Math 316 Fall 2024 **Midterm Exam 1** **September 10, 2024**

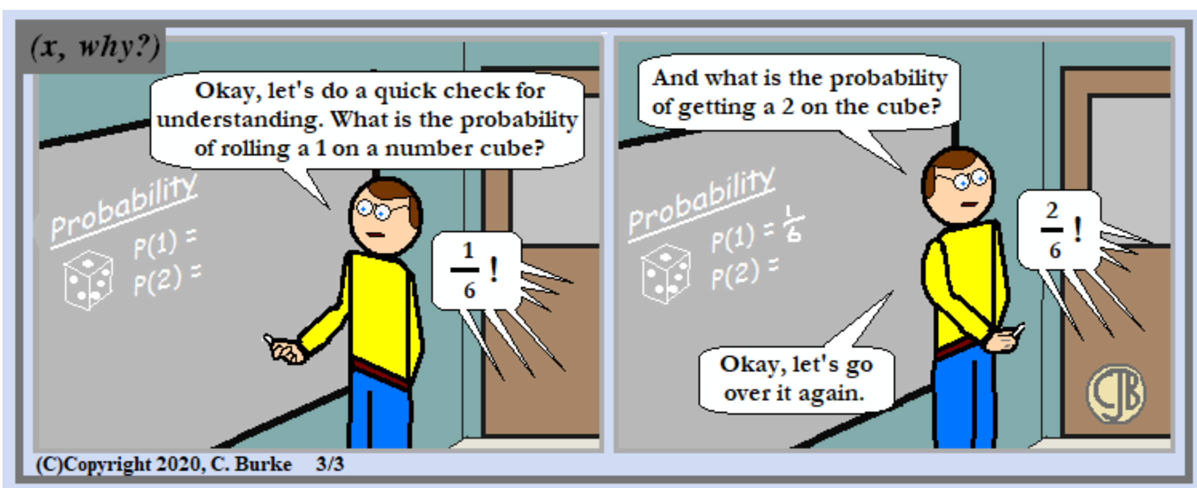
Name: Solutions

Problem	1/2/3	4/5	6	7/8	9	Total
Possible	23	27	14	20	16	100
Received						

DO NOT OPEN YOUR EXAM UNTIL TOLD TO DO SO.

You may use a 3 x 5 card (both sides) of handwritten notes and a calculator, and the provided Table 3 sheet for Areas Under the Normal Curve.

FOR FULL CREDIT, SHOW YOUR WORK FOR FINDING EACH SOLUTION.



8 points **1.** Find (and <u>show appropriate work</u>) the mean, variance and standard deviation of the following ten values:

2 2 2 2 2 5 5 10 10 10

$$\mu = \frac{2 \cdot 5 + 5 \cdot 2 + 10 \cdot 3}{10} = 5$$

$$\sigma^2 = \frac{(2-5)^2 \cdot 5 + (5-5)^2 \cdot 2 + (10-5)^2 \cdot 3}{10 \leftarrow (\text{or } 10-1)} = 12$$

So $\sigma = \sqrt{12}$.

9 points **2.** Approximately 84% of active duty United States Army is male. Suppose that three active duty Army personnel are randomly selected.

/2 What is the probability that all three are male?

$$(.84)^3 = .5927$$

/2 What is the probability that at least one is female? $1 - .5927 = .4073$

OR $_3C_2 (.84)^2(.16)^1 + {_3C_1} (.84)^1(.16)^2 + {_3C_0} (.84)^0(.16)^3 = \cdots = \uparrow$

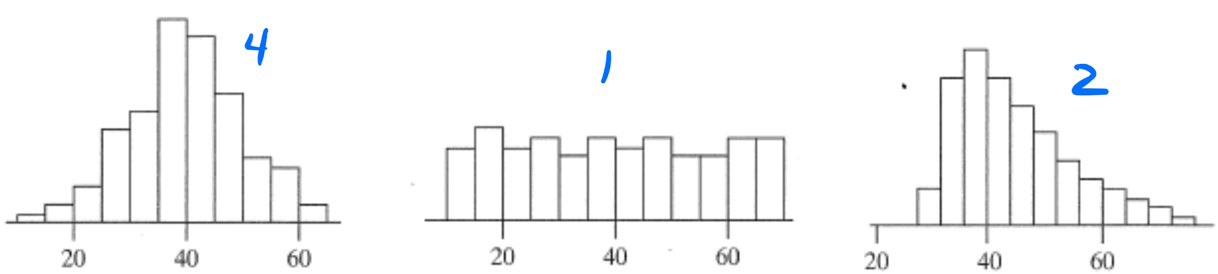
/3 What is the probability that two are male and one is female?

$$_3C_2 (.84)^2(.16)^1 = .3387$$

/2 What is the expected number of males? (That is, if I repeated randomly selected three persons, on average how many of them would be male?)

$$n \cdot p = 3(.84) = 2.52$$

6 points **3.** Match each histogram (a), (b) and (c) with case 1, 2, 3 or 4.

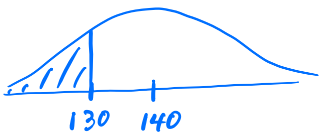


(a) (b) (c)

Case	1	2	3	4
Mean	40	42	38	40
Median	40	40	40	40
St. Dev.	18	13	13	10
	(b)	(c)	Too large for (a)	(a)

16 points **4.** The heights of a certain population of corn plants follow a normal distribution with mean 140 cm and standard deviation 20 cm.

/4 What percentage of the plant heights are 130 cm or less?



$z = \frac{130-140}{20} = -.5 \quad A(-.5) = .3085$

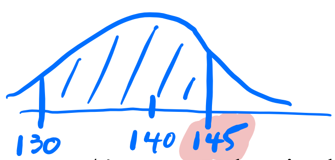
/2 What percentage of the plant heights are 130 cm or more?

$1 - .3085 = .6915$

/1 What percentage of the plant heights are exactly 130 cm?

0

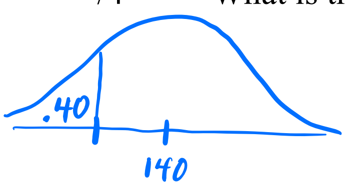
/5 What percentage of the plant heights are between 130 and 145 cm?



$z = \frac{145-140}{20} = .25 \quad A(.25) = .5987$

$.5987 - .3085 = .2902$

/4 What is the plant height at the 40th percentile?



$A(z) = .40 \Rightarrow z \approx -.25$ from Table 3

so $140 - .25(20) = 135$

11 points **5.** For the same corn plants just described above ($\mu = 140$ cm, $\sigma = 20$ cm), a sample of four corn plants is taken.

/8 What is the probability that the sample mean is between 130 and 145 cm?

$Pr\{130 \le \bar{Y} \le 145\} = \left\{ \frac{130-140}{20/\sqrt{4}} \le z \le \frac{145-140}{20/\sqrt{4}} \right\}$

$= Pr\{-1 \le z \le .5\} = A(.5) - A(-1)$

$= .6915 - .1587 = .5328$.

/3 If the sample size were larger than 4, then would the probability that the sample mean is between 130 and 145 cm be smaller or larger than what you found in the previous question (the 8 point question right above this one)? This new probability would be:

Larger Smaller

14 points **6.** Suppose that you have two investments with the following possible outcomes:

Investment A

Return	Probability
$2000	.30
$3000	.70

Investment B

Return	Probability
-$1000	.20
$5000	.80

/6 What are the expected returns/values of the two investments?

$\mu_A = \$2000(.30) + \$3000(.70) = \$2700$

$\mu_B = \$-1000(.20) + \$5000(.80) = \$3800$

/8 Find the standard deviation of each investment's return.

$\sigma_A^2 = (2000-2700)^2(.30) + (3000-2700)^2(.70) = 210\,000$

$\sigma_A = \sqrt{210\,000}$

$\sigma_B^2 = (-1000-3800)^2(.20) + (5000-3800)^2(.80) = 5\,760\,000$

$\sigma_B = \sqrt{5\,760\,000}$

$np(1-p)$ doesn't apply to this problem.

13 points **7.** Consider the following data from a study of 70 randomly selected adults in California regarding smoking habits and alcohol consumption.

	Alcohol consumption				
	None	Low	Moderate	High	**Total**
Smoke	2	2	4	7	**15**
Don't smoke	28	18	21	18	**85**
Total	**30**	**20**	**25**	**25**	**100**

/2 What is the probability that someone in this study is a high consumer of alcohol?

$\frac{25}{100} = .25$

/2 What is the probability that someone in this study is a smoker?

$\frac{15}{100} = .15$

/3 What is the probability that someone who smokes is a high alcohol consumer?

$\frac{7}{15} \approx .47$ (> .25, almost double)

/3 What is the probability that someone who is a high alcohol consumer is a smoker?

$\frac{7}{25} = .28$ (> .15, almost double)

/3 Is smoking independent of being a high alcohol consumer? Why or why not?

No. $Pr(HA|S) \neq Pr(HA)$, etc.

7 points **8.** Suppose that in a certain population of married couples we know that 40% of the husbands watch TV daily (that is, at least once a day), 30% of the wives watch TV daily, and that in 20% of the couples both the husband and wife watch TV daily.

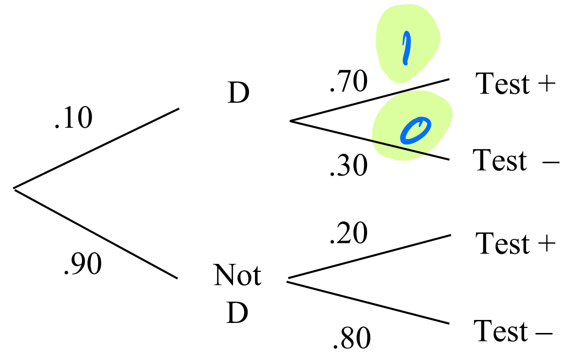
/5 What is the probability that a woman watches TV daily if we know that her husband watches TV daily?

$$Pr\{W|M\} = \frac{Pr\{W \text{ and } M\}}{Pr\{M\}} = \frac{.20}{.40} = .50$$

/2 Are the TV-watching habits (of watching TV daily or not) of married men independent of the TV-watching habits of their wives?

No. $Pr\{W|M\} \neq Pr\{W\}$

16 points **9.** Suppose that a medical test has a 70% chance of detecting a disease (D) if the person actually has it (so there is a 30% of a false negative) and a 80% chance of correctly indicating that the disease is absent if the person does not have the disease (so there is a 20% chance of false positive). Finally, suppose that 10% of the population actually has the disease.



/7 What is the probability that a randomly chosen person who tests <u>negative</u> <u>has</u> the disease?

$$\Pr\{D \mid -\} = \frac{\Pr\{D \text{ and } -\}}{\Pr\{-\}} = \frac{(.10)(.30)}{(.10)(.30) + (.90)(.80)}$$

$$= \frac{.03}{.75} = .04.$$

/2 What is the probability that a randomly chosen person who tests <u>negative</u> does <u>not</u> have the disease?

Easier approach: $1 - .04 = .96$

/7 Finally, suppose that this test did not return false negatives, that is anyone with the disease would always test positive (so the .70 and .30 above would now be 1 and 0). What would the probability be that a randomly chosen person who tests <u>positive</u> does <u>not</u> have the disease?

$$\Pr\{D' \mid +\} = \frac{\Pr\{D' \text{ and } +\}}{\Pr\{+\}}$$

$$= \frac{(.90)(.20)}{(.10)(1) + (.90)(.20)}$$

$$= \frac{.18}{.28} \approx .643.$$