Homework 26 Solutions Math 141

1. $f(x, y) = x^2 - 3xy - y^2$
$f(5,0) = 5^2 - 3(5)(0) - 0^2 = 25$
$f(5,-2) = 5^2 - 3(5)(-2) - (-2)^2 = 51$
$f(a, b) = a^2 - 3ab - b^2$

2. $g(x, y) = \sqrt{x^2 + 2y^2}$
$g(1, 1) = \sqrt{1^2 + 2(1^2)} = \sqrt{3}$
$g(0, -1) = \sqrt{0^2 + 2(-1^2)} = \sqrt{2}$
$g(a, b) = \sqrt{a^2 + 2b^2}$

3. $g(x, y, z) = \dfrac{x}{y - z}$
$g(2,3,4) = \dfrac{2}{3-4} = -2$
$g(7, 46, 44) = \dfrac{7}{46-44} = \dfrac{7}{2}$

4. $f(x, y, z) = x^2 e^{\sqrt{y^2+z^2}}$
$f(1,-1,1) = (1^2)e^{\sqrt{(-1)^2+1^2}} = e^{\sqrt{2}}$
$f(2, 3, -4) = (2^2)e^{\sqrt{3^2+(-4)^2}} = 4e^5$

5. $f(x,y) = xy \Rightarrow$
$f(2+h,3) = (2+h)3 = 6+3h$
$f(2,3) = (2)3 = 6$
$f(2+h,3) - f(2,3) = (6+3h) - 6 = 3h$

6. $f(x,y) = xy \Rightarrow$
$f(2,3+k) = 2(3+k) = 6+2k$
$f(2,3) = (2)3 = 6$
$f(2+h,3) - f(2,3) = (6+2k) - 6 = 2k$

7. $C(x, y, z)$ is the cost of materials for the rectangular box with dimensions x, y, z in feet. The area of the top and the bottom together is $2xy$, so the cost is $3(2xy) = 6xy$. The area of the front and back together is $2xz$, so the cost is $5(2xz) = 10xz$. The area of the right and left side together is $2yz$, so the cost is $5(2yz) = 10yz$.
Thus, $C(x, y, z) = 6xy + 10xz + 10yz$.

8. $C(x, y, z)$ is the cost of material.
Using the same reasoning as in exercise 7, we have $C(x, y, z) = 3xy + 5xz + 10yz$.

9. $f(x, y) = 20x^{1/3}y^{2/3}$
$f(8,1) = 20(8^{1/3})(1^{2/3}) = 40$
$f(1,27) = 20(1^{1/3})(27^{2/3}) = 180$
$f(8,27) = 20(8^{1/3})(27^{2/3}) = 360$
$f(8k, 27k)$
$= 20(8k)^{1/3}(27k)^{2/3}$
$= 20(8^{1/3})(k^{1/3})(27^{2/3})(k^{2/3})$
$= k(20)(8^{1/3})(27^{2/3}) = kf(8, 27)$

10. $f(x, y) = 10x^{2/5}y^{3/5}$
$f(3a, 3b) = 10(3a)^{2/5}(3b)^{3/5}$
$= 10(3^{2/5})(a^{2/5})(3^{3/5})(b^{3/5})$
$= 3(10)(a^{2/5})(b^{3/5})$
$= 3f(a,b)$

11. $P(A, t) = Ae^{-0.05t}$
$P(100, 13.8) = 100e^{-0.05(13.8)} = 100e^{-0.69}$
≈ 50.16
\$50 invested at 5% continuously compounded interest will yield \$100 in 13.8 years.

12. $C(x, y)$ is the cost of utilizing x units of labor and y units of capital. $C(x, y) = 100x + 200y$

13. $T = f(r,v,x) = \dfrac{r}{100}(0.40v - x)$

a. $v = 200{,}000$, $x = 5000$, $r = 2.5$
$T = \dfrac{r(0.4v - x)}{100}$
$= \dfrac{2.5(0.4(200{,}000) - 5000)}{100}$
$T = \$1875$

b. If $v = 200{,}000$, $x = 5000$, $r = 3$:
$T = \dfrac{r(0.4v - x)}{100} = \dfrac{3(0.4(200{,}000) - 5000)}{100}$
$= \$2250$
The tax due also increases by 20% since $1875 + (0.2)(1875) = \$2250$.

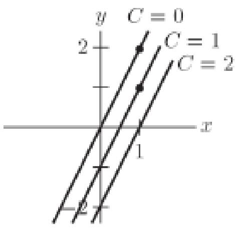
14. a. $v = 100{,}000$, $x = 5000$, $r = 2.2$
$T = \dfrac{r(0.4v - x)}{100}$
$= \dfrac{2.2(0.4(100{,}000) - 5000)}{100} = \$770$

b. If $v = 120{,}000$, $x = 5000$, $r = 2.2$:

$$T = \frac{r(0.4v - x)}{100} = \frac{2.2(0.4(120{,}000) - 5000)}{100} = \$946$$

20% of \$770 is \$154, so tax due does not increase by 20%.

15. $C = 2x - y$, so $y = 2x - C$
The level curves are $y = 2x$, $y = 2x - 1$, and $y = 2x - 2$.



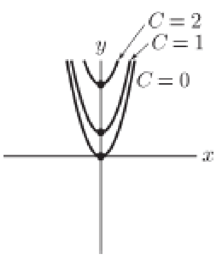
16. $C = -x^2 + 2y$, so $y = \frac{x^2}{2} + \frac{C}{2}$.

The level curves are

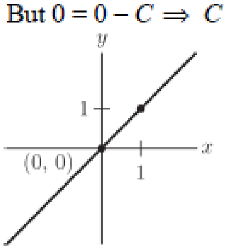
$$-x^2 + 2y = 0 \Rightarrow y = \frac{x^2}{2}$$

$$-x^2 + 2y = 1 \Rightarrow y = \frac{x^2}{2} + \frac{1}{2}$$

$$-x^2 + 2y = 2 \Rightarrow y = \frac{x^2}{2} + 1$$

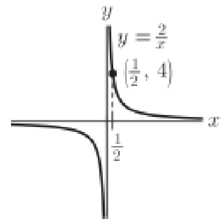


17. $C = x - y$, $y = x - C$
But $0 = 0 - C \Rightarrow C = 0$, so $y = x$.



18. $C = xy \Rightarrow y = \frac{C}{x}$

But $4 = \frac{C}{1/2} \Rightarrow C = 2$, thus $y = \frac{2}{x}$.



19. $y = 3x - 4 \Rightarrow y - 3x = -4$, so $y - 3x = C \Rightarrow$ $f(x, y) = y - 3x$.

20. $y = \frac{2}{x^2} \Rightarrow yx^2 = 2$

Thus, $yx^2 = C \Rightarrow f(x, y) = x^2 y$.

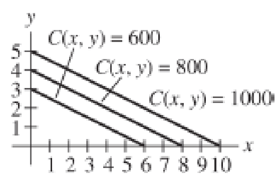
21. They correspond to the points having the same altitude above sea level.

22. $C(x, y) = 100x + 200y$ is the cost of using x units of labor and y units of capital.
If $C(x, y) = 600$, then $100x + 200y = 600 \Rightarrow$ $y = 3 - \frac{1}{2}x$.

If $C(x, y) = 800$, then $y = 4 - \frac{1}{2}x$.

If $C(x, y) = 1000$, then $y = 5 - \frac{1}{2}x$.

Points on the same level curve correspond to production amounts that have the same total cost.



23. (d) **24.** (b)

25. (c) **26.** (a)