(c) ${}_4C_2(0.44^2)(0.56^2) = 6(0.44^2)(0.56^2) = 0.3643$

(d) 0.0983 + 0.3091 + 0.3643 = 0.7717

(e) 0.3091 + 0.3643 = 0.6734

3.6.4 (a) $0.9^{20} = 0.1216$

(b) $20(0.9^{19})(0.1) = 0.2702$

(c) ${}_{20}C_2(0.9^{18})(0.1^2) = 190(0.9^{18})(0.2^2) = 0.2852$

(d) $190(0.9^{18})(0.2^2) = 0.2852$

3.6.5 (a) ${}_{10}C_5(0.6^5)(0.4^5) = 252(0.6^5)(0.4^5) = 0.2007$

(b) ${}_{10}C_6(0.6^6)(0.4^4) = 210(0.6^6)(0.4^4) = 0.2508$

(c) ${}_{10}C_7(0.6^7)(0.4^3) = 120(0.6^7)(0.4^3) = 0.2150$

3.6.6 (a) np = (10)(0.6) = 6

(b) $\sqrt{np(1-p)} = \sqrt{(10)(0.6)(0.4)} = 1.55$

3.6.7 (a) The mean is np = (20)(0.08) = 1.6

(b) The SD is $\sqrt{np(1-p)} = \sqrt{(20)(0.08)(0.92)} = 1.213$

• **3.6.8** On average, there are 105 males to every 100 females. Thus, Pr{male} = $\frac{105}{205}$ and Pr{female} = $\frac{100}{205}$. To use the binomial distribution, we arbitrarily identify "success" as "female."

(a) We have n = 4 and p = $\frac{100}{205}$. To find the probability of 2 males and 2 females, we set j = 2, so n - j = 2. The binomial formula gives Pr{2 males and 2 females} =

$${}_4C_2\left(\frac{100}{205}\right)^2\left(\frac{105}{205}\right)^2 = 0.3746.$$

(b) To find the probability of 4 males, we set j = 0, so n - j = 4. The binomial formula gives

$$\Pr\{4 \text{ males}\} = {}_4C_0\left(\frac{100}{205}\right)^0\left(\frac{105}{205}\right)^4 = (1)(1)\left(\frac{105}{205}\right)^4 = 0.0688.$$

(c) The condition that all four infants are the same sex can be satisfied two ways: All four could be male or all four could be female. The probability that all four are male has been computed in part (b) to be 0.0688. To find the probability that all four are females, we set j = 4, so n - j = 0.

$$\Pr\{4 \text{ females}\} = {}_4C_4\left(\frac{100}{205}\right)^4\left(\frac{105}{205}\right)^0 = (1)(1)\left(\frac{100}{205}\right)^4 = 0.0566.$$

Thus, we find that Pr{all four are the same sex} = 0.0566 + 0.0688 = 0.1254.

3.6.9 There are several possible answers. What is needed is a situation in which there are n=7 independent trials, each with probability of success p=0.8, and we wish to find the probability of exactly 3 successes. For example, suppose that the probability of a certain gene is 0.8 in a given breed of chickens and we take a random sample of n=7 chickens from that breed. The question "What is the probability that exactly 3 of the 7 sampled chickens has this gene?" has answer ${}_7C_3(0.8^3)(0.2^4)$.

3.6.10 (a) $0.7^8 = 0.0576$

(b) $(8)(0.7^7)(0.3^1) = 0.1977$

(c) 1 - 0.0576 - 0.1977 = 0.7447

• **3.6.11** (a) $0.75^6 = 0.1780$

(b) 1 - 0.1780 = 0.8220

3.6.12 Pr{high blood level} = $\frac{1}{8}$ = p. To apply the binomial formula, we arbitrarily identify "success" as "high blood lead." Then n = 16 and p = $\frac{1}{8}$.

(a) To find the probability that none has high blood lead, we set j = 0, so n - j = 16. The binomial formula gives Pr{none has high blood lead} = ${}_{16}C_0\left(\frac{1}{8}\right)^0\left(\frac{7}{8}\right)^{16} = (1)(1)\left(\frac{7}{8}\right)^{16} = 0.1181.$

(b) To find the probability that one has high blood lead, we set j = 1, so n - j = 15. The binomial formula gives Pr{one has high blood lead} = ${}_{16}C_1\left(\frac{1}{8}\right)^1\left(\frac{7}{8}\right)^{15} = (16)\left(\frac{1}{8}\right)^1\left(\frac{7}{8}\right)^{15} = 0.2699.$

(c) To find the probability that two have high blood lead, we set j = 2, so n - j = 14. The binomial formula gives Pr{two have high blood lead} = ${}_{16}C_2\left(\frac{1}{8}\right)^2\left(\frac{7}{8}\right)^{14} = (120)\left(\frac{1}{8}\right)^2\left(\frac{7}{8}\right)^{14} = 0.2891.$

(d) Pr{three or more have high blood lead} = 1 - Pr{2 or fewer have high blood lead}

$= 1 - [0.1181 + 0.2699 + 0.2891] = 0.3229.$

• **3.7.1** The first step is to determine the best-fitting value for p = Pr{boy}. The total number of children in all the families is

(6)(72,069) = 432,414.

The number of boys is

(0)(1,096) + (1)(6,233) + ... + (6)(1,579) = 222,638.

Thus, the value of p that fits the data best is

$$p = \frac{222638}{432414} = 0.514872321.$$

To compute the probabilities of various sex ratios, we apply the binomial formula with n = 6 and p = 0.514872321. Then we multiply each probability by 72,069 to obtain the expected frequency:

Number of boys (j)	Expected frequency		
0	$(72{,}069)(1)(1-p)^6$	=	939.5
1	$(72{,}069)(6)(p)(1-p)^5$	=	5,982.5
2	$(72{,}069)(15)(p^2)(1-p)^4$	=	15,873.1
3	$(72{,}069)(20)(p^3)(1-p)^3$	=	22,461.8
4	$(72{,}069)(15)(p^4)(1-p)^2$	=	17,879.3
5	$(72{,}069)(6)(p^5)(1-p)^1$	=	7,590.2
6	$(72{,}069)(1)(p^6)$	=	1,342.6

The following table compares the observed and expected frequencies:

Number of boys	Number of girls	Observed frequency	Expected frequency	Sign of (Obs - Exp)
0	6	1,096	939.5	+
1	5	6,233	5,982.5	+
2	4	15,700	15,873.1	-
3	3	22,221	22,461.8	-
4	2	17,332	17,879.3	-
5	1	7,908	7,590.2	+
6	0	1,579	1,342.6	+
		72,069	72,069.0	

We note that there is reasonable agreement between the observed and expected frequencies. However, the observed frequencies exceed the expected frequencies for the preponderantly unisex siblingships (those with 0, 1, 5, or 6 boys), whereas the observed frequencies are less than the expected frequencies for the more balanced siblingships (2, 3, or 4 boys). This pattern is similar to that seen in Example 3.7.1.

3.7.2 (a) Total number of embryos = (310)(9) = 2790.

Number of dead embryos = (0)(136) + (1)(103) + ... + (6)(1) = 277.

$$\text{Fitted value of } p = \frac{277}{2790} = 0.099283154.$$

Number dead	Number living	Observed frequency	Expected frequency	Sign of (Obs - Exp)
0	9	136	121.0	+
1	8	103	120.0	-
2	7	50	52.9	-
3	6	13	13.6	-
4	5	6	2.2	+
5	4	1	0.2	+
6	3	1	0.0	+
7	2	0	0.0	0
8	1	0	0.0	0
9	0	0	0.0	0
		310	309.9	

(b) The observed distribution has heavier tails than the fitted distribution: there are more females with 0 dead embryos and more females with 4 or more dead embryos than expected. This pattern suggests that the embryos are more similar within litters than between litters, and thus casts doubt on the classical assumption.

3.7.3 (a) Total number of seeds = (280)(5) = 1400.

Number germinated = (0)(17) + (1)(53) + ... + (5)(4) = 630.

$$\text{Fitted value of } p = \frac{630}{1400} = 0.45.$$

Number germ.	Number not germ.	Observed frequency	Expected frequency	Sign of (Obs - Exp)
0	5	17	14.1	+
1	4	53	57.6	-
2	3	94	94.3	-
3	2	79	77.2	+
4	1	33	31.6	+
5	0	4	5.2	-
		280	280	

(b) The differences between the observed and expected frequencies are small and there is no pattern to the signs. Thus, the variation among the students is consistent with the hypothesis that chance alone determined the outcomes. Bob could plausibly be told that he received five poor seeds just by chance; it was not his fault that none germinated.

(c) Fictitious data, with overall 45% germination:

Number germ.	Number not germ.	Number or students
0	5	154
1	4	0
2	3	0
3	2	0
4	1	0
5	0	126
		280

These data do not fit the binomial distribution. Thus, the variation is not due to chance alone, and Bob should be told that it is his fault.

3.S.1 (a) $0.1^2 = 0.01$

(b) $(2)(0.1)(0.9) = 0.18$

3.S.2 (a) 0.36

(b) 0.19

• **3.S.3** The probability that a square has no centipedes is the relative frequency of squares with zero centipedes. Thus, Pr{no centipedes} = .45.
To apply the binomial formula, we identify "success" as "no centipedes." Then n = 5 and p = 0.45.
To find the probability that three squares have centipedes and two do not, we set j = 2, so n - j = 3.
Pr{3 with, 2 without} = ${}_5C_2(0.45^2)(0.55^3) = 10(0.45^2)(0.55^3) = 0.3369$.

3.S.4 Pr{at least one centipede} = 0.55 = p. The mean is np = (5)(0.55) = 2.75.

3.S.5 (a) $(10)(0.5^3)(0.5^2) = 0.3125$

(b) $(10)(0.5^3)(0.5^2) + (5)(0.5^4)(0.5^1) + (1)(0.5^5) = 0.5$

(c) $0.5^5 + 0.5^5 = 0.0625$

3.S.6 (a) $0.99^{50} = 0.6050$

(b) 1 - 0.6050 = 0.3950

• **3.S.7 (a)** $1 - 0.99^{100} = 0.6340$

(b) $1 - 0.99^n \geq 0.95$, so $n \geq \log(0.05)/\log(0.99)$, so $n \geq 299$.

3.S.8 The probability that an *innocent* subject will have a higher electrodermal response on any critical word (compared to the three controls for that critical word) is 25%. Thus,
Pr{failure on a critical word} = 0.25.
Let us identify "success" as "success on a critical word." Thus, p = 0.75 and

1 - p = 0.25. We then use the binomial formula with n = 6 and p = 0.75.
To be labeled "guilty" a subject must fail 4 or more critical words out of 6. We find the probability of 4, 5, or 6 failures by setting j = 2, 1, or 0, as follows:

Pr{2 successes, 4 failures} = ${}_6C_2p^2(1-p)^4 = (15)(0.75^2)(0.25^4) = 0.0330$
Pr{1 success, 5 failures} = ${}_6C_1p^1(1-p)^5 = (6)(0.75^1)(0.25^5) = 0.0044$
Pr{0 successes, 6 failures} = ${}_6C_0p^0(1-p)^6 = (1)(1)(0.25^6) = 0.0002$

To find the probability that an innocent subject will be labeled "guilty," we add these three values:
Pr{4 or more failures} = 0.0330 + 0.0044 + 0.0002 = 0.0376.

3.S.9 (a) 0.66

(b) 0.21

(c) 0.38

• **3.S.10 (a)** To use the binomial distribution here, let us identify "success" as "blood pressure > 140." Then, p = Pr{blood pressure > 140} = 0.25 + 0.09 + 0.04 = 0.38, which is the area under the density curve beyond 140 mm Hg.

The number of trials is n = 4. To find the probability that all four men have blood pressure higher than 140 mm Hg, we set j = 4, so n - j = 0.
Pr{all four have blood pressure > 140} = ${}_4C_4p^4(1-p)^0 = (1)(0.38^4)(1) = 0.0209$.

(b) Pr{three have blood pressure > 140} = ${}_4C_3p^3(1-p)^1 = (4)(0.38^3)(0.62^1) = 0.1361$.

3.S.11 (a) $(5)(0.09^4)(0.91) = 0.000298$

(b) $0.09^5 = 0.0000059$

(c) 1 – 0.0000059 = 0.9999941

• **3.S.12 (a)** The men is (50)(0.09) = 4.5

(b) The SD is $\sqrt{(50)(0.09)(0.91)} = 2.02$