Math 316 Spring 2022 **Midterm Exam 1** **February 1, 2022**

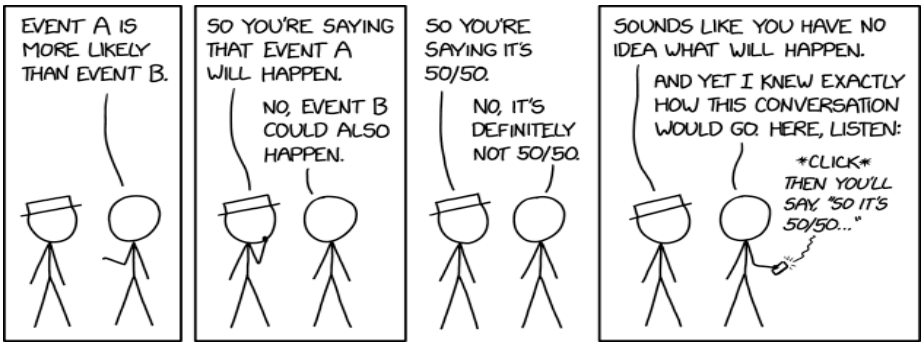
Name: Solutions

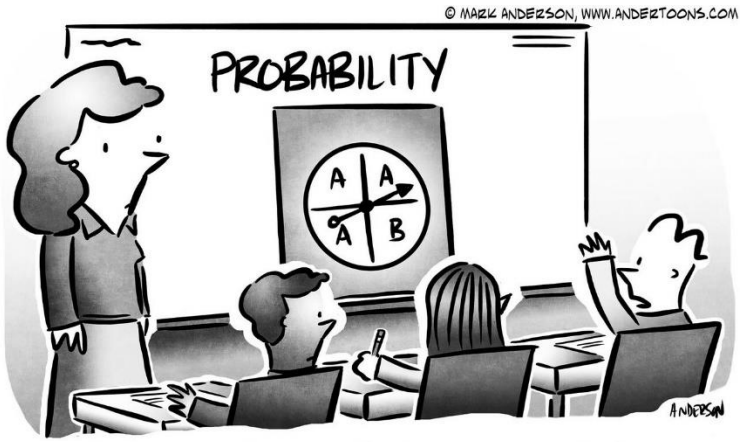
Problem	1	2 / 3	4	5 / 6	7 / 8	Total
Possible	20	25	20	21	14	100
Received						

DO NOT OPEN YOUR EXAM UNTIL TOLD TO DO SO.

You may use a 3 x 5 card (both sides) of handwritten notes and a calculator.

FOR FULL CREDIT, SHOW YOUR WORK.

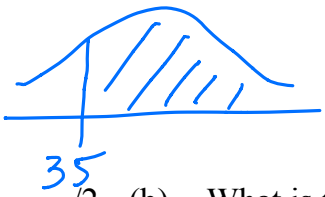




"I know mathematically that A is more likely, but I gotta say, I feel like B wants it more."

20 points **1**. Suppose that the average weight of a certain type of corn seed is normally distributed with a mean of 40 mg and a standard deviation of 10 mg.

/4 (a) What fraction of these seeds are heavier than 35 mg?



$z = \frac{35-40}{10} = -.5$

$1 - A(-.5) = 1 - .3085 = .6915$

/2 (b) What is the probability that an individual seed would be heavier than 35 mg?

Same as (a).

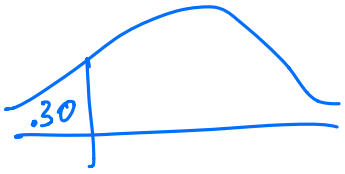
/4 (c) What fraction of seeds' weights are between 25 and 35 mg?

$A\left(\frac{35-40}{10}\right) = A(-.5) = .3085$

$A\left(\frac{25-40}{10}\right) = A(-1.5) = .0668$

$.3085 - .0668 = .2417$

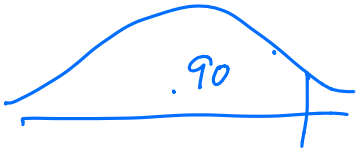
/4 (d) What seed weight is at the 30th percentile?



$A(z) = .30 \Rightarrow z \approx -.52$

so $40 - .52(10) = 34.8$

/4 (e) What seed weight is at the 90th percentile?

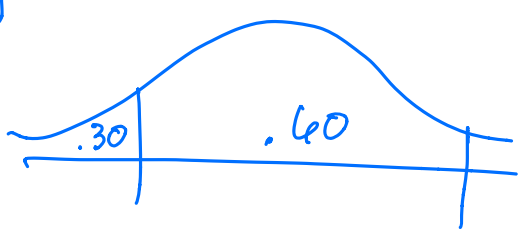


$A(z) = .90 \Rightarrow z \approx 1.28$

so $40 + 1.28(10) = 52.8$

/2 (f) What fraction of seeds' heights are between the heights you found in (d) and (e)?

Exactly 60%



15 points **2.** Same info from Problem 1: suppose that the average weight of a certain type of corn seed is normally distributed with a mean of 40 mg and a standard deviation of 10 mg.

/7 (a) If you take a sample of 9 seeds, find $\Pr\{37 \le \bar{Y} \le 46\}$, the probability that the sample mean $\bar{Y}$ will be between 37 and 46 mg.

$$\Pr\left\{\frac{37-40}{10/\sqrt{9}} \le z \le \frac{46-40}{10/\sqrt{9}}\right\} = \Pr\{-.9 \le z \le 1.8\}$$

$$= A(1.8) - A(-.9) = .9641 - .1841 = .7800$$

/8 (b) Given the four probabilities:

$a = \Pr\{35 \le \bar{Y} \le 45\}$, where $n = 100$

$b = \Pr\{30 \le \bar{Y} \le 50\}$, where $n = 100$

$c = \Pr\{30 \le \bar{Y} \le 50\}$, where $n = 200$

$d = \Pr\{35 \le Y \le 45\}$ for a single value Y

Write a, b, c, d in decreasing order (largest value to smallest):

c b a d

10 points **3.** Find the expected value μ_Y and standard deviation σ_Y given the following probability distribution for random variable Y. Show all pertinent work.

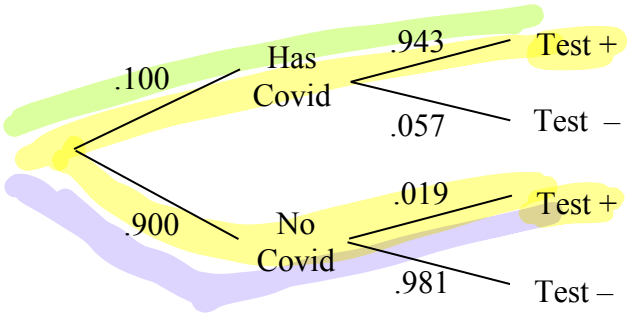
k	$\Pr\{Y = k\}$
-1	.4
3	.2
7	.4

$$\mu = (-1)(.4) + (3)(.2) + 7(.4) = 3$$

$$\sigma^2 = (-1-3)^2(.4) + (3-3)^2(.2) + (7-3)^2(.4) = 12.8$$

$$\sigma = \sqrt{12.8} \approx 3.58$$

20 points 4. For the iHealth Covid-19 Antigen Rapid Test, a positive result is accurate 94.3% of the time, and a negative test is accurate 98.1% of the time. Suppose that approximately 10% of the population currently has Covid.



Find the four missing values in the table below. **Show all pertinent work below the table.**

	Results of test: No Test	Positive	Negative
Probability person has Covid	.100	.8465	.0064
Probability person does not have Covid	.900	.1535	.9936

For example, this value is $\Pr\{No\ Covid \mid Test-\}$

Show all pertinent work below.

$$\Pr\{c \mid +\} = \frac{(.100)(.943)}{(.100)(.943)+(.900)(.019)} = \frac{.0943}{.1114} = .8465$$

$$\Pr\{no\ c \mid +\} = \frac{(.900)(.019)}{(.100)(.943)+(.900)(.019)} = \frac{.0171}{.1114} = .1535$$

$$\Pr\{c \mid -\} = \frac{(.100)(.057)}{(.100)(.057)+(.900)(.981)} = \frac{.0057}{.8886} = .0064$$

$$\Pr\{no\ c \mid -\} = \frac{(.900)(.981)}{(.100)(.057)+(.900)(.981)} = \frac{.8829}{.8886} = .9936$$

Notice sum of each column is 1.

13 points **5.** Suppose that approximately 10% of the population has Covid. You take a sample of 5 persons. Let Y denote the number of persons in the sample with Covid. Find each of the following.

/3 (a) $\Pr\{Y = 3\} =$ ${}_5C_3 (.10)^3(.90)^2 = .00810$

/7 (b) $\Pr\{Y > 3\} =$ ${}_5C_4 (.10)^4(.90)^1 + {}_5C_5(.10)^5(.90)^0$

$= .00045 + .00001$

$= .00046$

/3 (c) $\Pr\{Y < 3\} =$ ${}_5C_0 (.10)^0(.90)^5 + {}_5C_1(.10)^1(.90)^4 + {}_5C_2(.10)^2(.90)^3$

$= \ldots = .99144$

OR $= 1 - [.00810 + .00046] =$

8 points **6.** We are interested in hair color vs. eye color.

Hair color

Eye Color	Brown	Black	Red	Total
Brown	400	300	200	900
Blue	800	600	400	1,800
Total	1,200	900	600	2,700

/2 (a) Find $Pr\{Brown\ Eyes\}$.

$\frac{900}{2700} = \frac{1}{3}$

/3 (b) Find $\Pr\{Brown\ Eyes | Red\ Hair\}$.

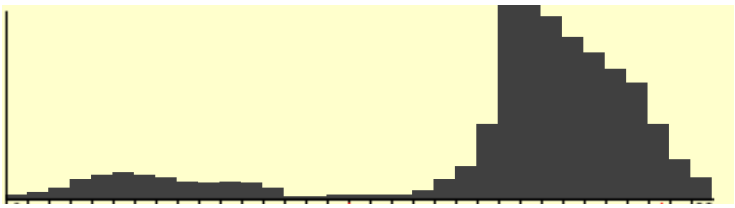
$\frac{200}{600} = \frac{1}{3}$

/3 (c) Are *Brown Eyes* and *Red Hair* independent traits or not? Explain/show work.

Yes.

$\Pr\{\text{Brown Eyes} | \text{Red Hair}\} = \Pr\{\text{Brown Eyes}\}$.

9 points **7**. The population distribution at right has a mean of 23 and a standard deviation of 7.



The sampling distributions using

$$n = 2,\ n = 10 \text{ and } n = 25$$

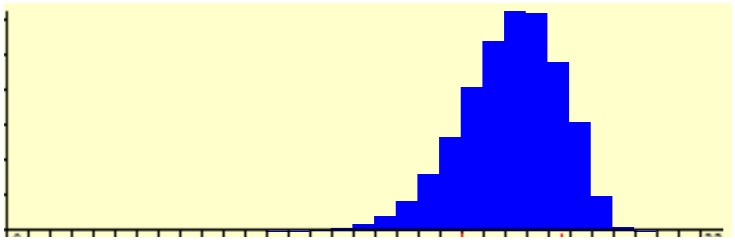
are shown below right (<u>not</u> in that order).

Determine the sample size in each case, and compute the sample mean and standard deviation for each sampling distribution.

$n =$ 10

$mean =$ 23

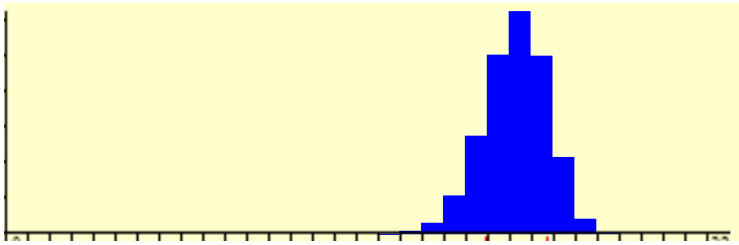
$SD =$ $7/\sqrt{10}$



$n =$ 25

$mean =$ 23

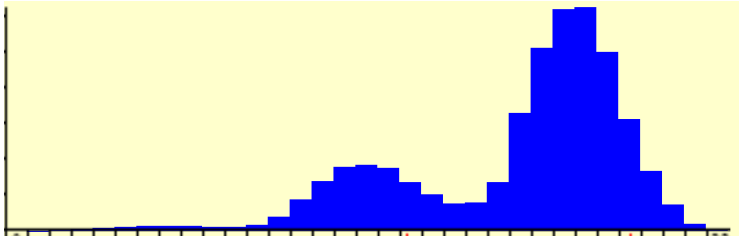
$SD =$ $7/\sqrt{25}$



$n =$ 2

$mean =$ 23

$SD =$ $7/\sqrt{2}$



5 points **8**. <u>Estimate</u> the mean and standard deviation of the data shown in the histogram at right.

$\mu \approx 1100$

$\sigma \approx$ a little larger than 50

so that $\approx \frac{2}{3}$ of data are between $\mu - \sigma$ and $\mu + \sigma$