1. **a.** $\frac{9}{17} \approx 0.5294$

b. $\frac{8}{17} \approx 0.4706$

c. $\Pr(\{3, 6, 9, 12, 15\}) = \frac{5}{17} \approx 0.2941$

d. $\Pr(\{1, 3, 5, 6, 7, 9, 11, 12, 13, 15, 17\}) = \frac{11}{17} \approx 0.6471$

2. a. $\frac{10}{100} = \frac{1}{10} = 0.1$

b. $\frac{50}{100} = \frac{1}{2} = 0.5$

c. $0.1 + 0.5 = 0.6$

3. **a.** $\frac{C(5, 2)}{C(11, 2)} = \frac{10}{55} = \frac{2}{11} \approx 0.1818$

b. $1 - \frac{C(5, 2)}{C(11, 2)} = 1 - \frac{10}{55} = \frac{9}{11} \approx 0.8182$

4. **a.** $\frac{C(7,3)}{C(12,3)} = \frac{35}{220} = \frac{7}{44} \approx 0.1591$

b. $1 - \frac{C(7,3)}{C(12,3)} = 1 - \frac{35}{220} = \frac{37}{44} \approx 0.8409$

5. **a.**
$$\frac{C(6,4) + C(7,4)}{C(13,4)} = \frac{15 + 35}{715} = \frac{50}{715} = \frac{10}{143} \approx 0.0699$$

b.
$$\frac{C(6,4) + C(6,3)C(7,1)}{C(13,4)} = \frac{15 + 20 \cdot 7}{715} = \frac{15 + 140}{715} = \frac{155}{715} = \frac{31}{143} \approx 0.2168$$

6. a. $\dfrac{C(8,3)+C(6,3)}{C(14,3)}=\dfrac{56+20}{364}$

$$=\frac{76}{364}=\frac{19}{91}\approx 0.2088$$

b. $\dfrac{C(6,3)+C(6,2)C(8,1)}{C(14,3)}=\dfrac{20+15\cdot 8}{364}$

$$=\frac{20+120}{364}$$

$$=\frac{140}{364}$$

$$=\frac{5}{13}\approx 0.3846$$

7. $1-\dfrac{C(5,\ 3)}{C(7,\ 3)}=1-\dfrac{10}{35}$

$$=\frac{5}{7}\approx 0.7143$$

8. $\dfrac{C(10,\ 4)\times C(5,\ 2)}{C(15,\ 6)}=\dfrac{60}{143}\approx 0.4196$

9. $1-\dfrac{C(9,\ 3)}{C(13,\ 3)}=1-\dfrac{84}{286}=\dfrac{101}{143}\approx 0.7063$

10. The total number of ways to select 5 senators is

$C(100,\ 5)=\dfrac{100!}{95!5!}=75{,}287{,}520.$

The total number of ways to select senators from different states is

$\dfrac{100\cdot 98\cdot 96\cdot 94\cdot 92}{5!}=67{,}800{,}320.$

Pr(no two members from same state)

$=\dfrac{67{,}800{,}320}{75{,}287{,}520}$

≈ 0.9006

11. $1-\dfrac{C(4,\ 3)}{C(10,\ 3)}=1-\dfrac{4}{120}=\dfrac{29}{30}\approx 0.9667$

12. $1-\dfrac{C(8,3)}{C(9,3)}=1-\dfrac{2}{3}=\dfrac{1}{3}\approx 0.3333$

13. $\dfrac{C(10,\ 7)}{C(22,\ 7)}=\dfrac{5}{7106}\approx 0.0007$

14. $\dfrac{12\cdot 11\cdot 10}{22\cdot 21\cdot 20}=\dfrac{1}{7}\approx 0.1429$

15. Ways for no girls to be chosen: $C(12,\ 7)$

Ways for exactly 1 girl to be chosen:

$C(12,\ 6)\times C(10,\ 1)$

$1-\dfrac{C(12,\ 7)+C(12,\ 6)\times C(10,\ 1)}{C(22,\ 7)}$

$=1-\dfrac{792+924\times 10}{170{,}544}=\dfrac{16}{17}\approx 0.9412$

16. Ways for no boy to be chosen: $C(10,7)=120$

Ways for exactly 1 boy to be chosen:

$C(10,6)C(12,1)=(210)(12)=2520$

Ways for exactly 2 boys to be chosen:

$C(10,5)C(12,2)=(252)(66)=16{,}632$

$1-\dfrac{120+2520+16632}{C(22,\ 7)}$

$=1-\dfrac{19272}{170{,}544}$

$=\dfrac{151{,}272}{170{,}544}$

$=\dfrac{573}{646}\approx 0.8870$

17. $1-\dfrac{7\cdot 6\cdot 5}{7\cdot 7\cdot 7}=1-\dfrac{210}{343}=\dfrac{133}{343}\approx 0.3878$

18. $1-\dfrac{12\cdot 11\cdot 10\cdot 9}{12\cdot 12\cdot 12\cdot 12}=1-\dfrac{11880}{20736}$

$$=\frac{8856}{20736}$$

$$=\frac{41}{96}\approx 0.4271$$

19. $1-\dfrac{30\times 29\times 28\times 27}{30^4}=\dfrac{47}{250}=0.188$

20. $1-\dfrac{16\times 15\times 14\times 13\times 12}{16^5}=\dfrac{4097}{8192}\approx 0.5001$

21. $1-\dfrac{P(20,8)}{20^8}\approx 0.8016$

22. $1-\dfrac{P(100,10)}{100^{10}}\approx 0.3718$

23. Pr(at least one birthday on June 13)

$$=1-\left(\frac{364}{365}\right)^{25} \approx 0.06629$$

Because in Table 1 no particular date is being matched. Any two (or more) identical birthdays count as a success.

24. Pr(at least one birthday on October 23)

$$=1-\left(\frac{364}{365}\right)^{100} \approx 0.2399$$

Johnny Carson's reasoning was wrong because he was looking for a particular date, the theory is true when looking for two people with the same date.

25. $\frac{6\cdot 5}{6^2}=\frac{30}{36}=\frac{5}{6}\approx 0.8333$

26. $\frac{6\cdot 5\cdot 4}{6\cdot 6\cdot 6}=\frac{120}{216}=\frac{5}{9}\approx 0.5556$

27. $\frac{3^4}{6^4}=\frac{81}{1296}=\frac{1}{16}\approx 0.0625$

28. $\frac{5^3}{6^3}=\frac{125}{216}\approx 0.5787$

29. $\frac{C(10, 4)}{2^{10}}=\frac{210}{1024}=\frac{105}{512}\approx 0.2051$

30. $\frac{C(7, 5)}{2^7}=\frac{21}{128}\approx 0.1641$

31. $1-\frac{7\times 6\times 5\times 4}{7^4}=\frac{223}{343}\approx 0.6501$

32. $\frac{5!}{5^5}=\frac{120}{3125}=\frac{24}{625}\approx .0384$

33. The tourist must travel 8 blocks of which 3 are south. Thus he has $C(8, 3) = 56$ ways to get to B from A.

a. To get from A to B through C there are $C(3, 1)\cdot C(5, 2) = 30$ ways.

The probability is $\frac{30}{56}=\frac{15}{28}\approx 0.5357$.

b. To get from A to B through D there are $C(5, 1)\cdot C(3, 2) = 15$ ways.

The probability is $\frac{15}{56}\approx 0.2679$.

c. To get from A to B through C and D there are $C(3, 1)\cdot C(2, 0)\cdot C(3, 2) = 9$ ways.

The probability is $\frac{9}{56}\approx 0.1607$.

d. The number of ways to get from A to B through C or D is $30 + 15 - 9 = 36$.

The probability is $\frac{36}{56}=\frac{9}{14}\approx 0.6429$.

34. The tourist must travel 10 blocks of which 4 are south. Thus he has $C(10, 4) = 210$ ways to get to B from A.

a. To get from A to B through C there are $C(5, 2)\cdot C(5, 2) = 100$ ways.

The probability is $\frac{100}{210}=\frac{10}{21}\approx 0.4762$.

b. To get from A to B through D there are $C(7, 3)\cdot C(3, 1) = 105$ ways.

The probability is $\frac{105}{210}=\frac{1}{2}=0.5$.

c. To get from A to B through C and D there are $C(5, 2)\cdot 2\cdot C(3, 1) = 60$ ways.

The probability is $\frac{60}{210}=\frac{2}{7}\approx 0.2857$.

d. The number of ways to get from A to B through C or D is $100 + 105 - 60 = 145$.

The probability is $\frac{145}{210}=\frac{29}{42}\approx 0.6905$.

35.
$$1-\frac{4\cdot 4\cdot 4}{5\cdot 5\cdot 5}=1-\frac{64}{125}=\frac{61}{125}\approx 0.488$$

36.
$$1-\frac{3\cdot 3\cdot 3}{4\cdot 4\cdot 4}=1-\frac{27}{64}=\frac{37}{64}\approx 0.5781$$

37.
$$1-\frac{12\cdot 11\cdot 10}{15\cdot 14\cdot 13}=1-\frac{1320}{2730}=\frac{1410}{2730}\approx 0.5165$$

His chances are increased.

38. $$1-\frac{9\cdot 8\cdot 7}{12\cdot 11\cdot 10}=1-\frac{504}{1320}=\frac{816}{1320}\approx 0.6182$$

His chances are increased.

39. $\frac{5\cdot 4\cdot 3}{5^3}=\frac{60}{125}=\frac{12}{25}=0.48$

40. $\frac{1\cdot 1}{7\cdot 6}=\frac{1}{42}\approx 0.0238$

41. There are 5! = 120 ways to arrange the family. First determine where the parents will stand. There are two ways with the man at one end. If the man doesn't stand at one end, there are $3\cdot 2=6$ ways for the couple to stand together. Next determine the order of the children. There are 3! = 6 ways to order the children. Thus there are $(2+6)\cdot 6=48$ ways to stand with the parents together.

The probability is $\frac{48}{120}=\frac{2}{5}=0.4$.

42. There are 5! or 120 ways to arrange 5 letters. In 36 of these, the 3 *E*'s will be adjacent.

Pr(all *E*'s adjacent) = $\frac{36}{120}=\frac{3}{10}=0.3$

43. There are $13\cdot 12=156$ ways to choose the two denominations, $C(4, 3)=4$ ways to choose the suits for the three of a kind and $C(4, 2)=6$ ways to choose the suits for the pair; thus there are $156\cdot 4\cdot 6=3744$ possible full house hands, so the probability is $\frac{3744}{C(52,\ 5)}\approx 0.0014$.

44. There are 13 choices for the denomination and $C(4, 3)=4$ choices for the suits for the three-of-a-kind; there are $C(12, 2)=66$ choices for the denominations and $4\cdot 4=16$ choices for the suits of the remaining two cards, for a total of $13\cdot 4\cdot 66\cdot 16=54{,}912$ possible three-of-a-kind poker hands; so the probability is $\frac{54{,}912}{C(52,\ 5)}\approx 0.0211$.

45. There are $C(13, 2)=78$ choices for the denominations of the two pairs, and $C(4, 2)=6$ choices each for their suits; the remaining card has $52-8=44$ choices, for a total of $78\cdot 6^2\cdot 44=123{,}552$ possible two pair hands, so the probability is $\frac{123{,}552}{C(52,\ 5)}\approx 0.0475$.

46. There are $13\cdot C(4, 2)=78$ pairs (13 choices for denomination, $C(4, 2)=6$ choices for suits); $C(12, 3)=220$ choices for the denominations of the remaining cards, and 4 choices each for their suits, which gives a total of $78\cdot 220\cdot 4^3=1{,}098{,}240$ one-pair hands, and so the probability is $\frac{1{,}098{,}240}{C(52,\ 5)}\approx 0.4226$.

47. **a.** There are 4 ways to select the suit of the 4 card group, $C(13, 4)$ ways to select their denominations, and $C(13, 3)^3$ ways to select 3 cards each from the remaining 3 suits. The probability of a 4-3-3-3 bridge hand is $\frac{4\cdot C(13,\ 4)\cdot C(13,\ 3)^3}{C(52,\ 13)}\approx 0.1054$.

b. There are $C(4, 2)=6$ ways to choose the suits of the 4-card groups and 2 ways to choose the suit of the 3-card group (then the suit of the 2-card group is uniquely determined). The denominations can then be chosen in $C(13,\ 4)^2\cdot C(13,\ 3)\cdot C(13,\ 2)$ ways, so the probability of a 4-4-3-2 bridge hand is $\frac{6\cdot 2\cdot C(13,\ 4)^2\cdot C(13,\ 3)\cdot C(13,\ 2)}{C(52,\ 13)}\approx 0.2155$.

48. **a.** $\frac{1}{C(69,5)\cdot 26}=\frac{1}{292{,}201{,}338}$

b. 1 to $292{,}201{,}337$

49. $\frac{2}{C(40,\ 6)}=\frac{1}{1{,}919{,}190}=0.0000005211$

50. Many people think multiples of 7 are lucky, and some might also pick 13 just to prove they're not superstitious! To avoid sharing, avoid "lucky" numbers.

51. $$\frac{C(5,3)\cdot C(34,2)}{C(39,\ 5)}=\frac{10\cdot 561}{575{,}757}=\frac{5610}{575{,}757}\approx 0.0097$$

52. $$\frac{C(47,6)}{C(53,\ 6)}=\frac{10{,}737{,}573}{22{,}957{,}480}\approx 0.4677$$