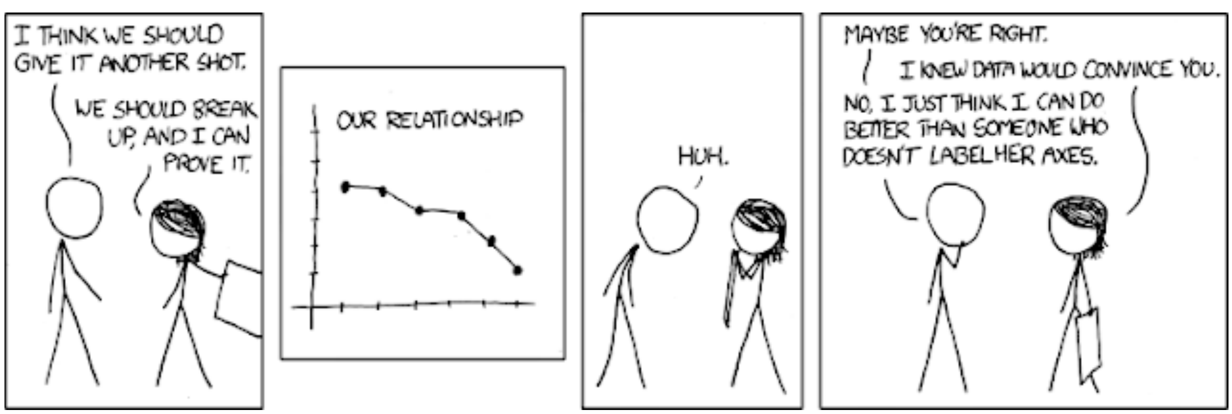
**Math 316 Spring 2023** **Exam 3** **March 27, 2023**

Name: Solutions

| Problem | 1 | 2 | 3 | 4 | 5 | 6 / 7 | Total |
|---|---|---|---|---|---|---|---|
| Possible | 15 | 15 | 14 | 23 | 19 | 14 | 100 |
| Received | | | | | | | |

FOR FULL CREDIT, SHOW ALL WORK RELATED TO FINDING EACH SOLUTION.

You may use a page of handwritten notes (both sides) and a calculator, and the supplied *Statistical Tables* and *Summary of Formulas and Tests*.




**15 points 1.** A volunteer at an animal shelter conducted a study of the effect of catnip (an herb, a member of the mint family) on cats at the shelter: she is wondering if catnip makes cats less sociable with other cats. She recorded the number of "negative interactions" each of 10 cats had with the other cats at the shelter in the 15 minutes *before* and then the 15 minutes *after* being given a teaspoon of catnip. The results are listed at right. **Notice we have paired data.**

| Cat | Before ($Y_1$) | After ($Y_2$) | Difference ($Y_1 - Y_2$) |
|---|---|---|---|
| 1 | 0 | 1 | −1 |
| 2 | 3 | 6 | −3 |
| 3 | 3 | 4 | −1 |
| 4 | 0 | 1 | −1 |
| 5 | 1 | 0 | 1 |
| 6 | 4 | 5 | −1 |
| 7 | 1 | 3 | −2 |
| 8 | 2 | 1 | 1 |
| 9 | 3 | 5 | −2 |
| 10 | 5 | 9 | −4 |
| Mean | 2.20 | 3.50 | −1.30 |
| SD | 1.69 | 2.84 | 1.57 |

/6 Construct a 95% confidence interval for the difference in mean number of negative interactions. (With pairing, you only need the Difference column values for this.)

$$\mu_D = -1.30 \pm 2.262 \underbrace{\left(\frac{1.57}{\sqrt{10}}\right)}_{\approx 1.123}$$

Table 4
$df = 10 - 1 = 9$

so $(-2.423, -.177)$

/3 For the confidence interval you just found, approximately how large would the sample size have needed to be so that the margin of error $t_{\alpha/2} SE_{\bar{D}}$ would have been $\leq 0.1$?

0.1 is much smaller than 1.123, so we'll need much larger n,
so from Table 4, 95%, use $t \approx 2$:

$$\frac{2(1.57)}{\sqrt{n}} \leq .1 \Rightarrow n \geq \left[\frac{2(1.57)}{.1}\right]^2 \approx 986.$$

/6 Compare the *before* and *after* populations using a $t$ test at $\alpha = 0.05$. Use a non-directional alternative. What do you conclude regarding $H_0$: *catnip has no effect* and $H_A$: *catnip has an effect*?

$$t_s = \frac{-1.30 - 0}{\frac{1.57}{\sqrt{10}}} \approx 2.62$$

Table 4, $df = 10 - 1 = 9$ : $.01 <$ one tail $< .02$
$\Rightarrow .02 < P < .04 < .05$

so reject $H_0$, accept $H_A$: there is significant evidence that catnip does have an effect.

**15 points 2.** Same data as in previous problem, but using different tests. **<u>We'll be ignoring pairing now</u>.** Use $df \approx 15$.

| Cat | Before ( $Y_1$ ) | After ( $Y_2$ ) | Difference ( $Y_1 - Y_2$ ) |
|---|---|---|---|
| 1 | 0 | 1 | −1 |
| 2 | 3 | 6 | −3 |
| 3 | 3 | 4 | −1 |
| 4 | 0 | 1 | −1 |
| 5 | 1 | 0 | 1 |
| 6 | 4 | 5 | −1 |
| 7 | 1 | 3 | −2 |
| 8 | 2 | 1 | 1 |
| 9 | 3 | 5 | −2 |
| 10 | 5 | 9 | −4 |
| Mean | 2.20 | 3.50 | −1.30 |
| SD | 1.69 | 2.84 | 1.57 |

/7 Construct a 95% confidence interval for the difference between Before and After, but **now ignoring pairing**. How does this interval differ from the interval you found earlier, when using pairing?

$$SE = \sqrt{\frac{1.69^2}{10} + \frac{2.84^2}{10}} \approx 1.045$$

$$\mu_D = -1.30 \pm 2.131\,(1.045)$$

Table 4, df = 15

so $(-3.527, 0.927)$

Without pairing, it is a wider interval ⍨

and in particular now includes 0.

/6 Compare the Before and After populations (still without pairing) using a <u>$t$ test</u> at $\alpha = 0.05$. Use a non-directional alternative. What do you conclude regarding $H_0$: *catnip has no effect* and $H_A$: *catnip has an effect*?

$$t_s = \frac{2.20 - 3.50}{1.045} \approx 1.24$$

Table 4, df = 15: $.10 <$ one tail $< .20$

$\Rightarrow .20 < P < .40$

$P \not< \alpha$, so we cannot reject $H_0$.

(There is some evidence, but not enough, to conclude that catnip has an effect.)

/2 Based on your above results, which test (**with pairing** in Problem 1 or **without paring** in this problem) would you say has more Power?

With pairing has more power:

we rejected $H_0$ and accepted $H_A$.

Without pairing we lost that power.

**14 points 3.** Same data as in previous problem, but now with different tests. **We'll not be assuming a normal distribution for these tests.**

| Cat | Before ( $Y_1$ ) | After ( $Y_2$ ) | Difference ( $Y_1 - Y_2$ ) |
|---|---|---|---|
| 1 | 0 | 1 | −1 |
| 2 | 3 | 6 | −3 |
| 3 | 3 | 4 | −1 |
| 4 | 0 | 1 | −1 |
| 5 | 1 | 0 | 1 |
| 6 | 4 | 5 | −1 |
| 7 | 1 | 3 | −2 |
| 8 | 2 | 1 | 1 |
| 9 | 3 | 5 | −2 |
| 10 | 5 | 9 | −4 |
| Mean | 2.20 | 3.50 | −1.30 |
| SD | 1.69 | 2.84 | 1.57 |

/5 Compare the before and after populations (**once again assuming pairing**) using a Sign Test at $\alpha = 0.05$. Again use a non-directional alternative for $H_A$.

$N_+ = 2$, $N_- = 8$ } $B_s = 8$

Table 7, $n = 10 \Rightarrow P = .109 > \alpha$, so do not reject $H_0$.

/3 Suppose that we have twenty cats instead of ten, but with the same outcomes twice. So the twenty differences would be

$$-1 \quad -1 \quad -3 \quad -3 \quad -1 \quad -1 \quad \ldots \quad -4 \quad -4.$$

instead of the original

$$-1 \quad -3 \quad -1 \quad \ldots \quad -4\,.$$

Find the new test statistic and corresponding $P$ value.

$N_+ = 4$, $N_- = 16$ } $B_s = 16$ Table 7, $n = 20 \Rightarrow P = .012 < \alpha$, so reject $H_0$, accept $H_A$.

/6 Compare the Before and After populations (**still assuming pairing**) using a Wilcoxon Signed-Rank Test at $\alpha = 0.01$. Again use a non-directional alternative.

| Differences | Rank | |
|---|---|---|
| 4 | 10 | |
| 3 | 9 | |
| 2 | 8 | Avg. 7.5 |
| 2 | 7 | |
| 1 | 6 | |
| 1 | 5 | |
| 1 | 4 | Avg. 3.5 |
| 1 | 3 | |
| 1 | 2 | |
| 1 | 1 | |

$W_+ = 3.5 + 3.5 = 7$

$W_- = 3.5(4) + 7.5(2) + 9 + 10 = 48$

$W_s = 48$

Table 8, $n = 10$:

$.02 < P < .049$

so $P \not< \alpha$,

so do not reject $H_0$.

**23 points** **4.** We are interested in the effects of a (still unproven) drug on limiting the severity of COVID symptoms. The hope is that this drug will prevent (or at least reduce) more serious symptoms, such as the need for a ventilator. 128 persons, all of whom tested positive for COVID, were randomly selected for a study. 98 persons received the drug, and 30 persons did not (these 30 received a placebo), with the following results.

**Use Wilson adjustment for $\tilde{p}$ values. Use four digits of accuracy after decimal point.**

| | | Drug | Placebo |
|---|---|---|---|
| Severity of symptoms? | Mild | 89 | 24 |
| | Serious | 9 | 6 |
| | Total | 98 | 30 |

/7 First, use these data to construct a 95% confidence interval for the proportion of all Drug patients who have (or would) experienced Serious symptoms. (You are not using the Placebo info for this question.)

$\tilde{p} = \frac{9+2}{98+4} \approx .1078 \qquad SE = \sqrt{\frac{(.1078)(1-.1078)}{98+4}} \approx .0307$

95%, Table 4 df = $\infty$ (or Table 3): $z = 1.96$

so $p_{Drug} = .1078 \pm 1.96(.0307)$, i.e. $(.0476, .1680)$

/3 Would this confidence interval be more precise (narrower) or less precise (wider) if the confidence level were instead 99%? Why? Comment on all values involved in computing how the interval would (or would not) change.

$\tilde{p}$ is approx. the same (slightly different), so SE is approx the same, but higher conf. $\Rightarrow$ larger $z$ $\Rightarrow$ wider CI.

/5 For the confidence interval you just found, approximately how large would the sample size have needed to be so that the margin of error would have been $\leq 0.01$?

95%

$1.96\sqrt{\frac{.1078(1-.1078)}{n+4}} \leq .01 \Rightarrow n \geq \left[\frac{(.1078)(.8922)(1.96)}{.01}\right]^2 - 4$

$\approx 3696$

/8 Use the above data to construct a 95% confidence interval for the difference in proportions of Drug and Placebo patients who have experienced serious symptoms. What is your conclusion about whether the Drug has an effect or not?

$\tilde{p}_1 = \frac{9+1}{98+2} = .10 \qquad \tilde{p}_2 = \frac{6+1}{30+2} \approx .2188$

$SE = \sqrt{\frac{(.1)(1-.1)}{98+2} + \frac{(.2188)(1-.2188)}{30+2}} \approx .0790$

$p_1 - p_2 = .1 - .2188 \pm 1.96(.0790)$, i.e. $(-.2736, .0360)$

**19 points 5.** A larger follow-up study was done to further test the COVID-fighting drug mentioned in the previous problem. In this new study, rather than simply Mild or Serious severity of symptoms, there were three categories: Mild (and then recovered), Serious (and then recovered), and Death.

| | Drug | Placebo | Total |
|---|---|---|---|
| Mild | 280 (270) | 170 (180) | **450** |
| Serious | 20 (24) | 20 (16) | **40** |
| Death | 0 (6) | 10 (4) | **10** |
| **Total** | **300** | **200** | **500** |

$\frac{200 \cdot 40}{500}$

/7 Fill in the missing expected values ( ) and use a chi-square test to determine whether there is a significant association between treatment (Drug or Placebo) and resulting severity of illness (Mild or Serious or Death). Use $\alpha = .01$.

$$\chi_s^2 = \frac{(280-270)^2}{270} + \cdots + \frac{(10-4)^2}{4} \approx 17.59$$

Table 9, $df = (3-1)(2-1) = 2$ : $.0001 < P < .001$

$P < \alpha$, so reject $H_0$ : the drug does have an effect.

If both sample sizes were halved (so 250 persons total), but the proportions remained the same (so, for example, there were 140 Mild/Drug persons, 10 Serious/Drug, etc.):

/3 What would the new test statistic $\chi_s^2$ be? Halved : $17.59/2 \approx 8.80$

/3 What would new the $P$-value be? Table 9: $.01 < P < .02$ (larger)

/3 According to this sample, how much more likely is someone who received the drug to experience only mild symptoms than someone who did not? That is, find

$$\frac{\hat{P}r\{\, Mild \mid Drug \,\}}{\hat{P}r\{\, Mild \mid Placebo \,\}} = \frac{280/300}{170/200} \approx 1.10$$

so about 10% more likely.

/3 Are severity of illness (Mild/Serious/Death) and treatment (Drug/Placebo) independent? Briefly explain.

No. $H_0$ was that and are independent.

This is what we rejected in the first question.

**8 points** **6.** *The following are (approximately) real data, gathered in March 2020 by Chinese scientists. I've changed the numbers to make the problem a bit simpler.* Early data suggests that COVID infection rates are affected by blood type. The values below are based on 400 persons. (*Their data was for nearly 4000 persons.*) The Expected numbers are based on the overall approximate blood type ratios in Wuhan, China of 3:2:1:2 for blood types A:B:AB:O.

| Blood type | A | B | AB | O |
|---|---|---|---|---|
| Wuhan residents with COVID (Observed) | 120 | 105 | 45 | 130 |
| General Wuhan population (Expected) | 150 | 100 | 50 | 100 |

In particular, it appears that observed blood type A was much lower than expected and type O was much higher than expected, while the other two observed blood type infection rates were close to what was expected. Determine whether the infection rate is affected by blood type using a chi-square test at $\alpha = .02$.

$$\chi_s^2 = \frac{(120-150)^2}{150} + \cdots + \frac{(130-100)^2}{100} = 15.75$$

Table 9, $df = 4-1 = 3$ : $.001 < P < .01$

$P < \alpha$ so reject $H_0$.

There is significant evidence that blood type does affect infection rate.

**6 points** **7.** Suppose we focus only on the 250 persons with blood types A and O:

| Blood type | A | O |
|---|---|---|
| Wuhan residents with COVID (Observed) | 120 | 130 |
| General Wuhan population (Expected) | 150 | 100 |

The direction in this sample is the same as what is being tested.

Using a chi-square test at $\alpha = .02$, determine whether type A persons have a lower (so this is a directional test) risk of being infected with COVID than type O persons.

$$\chi_s^2 = \frac{(120-150)^2}{150} + \frac{(130-100)^2}{100} = 15$$

Table 9, $df = 2-1 = 1$ :

$$\frac{.0001}{2} < P < \frac{.001}{2}$$