Math 316 Spring 2022 **Midterm Exam 2** **February 24, 2022**

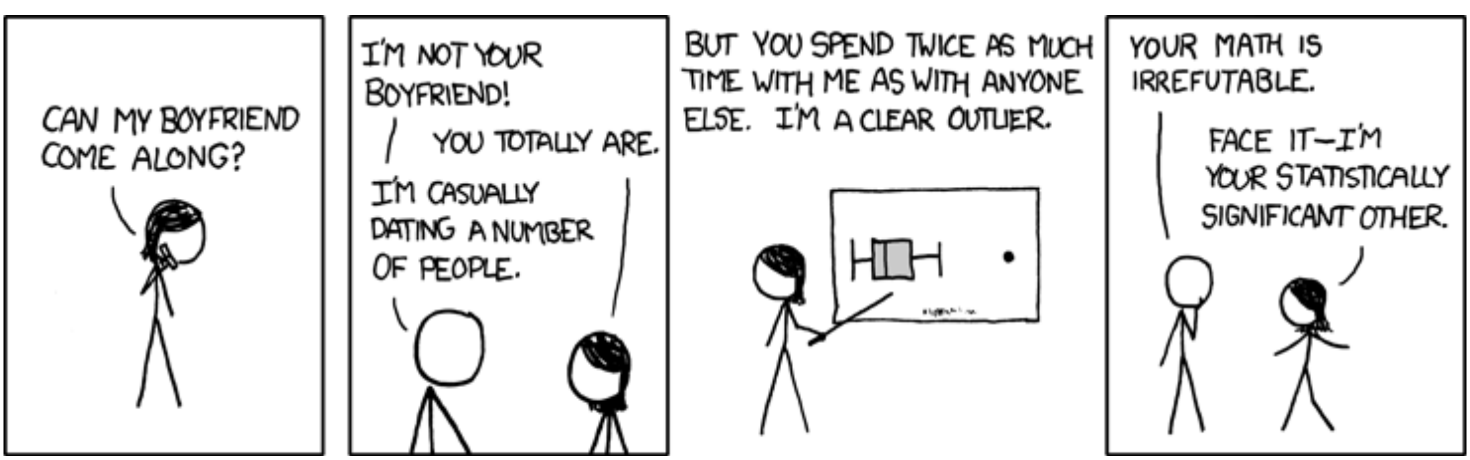
Name: Solutions

Problem	T / F	1	2	3	Total
Possible	30	25	30	15	100
Received					

DO NOT OPEN YOUR EXAM UNTIL TOLD TO DO SO.

You may use a 3 x 5 card (both sides) of handwritten notes and a calculator.

FOR FULL CREDIT, SHOW YOUR WORK.



True/False. Answer the following True/False questions. True means always true. False means it is simply false, or it could sometimes be true but not necessarily always. Each question is worth 2 points.

T **F** In general, the more different two sample means are, the larger their test statistic will be, and the more likely it is that we will conclude that the populations are different.

T **F** "Power" is the likelihood that we will conclude that the population means are ~~the same~~ when in fact they are.

different

T **F** If in testing the hypotheses $H_0: \mu_1 = \mu_2$, $H_A: \mu_1 \neq \mu_2$ we find that $P = .07$, then there is a probability of .07 that we are making a Type II error if we decide to ~~not~~ reject H_0.

I

T **F** Two different samples from the same population could have very different means and standard deviations.

Unlikely, but is possible.

T **F** Suppose that we are testing the hypotheses $H_0: \mu_1 = \mu_2$, $H_A: \mu_1 \neq \mu_2$. If there is not significance evidence to reject H_A, then we should accept H_0 and conclude that the population means are equal.

T **F** If in testing a hypothesis we choose $\alpha = 0.05$, then there is up to a 5% chance we might make a Type I error if we decide to accept $H_A: \mu_1 \neq \mu_2$.

T **F** A larger sample size n will typically result in a smaller sample standard deviation s.

No matter what sample sizes are,

Sample $s \approx$ Population σ

T F If using a sample to estimate a confidence interval for the population mean, more variation in the sample data (larger standard deviation) will typically result in a wider confidence interval.

T F If using a sample to estimate a confidence interval for the population mean, a larger sample will typically result in a wider confidence interval.

T F People who drink more also smoke more, thus drinking causes smoking.

Causation ≠ association.

T F If we have $P < \alpha$ for a directional test (e.g. $H_A: \mu_1 > \mu_2$ or $H_A: \mu_1 < \mu_2$), then we will have $P < \alpha$ for a non-directional test (e.g. $H_A: \mu_1 \neq \mu_2$).

Not neccesarily. The converse is true.

T F Given samples with means of 2 and 3, if using the t-test, it is possible to get value of $t_s = -0.2$, depending on what sample sizes and standard deviations are.

T F Given samples of sizes 7 and 3, if using the Wilcoxon-Mann-Whitney Test, it is possible to get a test statistic of $U_s = 6.5$.

max $U_s = 7 \cdot 3 = 21$

so min $U_s = \frac{21}{2} = 11.5$

T F It is possible that the difference between two samples is both "significant" and "important."

T F Larger effect size corresponds to ~~larger~~ smaller overlap between two samples' data.

25 points **1**. Consider the sample at right.

Sample	1 , 2, 3, 4, 5
n	5
$\bar{y}$	3.00
s	1.58

/7 (a) Find a two-sided 95% confidence interval for μ.

$3 \pm 2.776\left(\frac{1.58}{\sqrt{5}}\right) = 3 \pm 1.96$

$df = 5-1=4$ $\quad = (1.04, 4.96)$

/4 (b) How large would your sample need to be in order to have a confidence interval of total width 0.2 (i.e., so that $t \cdot \frac{s}{\sqrt{n}} = 0.1$)?

n will be much larger, so will have $t \approx 2$.

So $2 \cdot \frac{1.58}{\sqrt{n}} \le 0.1 \Rightarrow n \ge \left[\frac{2(1.58)}{0.1}\right]^2 \approx 999$.

/2 (c) If the sample were larger, but still had the same mean and standard deviation, the confidence interval would be:

Narrower The same Wider

/3 (d) Which one of the following could be a 95% confidence interval (same confidence level as in part (a)) for μ if the Sample value of 5 were replaced with a larger number?

(−7 , 13) (2.8 , 3.2) (0 , 4) (−0.4 , 8.4)

We will have $\bar{y} > 3$ and $s > 1.58$ so $\frac{s}{\sqrt{n}} > 1.96$

/3 (e) Which one of the following could possibly be a confidence interval for μ with a confidence level that is $<$ 95%?

(−7 , 13) (2.8 , 3.2) (0 , 4) (−0.4 , 8.4)

Still centered at 3 but lower confidence ⇒ narrower CI

/6 (f) Find a one-sided 95% (in either direction, your choice: either $\mu <$ *something* or $\mu >$ *something*) CI for μ using the original data shown in the above table.

$3 + 2.132\left(\frac{1.58}{\sqrt{5}}\right) = 4.51$, so $\mu < 4.51$

or $3 - 2.132\left(\frac{1.58}{\sqrt{5}}\right) = 1.49$, so $\mu > 1.49$

$t = 2.132$
$df = 4$

30 points **2**. Consider the two samples at right. We will test the hypothesis $H_0: \mu_1 = \mu_2$ vs. the non-directional alternative $H_A: \mu_1 \neq \mu_2$.

	Sample 1	Sample 2
	3 6 9	1 2 4 5
n	3	4
$\bar{y}$	6.00	3.00
s	3.00	1.82

/7 (a) Find a 95% confidence interval for $\mu_1 - \mu_2$. **Use** $\boldsymbol{df} \approx \mathbf{3}$.

$$\underbrace{6-3 \pm 3.182 \underbrace{\sqrt{\frac{3^2}{3} + \frac{1.82^2}{4}}}_{1.96}}$$

$$3 \pm 6.23$$

$(-3.23, 9.23)$, which contains 0,

so we cannot reject $H_0: \mu_1 - \mu_2 = 0$,

i.e. $H_0: \mu_1 = \mu_2$.

/7 (b) Find t_s and the corresponding value of P (again where our test is non-directional). Again **use** $\boldsymbol{df} \approx \mathbf{3}$.

$$t_s = \frac{6-3}{1.96} = 1.53 \Rightarrow .10 < \text{one tail} < .20$$

$$\Rightarrow .20 < P < .40$$

$P \not< .05$, so we cannot reject H_0.

/2 (c) Based on the confidence interval in (a), how did you know that P in (b) would be > .05?

Because 0 was in the CI in (a).

/2 (d) If the test were one-sided (directional), what would the P-value be?

$.10 < P < .20$

/3 (e) If the three values in Sample 1 were larger by the same amount (e.g. 4,7,10 or 5,8,11 or …), then:

t_s would be: Smaller Unchanged Larger Can't tell based on this information

/3 (f) If the sample sizes were both larger, but still had the same means and standard deviations, then t_s would be:

t_s would be: Smaller Unchanged Larger Can't tell based on this information

/3 (g) Suppose we took new (larger) samples and we wanted 95% Confidence ($\alpha = .05$) and 99% Power. How large would the two samples need to be?

Effect = $\frac{6-3}{3} = 1$. Table 5: $n \geq 38$.

Using effect $\frac{6-3}{1.82} = 1.65 \Rightarrow n \geq 17$ or 19. Be conservative: use $n \geq 38$.

/3 (h) If the samples had larger standard deviations but had the same means (for example, if the samples were 2,6,10 and 0,2,4,6), compared to the sample sizes you found in (f), to still get 95% Confidence and 99% Power, the new sample sizes would need to be:

Larger $s_1, s_2 \Rightarrow$ smaller effect $\Rightarrow$ Smaller The same Larger samples needed.

15 points **3**. Next we'll use the Wilcoxon-Mann-Whitney Test for the two samples at right.

Sample 1	Sample 2
2 3	1 0
4 6	2 0
4 9	4 1
$K_1 = 10$	5 1
	$K_2 = 2$

/6 (a) Find the test statistic U_s and corresponding P-value. Assume our test is non-directional.

So $U_s = 10$. (Notice $10 + 2 = 3 \cdot 4$.)

Table 6: $n = 4$, $n' = 3$.

While it is true that $P > .114$,

it is also true (and more correct/useful)

to realize $P > .20$.

/2 (b) How would U_s change if the 9 were changed to 10?

U_s would be: Smaller The same Larger

/7 (c) If the samples were doubled in size, but with the same values, find the test statistic U_s and corresponding P-value.

Sample 1	Sample 2
3	1
3	1
6	2
6	2
9	4
9	4
	5
$K_1 = 40$	5
	$K_2 = 8$

$U_s = 40$

Table 6: $n = 8$, $n' = 6$.

$P = .043$