Math 316 Fall 2024 **Exam 3** **October 29, 2024**

Name: Solutions

Problem	1	2	3	4 / 5	Total
Possible	31	28	20	21	100
Received					

FOR FULL CREDIT, SHOW ALL WORK RELATED TO FINDING EACH SOLUTION.

There are several 2-point problems in which I change the numbers/conditions a bit. The idea is that you should be able to figure out the resulting change without reworking the entire problem, especially since you will not have time to rework the problem.

Note: in the exam, CI means Confidence Interval.

Before giving out candy, the Gernsteads required that trick-or-treaters first watch a short video on dental care.

Deer Halloweens

31 points 1. In a sample of 100 children, 5 were found to be Vitamin D deficient. In your work below, use the Wilson-adjusted sample proportion $\tilde{p}$ rather than $\hat{p}$.

/10 (a) Find a <u>98%</u> CI for the proportion of all children who are Vitamin D deficient.

$$\hat{p} = \frac{5 + \frac{1}{2} \cdot 2.326^2}{100 + 2.326^2} \approx \frac{7.705}{105.410} \approx .073$$

$$SE = \sqrt{\frac{(.073)(1-.073)}{105.410}} \approx .0253$$

$$p = .073 \pm \underbrace{2.326\,(.0253)}_{\approx .059}$$

$$.014 < p < .1320$$

/2 (b) If the sample were 10 of 200 (rather than 5 of 100) Vitamin D deficient, compared to the CI you found in (b), the CI would now be (circle one):

Larger sample $\Rightarrow$ Narrower Wider Same

The $100 + 2.326^2$ would be $200 + 2.326^2$

/6 (c) Suppose that we want to find a 95% CI whose total width is .04, that is, we want $t_{\alpha/2} \cdot SE_{\tilde{p}} \leq .02$. What sample size n would we need? <u>In finding n, let's assume that we do not know what the sample proportion $\tilde{p}$ will be</u>. So use $\tilde{p} = .5$.

$$1.96 \sqrt{\frac{(.5)(1-.5)}{n+4}} \leq .02$$

For 95%

$$\frac{(.5)^2}{n+4} \leq \frac{(.02)^2}{(1.96)^2}$$

$$\frac{(.5)^2}{\frac{(.02)^2}{(1.96)^2}} \leq n+4$$

$$\Rightarrow n \geq \frac{(.5)^2(1.96)^2}{(.02)^2} - 4 \approx 2397$$

/9 (d) Suppose we believe that the true proportion of Vitamin D deficiency is .12 (i.e. 12%). Use the Chi-Square test with $\alpha = 0.05$ to test $H_0: p = .12$ with alternative $H_A: p \neq .12$.

	Vit. D Def.	Not def.
Obs.	5	95
Exp.	12	88

$$X_s^2 = \frac{(5-12)^2}{12} + \frac{(95-88)^2}{88} \approx 4.64$$

$df = 2 - 1 = 1$

Table 9: $.02 < \underbrace{P < .05}$

so reject H_0, accept H_A: There is sign. evidence that $p \neq .12$.

/2 (e) Using your work from (d), what is your conclusion if using alternative $H_A: p > .12$?

Wrong direction, since $\hat{p} < .12$.

/2 (f) Using your work from (d), what is your conclusion if using alternative $H_A: p < .12$?

Right direction, and we now have

$$\frac{.02}{2} < \underbrace{P < \frac{.05}{2}}$$

so $P < .05$

so reject H_0, accept H_A: there is sign. evidence that $p < .12$.

$\frac{100\cdot 100}{200}$ $\frac{50\cdot 100}{200}$ etc.

28 points 2. The genders of random samples, one from each of three local schools, are given below.

	Pepperdine	USC	UCLA	**Total**
Male	50 50	30 25	20 25	**100**
Female	50 50	20 25	30 25	**100**
Total	**100**	**50**	**50**	**200**

/12 (a) Use a Chi-Square test to determine whether there is significant evidence that gender ratios differ at the three schools. Use $\alpha = 0.10$.

$$\chi_s^2 = \frac{(50-50)^2}{50} + \frac{(50-50)^2}{50} + \frac{(30-25)^2}{25} + \frac{(20-25)^2}{25} + \frac{(20-25)^2}{25} + \frac{(30-25)^2}{25}$$

$$= 4.$$

Table 9: $df = (3-1)(2-1) = 2$ $\quad .10 < P < .20$

so do not reject H_0.

/2 (b) Suppose the results were instead as at right. Which of the following would be true? Just circle one of these four:

	Pepperdine	USC	UCLA
Male	50	40	10
Female	50	10	40

Larger χ_s^2 and larger P Smaller χ_s^2 and larger P

Larger χ_s^2 and smaller P Smaller χ_s^2 and smaller P

/12 (c) Focusing now on only Pepperdine and USC, find a 95% CI for the difference between the proportions of Pepperdine and USC Male students.

	Pepperdine	USC
Male	50	30
Female	50	20
Total	**100**	**50**

$$\tilde{p}_1 = \frac{50+1}{100+2} = .5 \qquad \tilde{p}_2 = \frac{30+1}{50+2} \approx .596$$

$$SE = \sqrt{\frac{(.5)(1-.5)}{100+2} + \frac{(.596)(1-.596)}{50+2}} \approx .0842$$

$$p_1 - p_2 = \underbrace{.5 - .596}_{-.096} \pm \underbrace{1.96(.0842)}_{\approx .165}$$

$$-.261 < p_1 - p_2 < .069$$

/2 (e) Suppose that the data were instead at right (samples doubled). Compared to the CI in (c), the new CI would be:

	Pepperdine	USC
Male	100	60
Female	100	40

Larger sample $\Rightarrow$ Narrower Wider Unchanged in width

20 points 3. During a weight loss study, each of 7 subjects was given a particular drug that is meant to decrease hunger levels, as well as a placebo (at some other time, obviously). The hunger level data are shown at right. **For problems on this page, we are assuming <u>pairing</u> and <u>normal distribution</u>.**

Person	With Drug	With Placebo	Difference ($Y_1 - Y_2$)
1	82	81	1
2	65	54	11
3	52	72	−20
4	15	25	−10
5	61	101	−40
6	107	99	8
7	77	114	−37
Mean	65.6	78.0	−12.4
SD	28.5	30.8	20.7

/8 (a) Construct a 90% CI for the difference in mean difference in change.

$df = 7 - 1 = 6$

$$\mu_D = -12.4 \pm \underbrace{1.943 \cdot \frac{20.7}{\sqrt{7}}}_{\approx 15.20}$$

$$-27.6 < \mu_D < 2.8$$

/4 (b) For the CI you just found, approximately how large would the sample size have needed to be so that the margin of error $t_{\alpha/2} \cdot SE_{\bar{D}}$ would have been ≤ 5?

Smaller than 15.20, so need smaller t, maybe $t \approx 1.7$ (in the 90% column)

$$1.7 \frac{20.7}{\sqrt{n}} \leq 5 \Rightarrow n \geq \left[\frac{(1.7)(20.7)}{5}\right]^2 \approx 49.5$$

/6 (c) Compare the Drug and Placebo populations using a **t −test** at $\alpha = 0.10$. Use a **non-directional** alternative. What do you conclude about *Drug vs. Placebo*: is there significant evidence that the drug changes hunger level?

$$t_s = \frac{-12.4 - 0}{\frac{20.7}{\sqrt{7}}} \approx -1.585 \qquad df = 7 - 1 = 6$$

Table 4: $\underbrace{2(.05) < P}_{} < 2(.10)$

so do not reject H_0.

(we would have needed $|t_s| > 1.943$ to get $P < .10$.)

/2 (d) Using the work from (c), is there significant evidence that the drug decreases hunger level? (So this is now a directional test.)

Right direction in sample

$.05 < \underbrace{P < .10}_{}$, so yes

Person	With Drug	With Placebo	Difference ($Y_1 - Y_2$)
1	82	81	1 1
2	65	54	11 4
3	52	72	−20 −5
4	15	25	−10 −3
5	61	101	−40 −7
6	107	99	8 2
7	77	114	−37 −6
Mean	65.6	78.0	−12.4
SD	28.5	30.8	20.7

14 points 4. Same data as Problem 3. **Assume pairing still, but do NOT assume normal distribution.**

/8 (a) Perform a Wilcoxon Signed Rank Test with $\alpha = .10$ to determine whether or not the drug affects hunger level.

$W_+ = 1 + 2 + 4 = 7$

$W_- = 3 + 5 + 6 + 7 = 21$

$\Big\}\ W_s = 21$

(Notice $7 + 21 = \frac{7 \cdot 8}{2} = 28$.)

Table 8, $n = 7$: $P > .20$ (not merely .156)

So not sign. evidence...

/2 (b) If Person 1 with Drug were 83 rather than 82, how would W_s change? W_s would be:

Larger Smaller No change

/2 (c) If Person 1 with Drug were 80 rather than 82, how would W_s change? W_s would be:

Larger Smaller No change

/2 (d) If Person 1 with Drug were 90 rather than 82, how would W_s change? W_s would be:

Larger Smaller No change

7 points 5. A study was conducted to determine whether a woman's risk of transmitting HIV to her unborn child is different for the younger vs. the older the child. A sample of 100 HIV-infected women who gave birth to two children found that, when only one of the two siblings had HIV, it occurred in 7 of the 100 older siblings and in 15 of the younger siblings, as shown in the table below.

		Younger sibling HIV? Yes	No
Older sibling HIV?	Yes	2	**7**
	No	**15**	76

/5 (a) We are not interested in the case when **both** siblings are HIV positive or the case when **both** are not HIV positive. We are only interested in the two cases in which one sibling is HIV positive and the other is not (the **7** and **15**). Test the hypothesis H_0: *the probability of being HIV positive is the same for older siblings as it is for younger siblings* along with the alternative H_A: *the probabilities are different*. Use $\alpha = .10$.

So non-directional test.

$$X_s^2 = \frac{(15-7)^2}{15+7} \approx 2.91$$

Table 9, $df = 1$: $.05 < P < .10$.

Yes there is sign. evidence to reject H_0, accept H_A.

/2 (b) Test the hypothesis if H_A is directional, that the younger sibling (**15** in the table) is more likely than the older sibling to have HIV. Use $\alpha = .10$. Right direction.

$$\frac{.05}{2} < P < \frac{.10}{2}$$

There is sign. evidence....