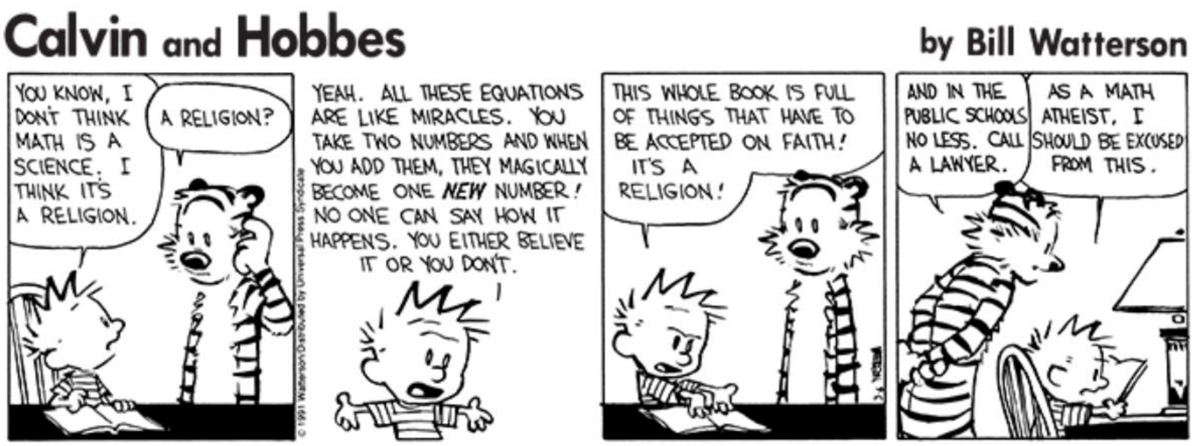
Math 316 Fall 2024 **Exam 2** **October 3, 2024**

Name: Solutions

Problem	T/F	1	2 / 3	4 / 5	Total
Possible	30	28	15	27	100
Received					

FOR FULL CREDIT, SHOW ALL WORK RELATED TO FINDING EACH SOLUTION.



True/False. Answer the following True/False questions (circle T or F), each worth 2 points.

T **F** In comparing two samples, if $|\bar{y}_1 - \bar{y}_2| \leq 0.1$, then the test statistic

$$t_s = \frac{\bar{y}_1 - \bar{y}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

Not neccesarily, e.g. not if s_1 or s_2 is large.

will be close to 0.

T **F** For a sample of some fish of a certain type, we find the mean length and standard deviation and we find the resulting 90% confidence interval of $(17.8, 22.2)$, thus we are 90% confident that the sample mean is between 17.8 and 22.2.

T F The smaller the Effect between two samples, the larger the sample sizes need to be in order to achieve a certain Confidence Level and Power Level. You can see this in Table 6.

T F Given samples of sizes 2 and 3, if using the t-test, it is possible to get a test statistic of $t_s = 6.5$. t_s could be any value

T **F** Given samples of sizes 2 and 3, if using the Wilcoxon-Mann-Whitney Test, it is possible to get a test statistic of $U_s = 6.5$. Max U_s value would be $2 \cdot 3 = 6$

T **F** It is possible to have a 95% confidence interval of $(12, 14)$ and a 90% confidence interval of $(11, 15)$. Smaller conf. <=> narrower interval.

T **F** A smaller sample size n will typically result in a smaller sample standard deviation s. For any n, $s \approx \sigma$.

T F If comparing two sample means, the sample sizes may be different.

True, but test is generally more effective if $n_1 \approx n_2$.

T **F** A confidence level of 95% is the same as a significance level of $\alpha = \cancel{.95}$ .05.

T **F** If $P > \alpha$, then we reject the alternative hypothesis $H_A: \mu_1 \neq \mu_2$ and accept the null hypothesis $H_0: \mu_1 = \mu_2$. Never!

T F If we have $P < \alpha$ for a non-directional test, then it is guaranteed that we will have $P < \alpha$ for a directional test. P for directional = $\frac{1}{2}$ P for non-directional

T **F** In creating a confidence interval, assuming all other sample values are the same (sample means and standard deviations), larger sample sizes will lead to ~~larger~~ narrower (~~wider, less precise~~) confidence intervals.

T **F** If creating a confidence interval for a population mean from a single sample, doubling the sample size will approximately halve the margin of error (i.e. the width) of the confidence interval. will reduce by approx. $\frac{1}{\sqrt{2}}$

T **F** "Power" is the likelihood that we will conclude that the population means are ~~the same~~ different when in fact they are.

T F If in testing a hypothesis we choose $\alpha = 0.05$, then there is up to a 5% chance we might make a Type I error if we decide to accept $H_A: \mu_1 \neq \mu_2$.

28 points 1. Biologists have theorized that male Monarch butterflies have, on average, a larger thorax than do females. A sample of male and female butterflies yielded the data at right. Use $df = 22$.

Show all work in answering these questions.

	Male	Female
n	11	15
$\bar{y}$	68	66
s	2.7	2.3

/9 Find a 95% confidence interval for the difference in the population means. At 95% confidence, would you conclude that male thorax size is different than female?

$$\mu_1 - \mu_2 = (68-66) \pm 2.074 \underbrace{\sqrt{\frac{2.7^2}{11} + \frac{2.3^2}{15}}}_{1.0077}$$

so $(-.09, 4.09)$, which contains 0,

so it is possible that $\mu_1 - \mu_2 = 0$, i.e. $\mu_1 = \mu_2$

so do not reject $H_0: \mu_1 = \mu_2$ (this does not mean that we accept/conclude $\mu_1 = \mu_2$).

/8 At $\alpha = .05$ significance, use a t test (and find the P-value) to determine whether we should reject the hypothesis that $H_0: \mu_{male} = \mu_{female}$ and accept the non-directional alternative $H_A: \mu_{male} \neq \mu_{female}$.

$$t_s = \frac{68-66}{\sqrt{\frac{2.7^2}{11} + \frac{2.3^2}{15}}} = \frac{2}{1.0077} = 1.985$$

From Table 4: $.025 < \text{area of one tail} < .03$

So $\underline{.05 < P} < .06$

so do not reject H_0.

/4 You should have found that we would *not* reject H_0. How large would t_s have needed to be for us to conclude that $\mu_{male} \neq \mu_{female}$?

Need $t_s > 2.074$

/4 At $\alpha = .05$ significance, use a t test (and find the P-value) to determine whether we should reject the hypothesis that $H_0: \mu_{male} = \mu_{female}$ and accept the directional alternative $H_A: \mu_{male} > \mu_{female}$.

$.025 < \underline{P < .03}$ so $P < .05$, so reject H_0, accept H_A.

/3 At $\alpha = .05$ significance, use a t test (and find the P-value) to determine whether we should reject the hypothesis that $H_0: \mu_{male} = \mu_{female}$ and accept the directional alternative $H_A: \mu_{male} < \mu_{female}$.

Sample $\bar{y}_1 > \bar{y}_2$ is wrong direction

(so $P > .5$)

9 points 2. In the previous problem, decide whether each of the following would result in a *smaller* or *larger* test statistic $t_s = \frac{\bar{y}_1 - \bar{y}_2}{SE_{\bar{Y}_1 - \bar{Y}_2}}$. Assume all other values, as given at right, remain the same. Just write "smaller" or "larger" in each box.
You should not have to (nor do you have time to) compute t_s for each situation; just think about what effect each change would have.

	Male	Female
n	11	15
$\bar{y}$	68	66
s	2.7	2.3

Change	Smaller or larger t_s ?
Decrease mean Female thorax weight from 66 to 65.	Larger
Decrease Male sample size.	Smaller
Increase standard deviation in Male sample.	Smaller

6 points 3. Suppose that the mean and standard deviation in Male and Female thorax weights (estimated from previous samples) are approximately

	Male	Female
μ	68	66
σ	2.7	2.3

/3 Suppose that we wanted to take samples from each population to determine whether mean thorax weights are different in Males vs. Females. In order to have

Two tailed, Significance $\alpha = .02$, Power = .95 Effect $= \frac{68-66}{2.7}$

how large would our samples need to be? $= .74$

Table 6: $n \geq 66$ (for Effect .70).

/3 In general, assuming the same Significance and Effect Size $\frac{|\mu_1 - \mu_2|}{\sigma}$ and two-tailed test, how would increasing the sample size change the Power level? Would Power level increase, decrease, or remain the same?

Sample size ↑ <=> Power ↑

$7 \cdot 8 = 56$

$n_1 = 7 \quad n_2 = 8$

13 points 4. Consider the data at right.

Y_1	Y_2
17	4
20	13
22	14
64	15
170	16
190	18
305	310
	390
$K_1 = 41$	$K_2 = ?$

/7 Find K_2, then use the Wilcoxon-Mann-Whitney Test to determine whether the populations from which the samples are taken have different means. Use $\alpha = 0.05$.

$K_2 = 15 \ (= 56 - 41)$

$U_s = 41$

$40 < U_s < 43$

$\Rightarrow \underline{.094 < P} < .189$

(Actually $.10 < P$)

So $P \not< .05$, so do not reject $H_0 : \mu_1 = \mu_2$.

/2 If the data values were different (but still 7 values in the first column and 8 values in the second column), what is the smallest U_s could possibly be, and what is the largest U_s could possibly be?

Max is $7 \cdot 8 = 56$, Min is $\frac{56}{2} = 28$.

/4 Suppose that the value of 18 in the second column were instead 21. Would the resulting U_s and P values be larger or smaller? (Just circle Larger or Smaller for each.)

U_s	Larger	Smaller
P	Larger	Smaller

New values:

$K_1 = 40$, $K_2 = 16$

$U_s = 40$

For all tests: Larger test statistic $\Rightarrow$ Smaller P value.

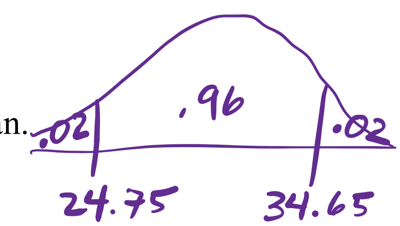
14 points 5. Six 3-year-old sheep were injected with an antibiotic. Their blood serum concentrations of Gentamicin 1.5 hours after injection were as follows:

34 26 34 31 23 30

For these data, the mean is 29.7 and the standard deviation is 4.4.

/7 Construct a (two-sided) 96% confidence interval for the population mean.

$\mu = 29.7 \pm \underbrace{2.757 \left(\frac{4.4}{\sqrt{6}}\right)}_{4.95}$, i.e. $24.75 < \mu < 34.65$



/3 Find both one-sided 98% confidence intervals for the population mean.

$24.75 < \mu$

$\mu < 34.65$

/4 Suppose we want a more precise (narrower) interval. Assuming that the mean and standard deviation would remain approximately 29.7 and 4.4, respectively, how many sheep would we need in our new sample to have a margin of error $t \cdot \frac{s}{\sqrt{n}} \leq 1$.

Table 4, 96% column: new t will be < 2.757 and > 2.056, so maybe around 2.1.

$2.1 \left(\frac{4.4}{\sqrt{n}}\right) \leq 1 \Rightarrow n \geq 85.4$.