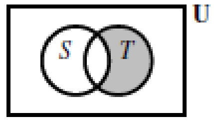
1. $\varnothing, \{a\}, \{b\}, \{a, b\}$

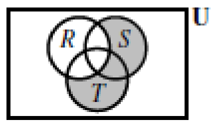
2. $(S \cup T')' = S' \cap T$



3. $C(16, 2) = \dfrac{16!}{2!14!} = 120$ possibilities

4. $2 \cdot 5! = 240$ ways

5.



6. $\binom{12}{0}x^{12} + \binom{12}{1}x^{11}(-2y) + \binom{12}{2}x^{10}(-2y)^2$

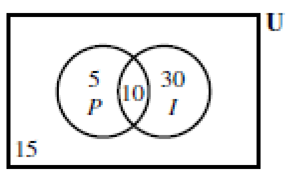
$= x^{12} - 24x^{11}y + 264x^{10}y^2$

7. $C(8, 3) \cdot C(6, 2) = \dfrac{8!}{3!5!} \cdot \dfrac{6!}{2!4!} = 56 \cdot 15 = 840$

8. Let U = {people given pills}, P = {people who received placebos}, I = {people who showed improvement}.

$n(U) = 60$; $n(P) = 15$; $n(I) = 40$; $n(P' \cap I) = 30$

Draw and complete a Venn diagram as shown.



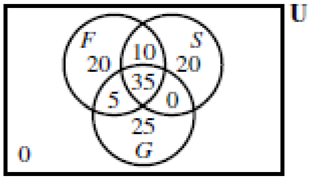
$n(P' \cap I') = 15$

Fifteen of the people who received the drug showed no improvement.

9. $7 \cdot 5 = 35$ combinations

10. $\binom{12}{2,\ 4,\ 6} = \frac{12!}{2!4!6!} = 13{,}860$

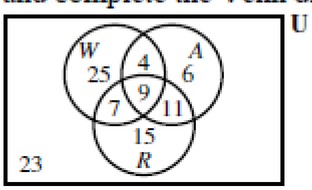
11. Let U = {applicants}, F = {applicants who speak French}, S = {applicants who speak Spanish}, and G = {applicants who speak German}.
$n(U) = 115$; $n(F) = 70$; $n(S) = 65$; $n(G) = 65$; $n(F \cap S) = 45$; $n(S \cap G) = 35$; $n(F \cap G) = 40$;
$n(F \cap S \cap G) = 35$. Draw and complete a Venn diagram.



$n((F \cup S \cup G)') = 0$
None of the people speak none of the three languages.

12. $\binom{17}{15} = \binom{17}{2} = \frac{17 \cdot 16}{2 \cdot 1} = 136$

For Exercises 13–20, let U = {members of the Earth Club}, W = {members who thought the priority is clean water}, A = {members who thought the priority is clean air}, R = {members who thought the priority is recycling}. Then $n(U) = 100$, $n(W) = 45$, $n(A) = 30$, $n(R) = 42$, $n(W \cap A) = 13$, $n(A \cap R) = 20$, $n(W \cap R) = 16$, $n(W \cap A \cap R) = 9$. Draw and complete the Venn diagram as follows.



13. $n(A \cap W' \cap R') = 6$

14. $n((W \cap A') \cup (W' \cap A)) = (25 + 7) + (6 + 11) = 32 + 17 = 49$

15. $n((W \cup R) \cap A') = 25 + 15 + 7 = 47$

16. $n(A \cap R \cap W') = 11$

17. $n((W \cap A' \cap R') \cup (W' \cap A \cap R') \cup (W' \cap A' \cap R)) = 25 + 6 + 15 = 46$

18. $n(R') = 23 + 25 + 6 + 4 = 58$

19. $n(R \cap A') = 15 + 7 = 22$

20. $n((W \cup A \cup R)') = 23$

21. $C(9, 4) = C(9, 5) = 126$

22. $2^{20} = 1{,}048{,}576$ ways

23. Let $S = \{\text{students who ski}\}$, $H = \{\text{students who play ice hockey}\}$. Then

$$n(S \cup H) = n(S) + n(H) - n(S \cap H)$$
$$= 400 + 300 - 150$$
$$= 550.$$

24. $6 \cdot 10 \cdot 8 = 480$ meals

25. $\binom{5}{1,\ 3,\ 1} = \frac{5!}{1!3!1!} = 20$ ways

26. The first digit can be anything but 0, the hundreds digit must be 3, and the last digit must be even.

$9 \cdot 1 \cdot 5 \cdot 10^4 = 450,000$

27. $9^2 \cdot 10^8 = 8,100,000,000$

28. $P(7,3) = 7 \cdot 6 \cdot 5 = 210$ ways

29. Strings of length 8 formed from the symbols *a*, *b*, *c*, *d*, *e*:

$5^8 = 390,625$ strings

Strings of length 8 formed from the symbols *a*, *b*, *c*, *d*:

$4^8 = 65,536$ strings

Strings with at least one *e*:

$390,625 - 65,536 = 325,089$ strings

30. $C(12, 5) = \frac{12!}{5!7!} = 792$

31. $C(30, 14) = \frac{30!}{14!16!}$

$= 145,422,675$ groups

32. $U = \{\text{households}\}$

$F = \{$households that get *Fancy Diet Magazine*$\}$

$C = \{$households that get *Clean Living Journal*$\}$

$$n(F \cup C) = n(F) + n(C) - n(F \cap C)$$
$$= 4000 + 10,000 - 1500$$
$$= 12,500$$

$$n((F \cup C)') = n(U) - n(F \cup C)$$
$$= 40,000 - 12,500$$
$$= 27,500$$

27,500 households get neither.

33. $3^{10} = 59,049$ paths

34. $C(60, 10) = \frac{60!}{10!50!} = 75,394,027,566$ ways

35. $5^{10} = 9,765,625$ tests

36. $21^6 = 85,766,121$ strings

37. $C(10, 4) = \frac{10!}{4!6!} = 210$ ways

38. $\frac{1}{3!} \cdot \frac{21!}{(7!)^3} = 66,512,160$ ways

39. $14! = 87,178,291,200$ ways

40. $\frac{1}{4!} \cdot \frac{20!}{(5!)^4} = 488,864,376$ ways

41. $3 \cdot 5 \cdot 4 = 60$ ways

42. $3 \cdot C(10, 2) = 135$

43. A diagonal corresponds to a pair of vertices, except that adjacent pairs must be excluded. Hence there are

$C(n, 2) - n = \frac{n(n-1)}{2} - n = \frac{n(n-3)}{2}$ diagonals.

44. $8 \cdot 6 = 48$

45. $5! \cdot 4! \cdot 3! \cdot 2! \cdot 1! = 120 \cdot 24 \cdot 6 \cdot 2 \cdot 1 = 34,560$

46. $P(12, 5) = 12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 = 95,040$

47. There are four suits. Once a suit is chosen, select 5 of the 13 cards.

Poker hands with cards of the same suit:

$4 \cdot C(13, 5) = 4 \cdot 1287 = 5148$ hands

48. $C(4, 3) \cdot C(48, 2) = 4 \cdot 1128 = 4512$ hands

49. Exactly the first two digits alike: $9 \cdot 1 \cdot 9 = 81$

First and last digit alike: $9 \cdot 9 \cdot 1 = 81$

Last two digits alike: $9 \cdot 9 \cdot 1 = 81$

Numbers with exactly two digits alike:

$81 + 81 + 81 = 243$

50. $9 \cdot 9 \cdot 8 = 648$ numbers

51. $3 \cdot 3 = 9$ pairings

52. $6 \cdot 3 \cdot 2 \cdot 2 \cdot 1 \cdot 1 = 72$ ways

53. $24 \cdot 23 + 24 \cdot 23 \cdot 22 = 552 + 12,144$
$= 12,696$ names

54. $3 \cdot 4! \cdot 2 = 3 \cdot 24 \cdot 2 = 144$ ways

55. Any two lines will intersect, so the number of intersections is $C(10, 2) = 45$.

56. First teacher: $\frac{1}{4!} \cdot \frac{24!}{(6!)^4} = 96,197,645,544$ ways

Second teacher:
$\frac{1}{6!} \cdot \frac{24!}{(4!)^6} = 4,509,264,634,875$ ways
The second teacher has more options.

57. $\binom{10}{3, 4, 3} = \frac{10!}{3!4!3!} = 4200$

58. Let n be the number of books.
$n! = 120$, so $n = 5$. There are five books.

59. $7 \cdot 6 \cdot 5 \cdot 1 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 1 = 5040$ orders

60. **a.** $C(12,2) \cdot C(12,3)$

b. Select five out of twelve states. Then determine the junior or senior senator from each state.
$C(12, 5) \cdot 2^5 = 25,344$

61. $C(7, 2) + C(7, 1) + C(7, 0) = 29$ ways

62. There are two choices for every row, so the number of paths from the A at the top to the final A is $2^6 = 64$ ways.

63. There are $2 \cdot 26^2$ 3-letter call letters and $2 \cdot 26^3$ 4-letter call letters, so $2 \cdot 26^2 + 2 \cdot 26^3 = 36,504$ call letters are possible.

64. Find n such that $n! = 479,001,600$.
Testing numbers on computer, $n = 12$.

65. $\binom{25}{10, 9, 6}$=16,360,143,800 ways

66. First choose the three letters. There are $C(26, 3)$ ways to do this. Then choose the three numbers. There are $C(10, 3)$ ways to do this. Then order the six. There are 6! ways to do this.
The number of license plates is
$C(26, 3) \cdot C(10, 3) \cdot 6! = 224,640,000$.

Conceptual Exercises

67. If $A \cap B = \varnothing$ then A and B have no elements in common.

68. If $n(A \cup B) = n(A) + n(B)$, then A and B are disjoint sets and have no elements in common.

69. The intersection of sets S and T will be the same as set T when all the elements of T are also in set S, in symbols, when $T \subseteq S$.

70. The union of sets S and T will be the same as set T when all the elements of S are also in set T, in symbols, when $S \subseteq T$.

71. True;
$$n(S \cup T) = n(S) + n(T) - n(S \cap T)$$
$$n(S \cup T) + n(S \cap T) = n(S) + n(T)$$

72. True; the empty set is a subset of every set.

73. $n(n-1)! = n(n-1)(n-2)(n-3)...1 = n!$
For example,
$5 \cdot 4! = 5(4 \cdot 3 \cdot 2 \cdot 1) = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 5!$.

74. Let $n = 1$. We have $1(1-1)! = 1(0)! = 1! = 1$ So, since 1 times any number is the number, we have $0! = 1! = 1$.

75. A permutation is an arrangement of items in which order is important. A combination is a subset of objects taken from a larger set in which the order of the objects does not make a difference.

76. The number of committees of size 6 is the same as the number of committees of size 4. For each committee of size 6, there is a committee consisting of the four people who were not chosen. For example, use letters A, B,C, D, E, F, G, H, I, J to represent the people. For the committee
{A, B, C, D, E, F} the complementary committee is {G, H, I, J}.

77. $C(10, 3) = C(10, 7)$. The number of subsets of size 3 taken from a 10 member set is the same as the number of subsets of size 7. For example,