Math 316 Spring 2023 **Midterm Exam 1** **January 31, 2023**

Name: Solutions

Problem	1 / 2	3 / 4	5 / 6	7	8 / 9	Total
Possible	15	26	18	20	21	100
Received						

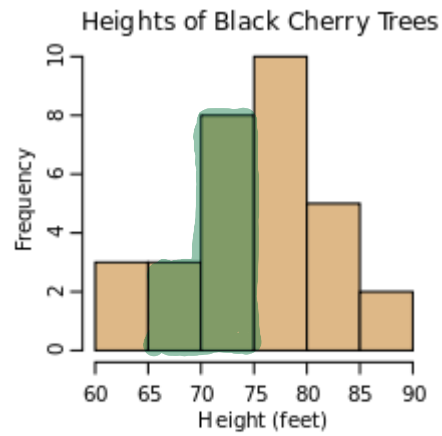
DO NOT OPEN YOUR EXAM UNTIL TOLD TO DO SO.

You may use a 3 x 5 card (both sides) of handwritten notes and a calculator.

FOR FULL CREDIT, SHOW YOUR WORK FOR TO FINDING EACH SOLUTION.

Math phobic's nightmare

6 points **1.** Consider the frequency distribution for 31 black cherry tree heights.



<u>Estimate</u> (don't try to compute exactly) the mean and standard deviation for this distribution (*circle one of the given values*):

/2 Mean: 70 75 80 82

/2 Standard deviation: 2 4 7 12

/2 Find the probability that a randomly selected tree (of these 31 trees) would be between 65 and 75 feet tall.

$$\frac{3+8}{31}$$

9 points **2.** Find (and <u>show appropriate work</u>) the mean, variance and standard deviation of the following ten values:

0 1 1 4 4 5 5 10 10 10

$$\mu = \frac{0\cdot 1 + 1\cdot 2 + 4\cdot 2 + 5\cdot 2 + 10\cdot 3}{10} = 5$$

$$\sigma^2 = \frac{(0-5)^2\cdot 1 + (1-5)^2\cdot 2 + \cdots + (10-5)^2\cdot 3}{10} = 13.4$$

Or use 10 − 1

$$\sigma = \sqrt{13.4}$$

9 points **3.** Suppose that in families with two or fewer kids we have the following information:

# kids	Fraction of families
0	.10
1	.30
2	.60

/3 What is the average number of kids in these families?

$\mu = 0(.1) + 1(.3) + 2(.6) = 1.5$

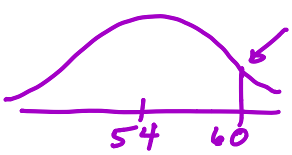
/6 What is standard deviation of the number of kids in these families?

$\sigma^2 = (0-1.5)^2(.1) + (1-1.5)^2(.3) + (2-1.5)^2(.6) = .45$

$\sigma = \sqrt{.45}$

17 points **4.** Suppose in a certain fish population (with normal distribution) that the average fish length is 54 mm with a standard deviation of 4 mm.

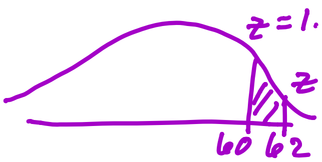
/4 What percentage of the fish are longer than 60 mm?



$z = \frac{60-54}{4} = 1.5$ $A(1.5) = .9332$ $1 - .9332 = .0668$

/2 What percentage of the fish are shorter than than 60 mm?

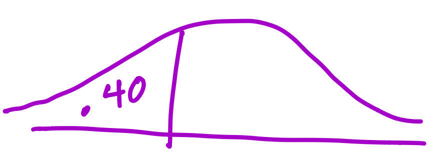
/6 What percentage of the fish are between 60 and 62 mm?



$A(2) - A(1.5) = .9772 - .9332$

$= .0440$

/5 What is the fish length at the 40^{th} percentile?



$A(z) = .40$

$\Rightarrow z \approx -.25$

So $y^* = 54 - .25(4) = 53$

12 points **5.** In the United States, 38% of the population has type O+ blood. Suppose we take a random sample of 10 persons, and we are interested in how many of them have O+ blood. There are 11 possible outcomes, from 0 to all 10 of them having O+ blood, as shown at right. (Variable Y is the number of persons of the 10 who have O+ blood.)

k	$Prob\{Y = k\}$
0	0.0084
1	0.0514
2	0.1419
$\vdots$	
6	?
$\vdots$	
10	0.0001

/6 Find the probability that 6 of the 10 persons have O+ blood.

$\underbrace{{}_{10}C_{6}}_{210}(.38)^6(.62)^4 = .0934$

/2 Of this sample of 10, what is the expected number μ_Y of persons with O+ blood?

$\mu = np = 10(.38) = 3.8$

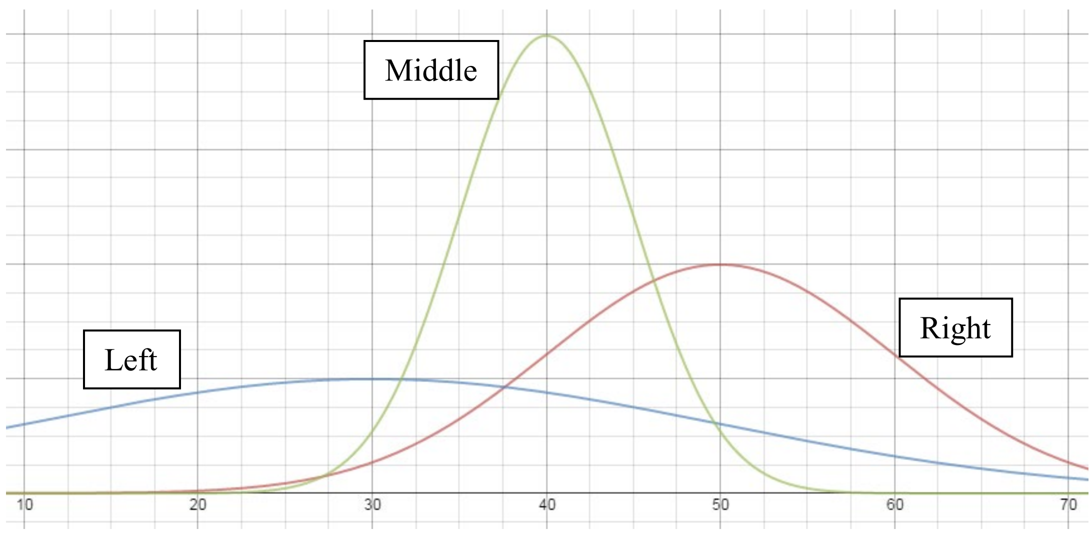
OR: $\mu = 0(.0084) + \cdots + 10(.0001) = 3.8$ if you like to do more work!

/4 What is the standard deviation σ_Y of this distribution of outcomes?

$\sigma = \sqrt{np(1-p)} = \sqrt{10(.38)(.62)} \approx 1.53$

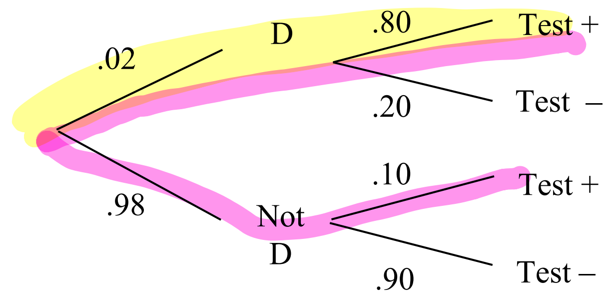
OR: $\sigma = (0-3.8)^2(.0084) + \cdots + (10-3.8)^2(.0001)$
etc.

6 points **6.** Three normal curves are plotted. Given that the standard deviations of these three curves are $5, 10$ and 20 (not necessarily in that order), write the mean and standard deviation for each.



Curve	Mean	Std Dev
Left	30	20
Middle	40	5
Right	50	10

20 points **7.** Suppose that a medical test has a 80% chance of detecting a disease (D) if the person actually has it (thus there is a 20% of a false negative) and a 90% chance of correctly indicating that the disease is absent if the person really does not have the disease (so there is a 10% chance of false positive). Finally, suppose that 2% of the population actually has the disease.



/2 What is the probability that a randomly chosen person both has the disease and tests positive?

$(.02)(.80) = .016$

/4 What is the probability that a randomly chosen person would test positive?

$(.02)(.80) + (.98)(.10) = .114$

/4 What is the probability that a randomly chosen person who tests positive actually has the disease?

$\Pr\{D \mid +\} = \dfrac{\Pr\{D \text{ and } +\}}{\Pr\{+\}} = \dfrac{.016}{.114} = .140$

/3 What is the probability that a randomly chosen person who tests positive does <u>not</u> have disease?

$\Pr\{\text{no } D \mid +\} = \dfrac{\Pr\{\text{no } D \text{ and } +\}}{\Pr\{+\}} = \ldots$ OR: $1 - .140 = .860$

/7 Finally, suppose that this test did not return false positives, that is anyone without the disease would always test negative (so the .10 and .90 above would now be 0 and 1). What would the probability be that a randomly chosen person who tests positive actually has the disease?

$\Pr\{D \mid +\} = \dfrac{\Pr\{D \text{ and } +\}}{\Pr\{+\}} = \dfrac{(.02)(.80)}{(.02)(.80) + (.98)(0)} = 1.$

If there are no false positives, then someone who tests positive <u>must</u> have the disease.

13 points **8.** Consider the following data from a study of 65 randomly selected adults in California regarding smoking habits and income level.

	Income			
	Low	Medium	High	**Total**
Smoke	8	3	2	**13**
Don't smoke	17	17	18	**52**
Total	**25**	**20**	**20**	**65**

/2 What is the probability that someone in this study is a smoker?

$\frac{13}{65}$

/2 What is the probability that someone in this study has low income?

$\frac{25}{65}$

/3 What is the probability that someone who has low income is a smoker?

$\frac{8}{25}$

/3 What is the probability that someone who is a smoker has low income?

$\frac{8}{13}$

/3 Is being a smoker independent of having a low income? Why or why not?

No: $Pr\{S \mid LI\} \neq Pr\{S\}$
$Pr\{LI \mid S\} \neq Pr\{LI\}$

$Pr\{S \mid LI\} \neq Pr\{LI \mid S\}$ is not a reason.

8 points **9.** Suppose that in a certain population of married couples we know that 50% of the husbands watch TV daily (that is, at least once a day), 60% of the wives watch TV daily, and that in 40% of the couples both the husband and wife watch TV daily.

/5 What is the probability that a woman watches TV daily if we know that her husband watches TV daily?

$$Pr\{W \mid H\} = \frac{Pr\{W \text{ and } H\}}{Pr\{H\}} = \frac{.40}{.50} = .80$$

/3 Are the TV-watching habits (of watching TV daily or not) of married men independent of the TV-watching habits of their wives? Explain/show why or why not.

No: $Pr\{W \mid H\} \neq Pr\{W\}$