21. $C(10,6)-C(7,4)=$
$210-35=175$ ways

22. $C(7,3)=35$ ways

23. **a.** $C(12,5)=792$ samples

b. $C(7,5)=21$ samples

c. $C(7,2)\cdot C(5,3)=21\cdot 10=210$ samples

d. $C(7,4)\cdot C(5,1)+21=$
$35\cdot 5+21=196$ samples

24. **a.** $C(15,6)=5005$ ways

b. $C(9,6)=84$ ways

c. $C(6,2)\cdot C(9,4)=15\cdot 126=1890$ ways

d. $5005-(84+C(9,5)\cdot C(6,1))$
$5005-(84+126\cdot 6)$
$5005-(84+756)$
$5005-840$
4165 ways

25. **a.** $C(10,3)=120$ ways

b. $C(8,3)=56$ ways

c. $120-56=64$

26. **a.** $C(100,5)=75,287,520$ ways

b $C(10,2)\cdot C(90,3)=$
$45\cdot 117,480=5,286,600$ ways

c. $75,287,520-C(90,5)$
$75,287,520-43,949,268$
$31,338,252$ ways

27. $C(4,2)\cdot C(6,2)=6\cdot 15=90$ ways

28. $C(9,5)\cdot C(7,6)=126\cdot 7=882$ ways

29. $C(4,3)\cdot C(4,2)=4\cdot 6=24$ ways

30. $C(4,2)\cdot 12C(4,2)\cdot 44=6\cdot 12\cdot 6\cdot 44$
$=19,008$ ways

31. $13C(4,3)\cdot 12C(4,2)=13\cdot 4\cdot 12\cdot 6$
$=3744$ ways

32. $C(13,2)\cdot C(4,2)\cdot C(4,2)\cdot 44=78\cdot 6\cdot 6\cdot 44$
$=123,552$ ways

33. $C(7,5)\cdot 5!\ \cdot 21\cdot 20=21\cdot 120\cdot 21\cdot 20$
$=1,058,400$ ways

34. $C(9,5)\cdot P(21,4)=126\cdot 143,640$
$=18,098,640$ ways

35. $C(10,5)\cdot P(21,5)=252\cdot 2,441,880$
$=615,353,760$ ways

36. $C(8,5)\cdot 5!\ \cdot P(21,3)=56\cdot 120\cdot 7980$
$=53,625,600$ ways

37. $6!\ \cdot 7\cdot 3!=720\cdot 7\cdot 6$
$=30,240$ ways

38. $4!\ \cdot 5\cdot 2!=24\cdot 5\cdot 2$
$=240$ ways

39. $C(9,5)=126$ ways

40. $C(10,4)=210$ ways

41. $C(26,22)\cdot C(10,7)=14,950\cdot 120$
$=1,794,000$ ways

42. $C(26,24)\cdot C(10,6)=325\cdot 210$
$=68,250$ ways

43. $C(12,6)=924$ ways

44. $C(8,5)=56$ ways

45. $\dfrac{C(100,50)}{2^{100}}\approx 0.07959=7.96\%$

46. $\dfrac{C(200,100)}{2^{200}}\approx 0.05634=5.63\%$

1. $\frac{5!}{3!1!1!}=20$

2. $\frac{5!}{2!1!2!}=30$

3. $\frac{6!}{2!1!2!1!}=180$

4. $\frac{6!}{3!3!}=20$

5. $\frac{7!}{3!2!2!}=210$

6. $\frac{7!}{4!1!2!}=105$

7. $\frac{12!}{4!4!4!}=34,650$

8. $\frac{8!}{3!3!2!}=560$

9. $\frac{12!}{5!3!2!2!}=166,320$

10. $\frac{8!}{2!2!2!2!}=2520$

11. $\frac{1}{5!}\cdot\frac{15!}{(3!)^5}=1,401,400$

12. $\frac{1}{2!}\cdot\frac{10!}{(5!)^2}=126$

13. $\frac{1}{3!}\cdot\frac{18!}{(6!)^3}=2,858,856$

14. $\frac{1}{3!}\cdot\frac{12!}{(4!)^3}=5775$

15. $\binom{20}{7,\ 5,\ 8}=\frac{20!}{7!5!8!}=99,768,240$ reports

16. $\binom{15}{5,\ 5,\ 5}=\frac{15!}{5!5!5!}=756,756$ ways

17. $\binom{8}{2,1,4,1}=\frac{8!}{2!1!4!1!}=840$ words

18. $\binom{10}{1,3,4,1,1}=\frac{10!}{1!3!4!1!1!}=25,200$ words

19. $\binom{9}{3,2,4}=\frac{9!}{3!2!4!}=1260$ ways

20. $\binom{6}{3,2,1}=\frac{6!}{3!2!1!}=60$ ways

21. $\frac{1}{5!}\cdot\frac{20!}{(4!)^5}=2,546,168,625$ ways

22. $\frac{1}{4!}\cdot\frac{20!}{(5!)^4}=488,864,376$ ways

23. $\binom{30}{10,\ 2,\ 18}=\frac{30!}{10!2!18!}$

$= 5,708,552,850$ ways

24. $\binom{9}{3,\ 3,\ 3}=\frac{9!}{3!3!3!}=1680$ ways

25. $\binom{4}{1,\ 1,\ 2}=\frac{4!}{1!1!2!}=12$ ways

26. $\binom{10}{4,\ 4,\ 2}=\frac{10!}{4!4!2!}=3150$ ways

27. $\frac{1}{7!}\cdot\frac{14!}{(2!)^7}=135,135$ ways

28. First partition the six days traveling and four days at the office. There are $\binom{10}{6,\ 4} = 210$ ways to do this without any restriction. Next partition the six travel days among the three cities. There are $\binom{6}{2,\ 2,\ 2} = \frac{6!}{2!2!2!} = 90$ ways to do this.
Thus there are 210 · 90 = 18,900 ways for her to schedule her travel.

29. The number of ways ten students are to be divided into two five member teams for a basketball game is $\frac{10!}{(2)!(5!)^2} = 126.$

30. $\binom{12}{6,\ 4,\ 2} = \frac{12!}{6!4!2!} = 13{,}860$ ways

31. $\binom{n}{1, 1, \cdots, 1} = \frac{n!}{1!1!\cdots 1!} = n!$

32. Select the number of elements of S_1:

$$p_1 = \binom{n}{n_1} = \frac{n!}{n_1!(n-n_1)!} \text{ ways}$$

Select the number of elements of S_2:

$$p_2 = \binom{n-n_1}{n_2} = \frac{(n-n_1)!}{n_2!(n-n_1-n_2)!} \text{ ways}$$

Select the number of elements of $S_k (1 < k < m)$:

$$p_k = \binom{n-n_1-n_2-\cdots-n_{k-1}}{n_k} = \frac{(n-n_1-n_2-\cdots-n_{k-1})!}{n_k!(n-n_1-n_2-\cdots-n_{k-1}-n_k)!} \text{ ways}$$

Select the number of elements of S_m:

$$p_m = \binom{n-n_1-n_2-\cdots-n_{m-1}}{n_m} = \binom{n_m}{n_m} = \frac{n_m!}{n_m!} \text{ way}$$

Observe that for $1 < k < m$, the numerator of P_k is $(n-n_1-\cdots-n_{k-1})!$ which also occurs in the denominator of p_{k-1}, which is $n_{k-1}!(n-n_1-\cdots-n_{k-1})!$. Using the generalized multiplication principle, the number of ordered partitions is $p_1 \cdot p_2 \cdots p_m$. All of the numerators cancel in this multiplication for $1 < k \le m$, leaving $n_1! \cdot n_2! \cdots n_m!$ in the denominator. Thus,

$$p_1 \cdot p_2 \cdots p_m = \frac{n!}{n_1! \cdot n_2! \cdot n_3! \cdots n_m!}.$$

33. $\binom{38}{10,\ 12,\ 10,\ 6}$
$= \frac{38!}{10!12!10!6!}$
= 115,166,175,166,136,334,240 ways

34. $\frac{65!}{(13!)^5} \approx 8.81 \times 10^{41}$ ways

35. $\binom{52}{13, 13, 13, 13} = \frac{52!}{13!13!13!13!}$
$\approx 5.4 \times 10^{28}$

Therefore, there are more than one octillion.