Math 316 Spring 2023 **Exam 2** **February 23, 2023**

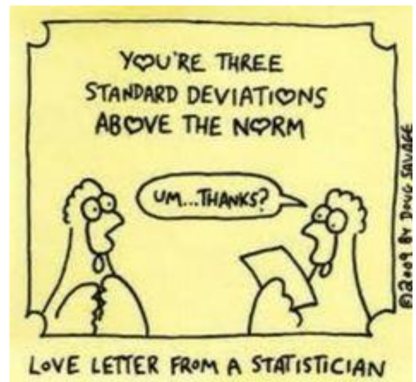
Name: Solutions

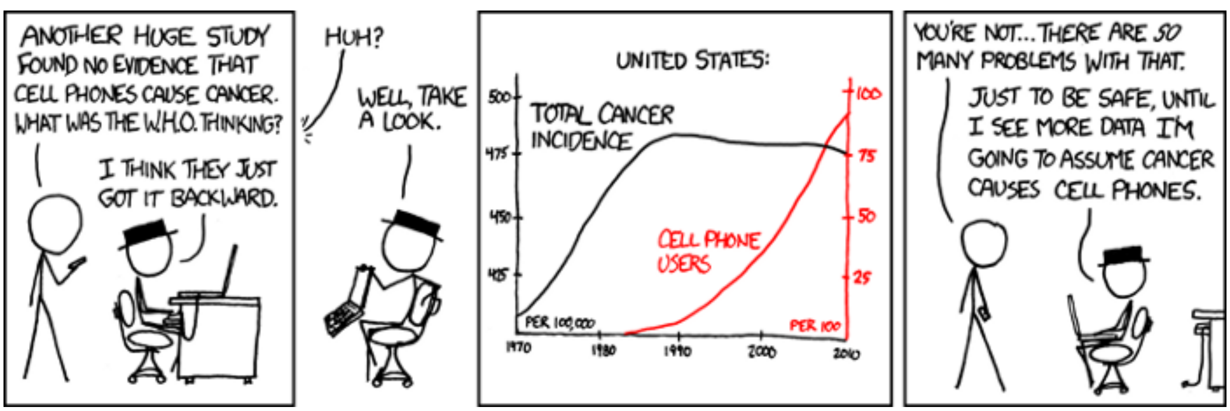
Problem	T/F	1	2	3	4	Total
Possible	40	8	13	23	16	100
Received						

DO NOT OPEN YOUR EXAM UNTIL TOLD TO DO SO.

You may use a 3 x 5 card of handwritten notes, a calculator, and the provided Tables.

FOR FULL CREDIT, SHOW ALL WORK RELATED TO FINDING EACH SOLUTION.





True/False. Answer the following True/False questions (circle **T** or **F**). Each question is worth 2 points.

T **F** If using a sample to estimate a confidence interval for the population mean, the lower the confidence level, the wider the confidence interval.

T **F** Assuming that all other values in a sample are unchanged, if the variation within the sample were larger, then the resulting confidence interval for μ would be wider.

T **F** Assuming that all other values in a sample are unchanged, if the sample size were larger, then the resulting confidence interval for μ would be wider.

T **F** If two samples come from the same population, then they would almost always have approximately the same sample mean. too strong a statement

T **F** If two samples come from two different populations that have the same mean, then the larger the samples, the more likely the sample means will be close to each other.

T **F** If a population has mean μ and standard deviation σ, then a sample of size n from that population would have mean $\bar{y} \approx \mu$ and standard deviation $s \approx \cancel{\frac{\sigma}{\sqrt{n}}}$

T **F** If in testing $H_0: \mu_1 = \mu_2$ we choose $\alpha = 0.05$, then we are willing to accept up to a 5% probability we might make a Type I error if we decide to accept $H_A: \mu_1 \neq \mu_2$.

T **F** "Power" is the likelihood that we will conclude that the population means are different when in fact they are.

T **F** If we conclude that two population means are significantly different, then that difference in means would also necessarily be considered important.

T **F** If in testing the hypotheses $H_0: \mu_1 = \mu_2$, $H_A: \mu_1 \neq \mu_2$ we find that $P = .04$, then there is a probability of .04 that we are making a Type II error if we decide to reject H_0 and accept H_A.

T **F** If in comparing two samples we do not conclude that $\mu_1 \neq \mu_2$, then we are concluding instead that $\mu_1 = \mu_2$. Never!

T **F** In finding a confidence interval for $\mu_1 - \mu_2$, if we have sample sizes that are large enough, then we can always guarantee that we will have a confidence interval which does not include 0 and thus conclude that the two populations from which the samples were taken have different means. We discussed this in class.

T **F** It is possible that two populations have approximately (or maybe even exactly) the same mean.

T **F** A two-tailed test is the same as a directional test.

T **F** The larger a test statistic, the more different the samples are, and the more likely we will conclude that the populations from which the samples are taken are different.

True of all tests, not only the two we've seen so far.

T F If two samples are taken of sizes 20 and 30, then a test statistic of $t_s = 301$ would result in an extremely small P-value.

$20 \cdot 30 = 600$

T F If two samples are taken of sizes 20 and 30, then a test statistic of $U_s = 301$ would result in an extremely small P-value.

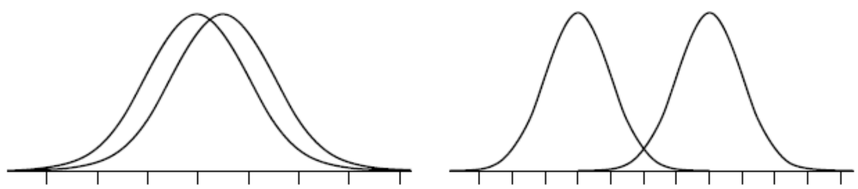
$\frac{600}{2} \leq U_s \leq 600$ Very small U_s, so P will be large.

T F If comparing the means of two populations, if the samples are of sizes 20 and 30, it would appropriate to use the t test.

True for large samples. 20 is probably large enough, but I accepted T or F.

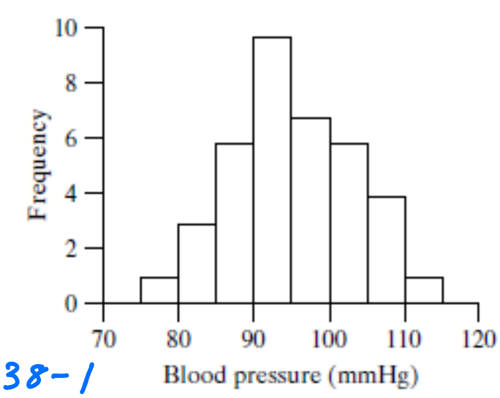
T F If comparing the means of two populations, if the samples are of sizes 20 and 30, it would be possible to use the Wilcoxon-Mann-Whitney Test, but the WMW Test might not be as powerful as the t test.

T F In the samples below, there is larger effect seen in the samples in (a) than the samples in (b).



(a) (b)

8 points 1. The blood pressure of each of 38 persons was measured. The mean blood pressure of these 38 persons was 95. A histogram of the data is shown at right.



/6 Find (approximately, i.e. estimate) a 99% two-sided confidence interval for the mean diameter.
Hint: first estimate the standard deviation of the data, using the histogram.

$s \approx 10$, or too be a bit more liberal, $7.5 < s < 12.5$.
$s = 5$: definitely too small
$s = 15$: " " large.

$df = 38 - 1 = 37$

$\mu = 95 \pm 2.750\left(\frac{10}{\sqrt{38}}\right)$
i.e. $(90.54, 99.46)$.

/2 If the 84% confidence interval for these 38 persons were $95 \pm a$ (where a is some number), if we instead had a sample of 152 (which is 4 times larger than 38) that had the same mean and standard deviation of the original 38 person sample, approximately what would be the 84% CI for this 152 person sample? (84% confidence is not a value given in Table 4. You do not need to know the t value to answer this question.)

If instead of $95 \pm t \cdot \frac{s}{\sqrt{n}}$ we had $95 \pm t \cdot \frac{s}{\sqrt{4n}} \leftarrow \frac{s}{2\sqrt{n}}$
then the interval of $95 \pm a$ would now be $\approx 95 \pm \underbrace{\frac{a}{2}}$
i.e. approx. half the orig. width

13 points 2. We are interested in the diameter of the stem of a certain type of wheat plant. We take a sample of 8 plants, which have the following stem diameters, with mean 2.15 and standard deviation 0.51:

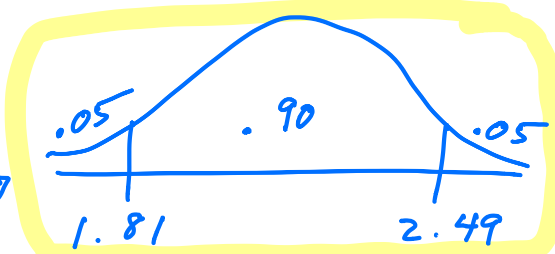
2.3 2.6 2.4 2.2 2.3 2.5 1.9 1.0

/5 Find a 90% two-sided confidence interval for the mean diameter.

$df = 8-1 = 7 \Rightarrow t_{\alpha/2} = 1.895.$

$\mu = 2.15 \pm 1.895\left(\frac{.51}{\sqrt{8}}\right)$

i.e. $(1.81, 2.49)$



/2 Find both 95% one-sided confidence intervals for the mean diameter.

From previous work:

$1.81 < \mu$

and $\mu < 2.49$

/4 If we wanted to end up with a 90% two-sided confidence interval with total width of 0.02, how large would our sample need to be? (We would expect the standard deviation of this new larger sample to be approximately the same as for the 8 plant sample.)

n will need to be much larger, so need $t \approx 1.7$ (or maybe slightly smaller)

$\frac{t \cdot s}{\sqrt{n}} \leq \frac{.02}{2} \Rightarrow n \geq \left(\frac{t \cdot s}{.01}\right)^2 = \left(\frac{(1.7)(.51)}{.01}\right)^2$

≈ 7517

/2 If the original data were changed so that the first and last values of 2.3 and 1.0 were now 2.4 and 0.9 (note the sample mean would still be 2.15), the new CI would be (circle one):

Wider The same width Narrower

Larger $s \Rightarrow$ larger $\frac{t \cdot s}{\sqrt{n}}$

23 points 3. We are comparing two populations from which we have the samples shown at right, each of size 4.
For all of the following problems we will use $\alpha = 0.10$, i.e. confidence level of 90%, and $df = 4$.

	Sample 1	Sample 2
	3.5	2
	3.5	3
	6.5	3
	6.5	4
Mean	**5.00**	**3.00**
SD	**1.73**	**0.82**

/5 Find a two-sided confidence interval for $\mu_1 - \mu_2$. What is your conclusion regarding $H_0: \mu_1 = \mu_2$ with $H_A: \mu_1 \neq \mu_2$?

$SE = \sqrt{\frac{1.73^2}{4} + \frac{.82^2}{4}} \approx .957$ $df = 4 \Rightarrow t = 2.132$

$\mu_1 - \mu_2 = \underbrace{5-3}_{2} \pm \underbrace{2.132(.957)}_{2.04}$

So $(-.04, 4.04)$, which includes 0 so do not reject $H_0: \mu_1 = \mu_2$.

/5 Next, use the t-test for $H_0: \mu_1 = \mu_2$, $H_A: \mu_1 \neq \mu_2$. Show pertinent work (t_s, P, etc.).

$t_s = \frac{5-3}{.957} = 2.09$ Table 4: $.05 < \text{one tail} < .10$
$\Rightarrow .10 < P < .20$

$P \not< \alpha$, so (as above)
do not reject H_0.

/2 What would your conclusion be if we instead had $H_A: \mu_1 < \mu_2$?

Wrong direction in sample: $\bar{y}_1 > \bar{y}_2$.
So definitely do not reject H_0 and accept

/2 What would your conclusion be if we instead had $H_A: \mu_1 > \mu_2$?

One tailed test: $.05 < P < .10$
$P < \alpha$, so yes reject H_0, accept

/4 Suppose that all of the values in Sample 1 were 0.1 larger, so those values were now 3.6, 3.6, 6.6 and 6.6. How would the values of t_s and P change?

t_s would be: Larger Same Smaller

P would be: Larger Same Smaller

/5 What sample size would be needed (assuming both sample sizes are still the same) in order to have a power level of 0.95, to go along with two-tailed confidence of 90%?

Effect $= \frac{5-3}{1.73} \approx \underbrace{1.15}$ Round down to be conservative, i.e. larger n

OR $\frac{5-3}{.82} \approx \underbrace{2.45}$ Use 2.4

Table 6: $n \geq 5$

Table 5: $n \geq 19$

Be conservative, use larger n, so $n \geq 19$.

16 points 4. Let's now use the Wilcoxon-Mann-Whitney Test, still with $\alpha = 0.10$.

Sample 1	Sample 2
3.5	2
3.5	3
6.5	3
6.5	4

/6 Test $H_0: \mu_1 = \mu_2$ with $H_A: \mu_1 \neq \mu_2$. Show pertinent work.

$U_s = 14$ Table 6:

$.10 < P < .20$.

You can tell $P > .10$:
We need (at least) $U_s \geq 15$
to get $P < .10$.

$K_1 = 14$ $K_2 = 2$
(Notice $K_1 + K_2 = 4 \cdot 4$)

/2 What would your conclusion be if we instead had $H_A: \mu_1 < \mu_2$?

Wrong direction. Do not reject H_0 or accept

/2 What would your conclusion be if we instead had $H_A: \mu_1 > \mu_2$?

One tailed: $.05 < P < .10$

$P < \alpha$, so reject H_0, accept

/4 Suppose that all of the values in Sample 1 were 0.1 larger, so those values were now 3.6, 3.6, 6.6 and 6.6. How would the values of U_s and P change?

U_s would be:	Larger	Same	Smaller
P would be:	Larger	Same	Smaller

Same K_1, K_2, U_s, P.

We see that the WMW Test is not as sensitive to changes in the data. (Not a good thing.)

/2 For the above data, which test is more appropriate, the t-test or the WMW Test? Why?

WMW. Very small samples,
and Sample 1 is not normal.