimental treatment is no more than 3.2 percentage points greater than the population mean for the control population mean.

- 7.7.1 From the preliminary data, we obtain 0.3 cm as a guess of  $\sigma$ .

(a) If the true difference is 0.25 cm, then the effect size is

$$\frac{|\mu_1 - \mu_2|}{\sigma} = \frac{0.25}{0.3} = 0.83.$$

We consult Table 5 for a two-tailed test at  $\alpha = 0.05$  and an effect size of  $0.83 \approx 0.85$ ; to achieve power 0.80, Table 5 recommends  $n = 23$ .

(b) If the true difference is 0.5 cm, then the effect size is

$$\frac{|\mu_1 - \mu_2|}{\sigma} = \frac{0.5}{0.3} = 1.67.$$

We consult Table 5 for a two-tailed test at  $\alpha = 0.05$  and an effect size of  $1.67 \approx 1.7$ ; to achieve power 0.95, Table 5 recommends  $n = 11$ .

7.7.2  $|\mu_1 - \mu_2|/\sigma = 44.4/69.6 = 0.64.$

(a) Table 5 gives  $n = 39$ .

(b) Table 5 gives  $n = 30$ .

7.7.3  $|\mu_1 - \mu_2|/\sigma = 2/0.8 = 2.5.$

(a) Table 5 gives  $n = 5$ .

(b) The required conditions are that the sampled populations are normal with equal standard deviations. The condition of normality can be checked from the pilot data.

(c) Table 5 gives  $n = 6$ .

- 7.7.4 We need to find  $n$  to achieve a power of 0.9. The effect size is

$$\frac{|\mu_1 - \mu_2|}{\sigma} = \frac{81 - 75}{11} = 0.55.$$

We consult Table 5.

(a) For a two-tailed test at  $\alpha = 0.05$ , Table 5 gives  $n = 71$ .

(b) For a two-tailed test at  $\alpha = 0.01$ , Table 5 gives  $n = 101$ .

(c) For a one-tailed test at  $\alpha = 0.05$ , Table 5 gives  $n = 58$ .

7.7.5 (a)  $|\mu_1 - \mu_2|/\sigma = 2/2.5 = 0.8$ ; Table 5 gives  $n = 26$ .

(b)  $|\mu_1 - \mu_2|/\sigma = 2/1.25 = 1.6$ ; Table 5 gives  $n = 8$ . Yes, the measure would be cost-effective, because the required number of rats would be reduced by more than half. (In fact, the modified experiment

would cost only 62% as much, because  $(2)(8)/26 = 0.62$ .)

- 7.7.6 The effect size is

$$\frac{|\mu_1 - \mu_2|}{\sigma} = \frac{4}{10} = 0.4.$$

From Table 5 we find that, for a one-tailed test with  $n = 35$  at significance level  $\alpha = 0.05$ , the power is 0.50 if the effect size is 0.4. Thus, the probability that Jones will reject  $H_0$  is equal to 0.50.

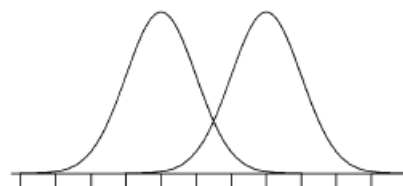
7.7.7 Considering  $H_A$  to be directional, as in 7.5.10(a), and with  $\alpha = 0.05$  and an effect size of 0.50, the closest sample size to 25 listed in Table 5 is 23. With these sample and effect sizes, the chance of finding a significant difference between the mean effectiveness of the two drugs is about 0.50, or 50%.

7.7.8  $|\mu_1 - \mu_2|/\sigma = 0.5$

(a) Table 5 gives  $n = 51$ .

(b) Table 5 gives  $n = 70$ .

7.7.9 (a)



(b)

