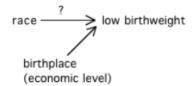
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Illinois. This would explain why U.S.-born black women have a high percentage of low birthweight babies.



- 7.4.9 (a) As an observational study, we will not be able to establish a causal relationship between the book release date and injuries. There could be other events on these weekends that are causing a drop. Note to students: If you could go back in time and re-release these books, could you think of an experiment to test the hypothesis that there are fewer injuries, on average, on Harry Potter weekends?
  - (b) H<sub>0</sub>: The mean number of injuries is the same for both types of weekends (μ<sub>1</sub> = μ<sub>2</sub>) H<sub>A</sub>: The mean number of injuries differs between the types of weekends (μ<sub>1</sub> ≠ μ<sub>2</sub>)

$$SE_{(\overline{Y}_1 - \overline{Y}_2)} = \sqrt{\frac{0.7^2}{2} + \frac{10.4^2}{24}} = 2.18$$

- t<sub>s</sub> = (36.5 67.4)/2.179 = -14.2. Using Table 4 with df=24 we have t<sub>0.0005</sub> = 3.745. Thus P-value < 0.0005x2 = 0.001, so we reject H<sub>0</sub>. There is strong evidence (P < 0.001) to conclude that the number of injuries is related to the type of weekend.</p>
- 7.4.10 Only 1 statement is true. (i) is true; this is what rejecting H<sub>0</sub> is all about. (ii) is false; the difference need not be important. (iii) is false; we cannot infer causation from an observational study.
- 7.5.1 (a) The observed t statistic is

$$t_{_{\mathbf{i}}} = \frac{\overline{y}_1 - \overline{y}_2}{SE_{(\overline{y}_1 - \overline{y}_2)}} = \frac{10.8 - 10.5}{0.23} = 1.30.$$

To check the directionality of the data, we note that  $\overline{y}_1 > \overline{y}_2$ . Thus, the data do deviate from  $H_0$  in the direction  $(\mu_1 > \mu_2)$  specified by  $H_A$ , and therefore the one-tailed P-value is the area under the t curve beyond 1.30.

From Table 4 with df = 18, we find the critical values  $t_{0.20} = 0.862$  and  $t_{0.10} = 1.330$ . Because  $t_s$  is between  $t_{0.10}$  and  $t_{0.20}$ , the one-tailed P-value must be between 0.10 and 0.20. Thus, the P-value is bracketed as 0.10 < P < 0.20.

(b) The observed t statistic is

$$t_s = \frac{\overline{y}_1 - \overline{y}_2}{SE_{(\overline{Y}_1 - \overline{Y}_2)}} = \frac{750 - 730}{11} = 1.82.$$

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To check the directionality of the data, we note that  $\overline{y}_1 > \overline{y}_2$ . Thus, the data do deviate from  $H_0$  in the direction  $(\mu_1 > \mu_2)$  specified by  $H_A$ , and therefore the one-tailed P-value is the area under the t curve beyond 1.82.

From Table 4 with df = 140 (the closest value to 180), we find the critical values  $t_{0.04} = 1.763$  and  $t_{0.03} = 1.896$ . Because  $t_s$  is between  $t_{0.03}$  and  $t_{0.04}$ , the one-tailed P-value must be between 0.03 and 0.04. Thus, the P-value is bracketed as 0.03 < P < 0.04.

7.5.2 (a) 
$$t_s = \frac{3.24 - 3.00}{0.61} = 0.39$$

With df = 17, Table 4 gives  $t_{0.20} = 0.863$ . Thus, P-value > 0.20.

**(b)** 
$$t_s = \frac{560 - 500}{45} = 1.33$$

With df = 8, Table 4 gives  $t_{0.20} = 0.889$  and  $t_{0.10} = 1.397$ . Thus,  $0.10 \le P$ -value  $\le 0.20$ .

(c) 
$$t_s = \frac{73 - 79}{2.8} = -2.14$$
.

Because  $\overline{y}_1 < \overline{y}_2$ , the data do <u>not</u> deviate from  $H_0$  in the direction specified by  $H_A$ . Thus, P > 0.50.

- 7.5.3 (a) Yes. t<sub>2</sub> is positive, as predicted by H<sub>A</sub>. Thus, the P-value is the area under the t curve beyond 3.75. With df = 19, Table 4 gives t<sub>0.005</sub> = 2.861 and t<sub>0.0005</sub> = 3.883. Thus, 0.0005 < P-value < 0.005. Since P < α, we reject H<sub>0</sub>.
  - (b) Yes.  $t_s$  is positive, as predicted by  $H_A$ . Thus, the P-value is the area under the t curve beyond 2.6 With df = 5, Table 4 gives  $t_{0.025}$  = 2.571 and  $t_{0.02}$  = 2.757. Thus,  $0.02 \le P$ -value  $\le 0.025$ . Since  $P \le \alpha$ , we reject  $H_0$ .
  - (c) Yes.  $t_s$  is positive, as predicted by  $H_A$ . Thus, the P-value is the area under the t curve beyond 2.1 With df = 7, Table 4 gives  $t_{0.04}$  = 2.046 and  $t_{0.03}$  = 2.241. Thus, 0.03 < P-value < 0.04. Since P <  $\alpha$ , we reject  $H_0$ .
  - (d) No.  $t_s$  is positive, as predicted by  $H_A$ . Thus, the P-value is the area under the t curve beyond 1.8. With df = 7, Table 4 gives  $t_{0.10}$  = 1.415 and  $t_{0.05}$  = 1.895. Thus,  $0.05 \le P$ -value  $\le 0.10$ . Since  $P \ge \alpha$ , we do not reject  $H_0$ .
- 7.5.4 (a) No. With df = 23, Table 4 gives  $t_{0.10} = 1.319$  and  $t_{0.05} = 1.714$ . Thus, 0.05 < P-value < 0.10. Since  $P > \alpha$ , we do not reject  $H_0$ .
  - (b) Yes. With df = 5, Table 4 gives  $t_{0.04} = 2.191$  and  $t_{0.03} = 2.422$ . Thus, 0.03 < P-value < 0.04.

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Since  $P < \alpha$ , we reject  $H_0$ .

- (c) No. Because t<sub>s</sub> > 0, the data do <u>not</u> deviate from H<sub>0</sub> in the direction specified by H<sub>A</sub>. Thus, P > .50 and we do not reject H<sub>0</sub>.
- (d) Yes. With df = 27, Table 4 gives  $t_{0.005}$  = 2.771 and  $t_{0.0005}$  = 3.690. Thus, 0.0005 < P-value < 0.005. Since P <  $\alpha$ , we reject  $H_0$ .
- 7.5.5 Let 1 denote infected and 2 denote noninfected.

$$H_0$$
: Malaria does not affect red cell count ( $\mu_1 = \mu_2$ )  
 $H_A$ : Malaria reduces red cell count ( $\mu_1 < \mu_2$ )

We note that  $\overline{y}_1 > \overline{y}_2$ , so the data do <u>not</u> deviate from  $H_0$  in the direction specified by  $H_A$ . Thus, P > 0.50.

- (a) Ho is not rejected. There is no evidence that malaria reduces red cell count in this population.
- (b) Same as part (a).

[Note: If  $H_A$  were reversed ( $\mu_1 > \mu_2$ ), then  $H_0$  would be rejected at  $\alpha$  = .10. Thus, this exercise illustrates the importance of the directionality check.]

7.5.6 Let 1 denote experimental (to be hypnotized) and 2 denote control

$$SE_{(\overline{Y}_1 - \overline{Y}_2)} = \sqrt{\frac{0.621^2}{8} + \frac{0.652^2}{8}} = 0.3183.$$

$$t = (6.169 - 5.291)/0.3183 = 2.76.$$

With df =  $n_1 + n_2 - 2 = 14$  (Formula (6.7.1) yields df = 13.97), Table 4 gives  $t_{0.01} = 2.624$  and  $t_{0.005} = 2.977$ .

(a) H<sub>0</sub>: Mean ventilation is the same in the "to be hypnotized" condition and in the "control" condition (u. = u.)

 $H_A$ : Mean ventilation is different in the "to be hypnotized" condition than in the "control" condition  $(\mu_1 \neq \mu_2)$ 

 ${\rm H_0}$  is rejected. There is sufficient evidence (0.01 < P < 0.02) to conclude that mean ventilation is higher in the "to be hypnotized" condition than in the "control" condition.

(b) H<sub>0</sub>: Mean ventilation is the same in the "to be hypnotized" condition and in the "control" condition (μ<sub>1</sub> = μ<sub>1</sub>)

 $H_A$ : Mean ventilation is higher in the "to be hypnotized" condition than in the "control" condition  $(\mu_1 > \mu_2)$ 

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 $H_0$  is rejected. There is sufficient evidence (0.005 < P < 0.01) to conclude that mean ventilation is higher in the "to be hypnotized" condition than in the "control" condition.

(c) The nondirectional alternative (part (a)) is more appropriate. According to the narrative, the researchers formulated the directional alternative in part (b) after they had seen the data. Thus, it would not be legitimate for them (or us) to use a directional alternative.

$$7.5.7$$
 (a)  $t_s = -1.429$ 

(b) We reject H<sub>0</sub> because the P-value is smaller than 0.10. We have sufficient evidence to say that mean leaf dry weight is greater when extra nitrogen is applied.

# 7.5.8 (a) $H_0$ : $\mu_1 = \mu_2$ and $H_A$ : $\mu_1 < \mu_2$

- (b) The small P-value means that if the two groups really were the same then it would be unlikely that the sample mean birthweight for females would exceed the mean for males by as much as it did in this study.
- (c) We reject H<sub>0</sub> in favor of H<sub>A</sub>. There is sufficient evidence that mean birthweight for females is greater than mean birthweight for males.
- 7.5.9 The null and alternative hypotheses are

$$H_0$$
:  $\mu_1 = \mu_2$   
 $H_A$ :  $\mu_1 \le \mu_2$ 

where 1 denotes wounded and 2 denotes control. These hypotheses may be stated as

 $\mathrm{H}_0$ : Wounding the plant has no effect on larval growth

Ha: Wounding the plant tends to diminish larval growth

To check the directionality of the data, we note that  $\overline{y}_1 < \overline{y}_2$ . Thus, the data do deviate from  $H_0$  in the direction  $(\mu_1 < \mu_2)$  specified by  $H_4$ . We proceed to calculate the test statistic.

The standard error of the difference is

$$SE_{(\overline{Y}_1 - \overline{Y}_2)} = \sqrt{\frac{9.02^2}{16} + \frac{11.14^2}{18}} = 3.46.$$

The test statistic is

$$t_s = \frac{\overline{y}_1 - \overline{y}_2}{SE_{(\overline{y}_1 - \overline{y}_2)}} = \frac{28.66 - 37.96}{3.46} = -2.69.$$

From Table 4 with df =  $16 + 18 - 2 = 32 \approx 30$  (Formula (6.7.1) yields df = 31.8), we find the critical values  $t_{0.01} = 2.457$  and  $t_{0.005} = 2.750$ . Thus, the P-value is bracketed as  $0.005 \le P$ -value  $\le 0.01$ .

Since the P-value is less than  $\alpha$  (0.05), we reject H<sub>0</sub>. There is sufficient evidence (0.005 < P < 0.01) to conclude that wounding the plant tends to diminish larval growth.

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• 7.5.10 (a) The null and alternative hypotheses are

$$H_0$$
:  $\mu_1 = \mu_2$   
 $H_A$ :  $\mu_1 > \mu_2$ 

where 1 denotes drug and 2 denotes placebo. These hypotheses may be stated as

H<sub>0</sub>: The drug is not effective H<sub>4</sub>: The drug is effective

To check the directionality of the data, we note that  $\overline{y}_1 > \overline{y}_2$ . Thus, the data do deviate from  $H_0$  in the direction  $(\mu_1 > \mu_2)$  specified by  $H_4$ . We proceed to calculate the test statistic.

The standard error of the difference is

$$SE_{(\overline{Y}_1 - \overline{Y}_2)} = \sqrt{\frac{12.05^2}{25} + \frac{13.78^2}{25}} = 3.66.$$

The test statistic is

$$t_s = \frac{\overline{y}_1 - \overline{y}_2}{SE_{(\overline{y}_1 - \overline{y}_2)}} = \frac{31.96 - 25.32}{3.66} = 1.81.$$

From Table 4 with df = 25 + 25 -2 = 48  $\approx$  50 (Formula (6.7.1) yields df = 47.2), we find the critical values  $t_{0.04} = 1.787$  and  $t_{0.03} = 1.924$ . Thus, the P-value is bracketed as

0.03 < P-value < 0.04.

Since the P-value is less than  $\alpha$  (0.05), we reject  $H_0$ . There is sufficient evidence (0.03 < P < 0.04) to conclude that the drug is effective at increasing pain relief.

- (b) The only change in the calculations from part (a) would be that the one-tailed area would be doubled if the alternative were nondirectional. Thus, the p-value would be between 0.06 and 0.08 and at  $\alpha = 0.05$  we would not reject  $H_a$ .
- 7.5.11 (a) H<sub>0</sub>: The mean time until first flatus is the same for when using a rocking chair as it is when not using a rocking chair. H<sub>a</sub>: The mean time until first flatus is lower when using a rocking chair.
  - (b) We reject H<sub>0</sub> because the P-value is smaller than 0.05. We have strong evidence (P=0.000496) to say that using a rocking chair reduces average time to first flatus.
  - (c) A non-directional test may have been more appropriate. If rocking caused an increase in POI duration, we would certainly want our test to detect this outcome. Significant evidence of an increase in POI would result in having patients avoid post-surgery rocking.
- 7.5.12 0.03 < P-value < 0.05. Thus, we would reject H<sub>0</sub> and conclude that ancy does, indeed, inhibit growth.
- 7.5.13 Let 1 denote 250 meters and 2 denote 800 meters.

 $H_0$ : Distance from the reef does not affect settler density ( $\mu_1 = \mu_2$ )  $H_4$ : Settler density decreases as distance from the reef increases ( $\mu_1 > \mu_2$ )

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$$SE_{(\overline{Y}_1 - \overline{Y}_2)} = \sqrt{\frac{0.514^2}{48} + \frac{0.413^2}{48}} = 0.095.$$

$$t_1 = (0.818 - 0.628)/0.095 = 2.$$

With df =80, Table 4 gives  $t_{0.025}$  = 1.990 and  $t_{0.02}$  = 2.088. Thus,  $0.02 \le P$ -value  $\le 0.025$  and we reject  $H_0$ . There is statistically significant evidence (0.02  $\le P \le 0.025$ ) to conclude that settler density decreases as distance from the reef increases.

## 7.5.14 (a) $H_0$ : $\mu_1 = \mu_2$

- (b) The dotplots provide visual evidence that Ho is false.
- (c) The small P-value means that if the two groups really were the same then it would be extremely unlikely that the sample mean quality of life score would differ by as much as they did in this experiment.
- (d) Since the P-value = 0.00866 < α = 0.01, we reject H<sub>0</sub> in favor of H<sub>A</sub> There is strong evidence that mean quality of life score is higher for those who exercise than for those those who don't. There is strong evidence that exercise improves quality of life. (A causal claim is possible because this study was an experiment).
- (e) The non-directinoal P-value is 2×0.00866 = 0.01732.
- (f) Since the non directional P-value is not less than 0.01, we do not reject H<sub>0</sub>. There is no significant evidence that the mean quality of life score differs for the two groups. There is no significant evidence that exercise affects quality of life.
- 7.5.15 (a) It is not proper to choose the direction for a directional H<sub>A</sub> based on the data being tested.
  - (b) No. The test statistic changes very little so the P-value will still be small (it turns out to be 0.0076) so there is still evidence for Ha.

### 7.5.16 (a) t = 2.54

- (b) P=0.03\*2 = 0.06. If there were no apriori reason to use a directional test then a non-directional test should have been done. For such a test the P-value includes the lower tail area below -1.97 (0.03) as well as the upper tail area above 1.97 (0.03).
- 7.6.1 The proponents are confused. They are speaking as if it is known that μ<sub>1</sub> μ<sub>2</sub> = 40 lb/acre, whereas the field trial indicates only that \$\overline{y}\_1\$ \$\overline{y}\_2\$ = 40 lb/acre. That statistician's data analysis indicates that the trial gives only weak information about μ<sub>1</sub> μ<sub>2</sub>; in fact, the results do not even show whether μ<sub>1</sub> μ<sub>2</sub> is positive, let alone that it is equal to 40 lb/acre.
- 7.6.2 The lack of a statistically significant difference in therapeutic responses does not show that the two medications are equally effective. (Such evidence could be obtained from either a confidence interval