2. $f(x) = x\ln x - x \quad \Rightarrow \quad f'(x) = x \cdot \dfrac{1}{x} + (\ln x) \cdot 1 - 1 = 1 + \ln x - 1 = \ln x$

5. $f(x) = \log_2(1 - 3x) \quad \Rightarrow \quad f'(x) = \dfrac{1}{(1-3x)\ln 2}\dfrac{d}{dx}(1-3x) = \dfrac{-3}{(1-3x)\ln 2}$ or $\dfrac{3}{(3x-1)\ln 2}$

7. $f(x) = \sqrt[5]{\ln x} = (\ln x)^{1/5} \quad \Rightarrow \quad f'(x) = \frac{1}{5}(\ln x)^{-4/5}\dfrac{d}{dx}(\ln x) = \dfrac{1}{5(\ln x)^{4/5}} \cdot \dfrac{1}{x} = \dfrac{1}{5x\sqrt[5]{(\ln x)^4}}$

8. $f(x) = \ln \sqrt[5]{x} = \ln x^{1/5} = \frac{1}{5}\ln x \quad \Rightarrow \quad f'(x) = \dfrac{1}{5} \cdot \dfrac{1}{x} = \dfrac{1}{5x}$

11. $F(t) = \ln \dfrac{(2t+1)^3}{(3t-1)^4} = \ln(2t+1)^3 - \ln(3t-1)^4 = 3\ln(2t+1) - 4\ln(3t-1) \quad \Rightarrow$

$F'(t) = 3 \cdot \dfrac{1}{2t+1} \cdot 2 - 4 \cdot \dfrac{1}{3t-1} \cdot 3 = \dfrac{6}{2t+1} - \dfrac{12}{3t-1}$, or combined, $\dfrac{-6(t+3)}{(2t+1)(3t-1)}$.

12. $h(x) = \ln\left(x + \sqrt{x^2-1}\right) \quad \Rightarrow \quad h'(x) = \dfrac{1}{x+\sqrt{x^2-1}}\left(1 + \dfrac{x}{\sqrt{x^2-1}}\right) = \dfrac{1}{x+\sqrt{x^2-1}} \cdot \dfrac{\sqrt{x^2-1}+x}{\sqrt{x^2-1}} = \dfrac{1}{\sqrt{x^2-1}}$

41. $f(x) = 12 + 4x - x^2$, $[0, 5]$. $f'(x) = 4 - 2x = 0 \quad \Leftrightarrow \quad x = 2$. $f(0) = 12$, $f(2) = 16$, and $f(5) = 7$.
So $f(2) = 16$ is the absolute maximum value and $f(5) = 7$ is the absolute minimum value.

45. $f(x) = x^4 - 2x^2 + 3$, $[-2, 3]$. $f'(x) = 4x^3 - 4x = 4x(x^2-1) = 4x(x+1)(x-1) = 0 \quad \Leftrightarrow \quad x = -1, 0, 1$.
$f(-2) = 11$, $f(-1) = 2$, $f(0) = 3$, $f(1) = 2$, $f(3) = 66$. So $f(3) = 66$ is the absolute maximum value and $f(\pm 1) = 2$ is the absolute minimum value.

47. $f(t) = t\sqrt{4-t^2}$, $[-1, 2]$.

$f'(t) = t \cdot \frac{1}{2}(4-t^2)^{-1/2}(-2t) + (4-t^2)^{1/2} \cdot 1 = \dfrac{-t^2}{\sqrt{4-t^2}} + \sqrt{4-t^2} = \dfrac{-t^2 + (4-t^2)}{\sqrt{4-t^2}} = \dfrac{4-2t^2}{\sqrt{4-t^2}}$.

$f'(t) = 0 \quad \Rightarrow \quad 4 - 2t^2 = 0 \quad \Rightarrow \quad t^2 = 2 \quad \Rightarrow \quad t = \pm\sqrt{2}$, but $t = -\sqrt{2}$ is not in the given interval, $[-1, 2]$.
$f'(t)$ does not exist if $4 - t^2 = 0 \quad \Rightarrow \quad t = \pm 2$, but -2 is not in the given interval. $f(-1) = -\sqrt{3}$, $f(\sqrt{2}) = 2$, and $f(2) = 0$. So $f(\sqrt{2}) = 2$ is the absolute maximum value and $f(-1) = -\sqrt{3}$ is the absolute minimum value.

48. $f(x) = \dfrac{x^2-4}{x^2+4}$, $[-4, 4]$. $f'(x) = \dfrac{(x^2+4)(2x) - (x^2-4)(2x)}{(x^2+4)^2} = \dfrac{16x}{(x^2+4)^2} = 0 \quad \Leftrightarrow \quad x = 0$. $f(\pm 4) = \frac{12}{20} = \frac{3}{5}$ and $f(0) = -1$. So $f(\pm 4) = \frac{3}{5}$ is the absolute maximum value and $f(0) = -1$ is the absolute minimum value.

50. $f(x) = x - \ln x$, $[\frac{1}{2}, 2]$. $f'(x) = 1 - \frac{1}{x} = \frac{x-1}{x}$. $f'(x) = 0 \quad \Rightarrow \quad x = 1$. [Note that 0 is not in the domain of f.]
$f(\frac{1}{2}) = \frac{1}{2} - \ln\frac{1}{2} \approx 1.19$, $f(1) = 1$, and $f(2) = 2 - \ln 2 \approx 1.31$. So $f(2) = 2 - \ln 2$ is the absolute maximum value and $f(1) = 1$ is the absolute minimum value.

51. $f(x) = \ln(x^2 + x + 1)$, $[-1, 1]$. $f'(x) = \dfrac{1}{x^2 + x + 1} \cdot (2x + 1) = 0 \quad \Leftrightarrow \quad x = -\frac{1}{2}$. Since $x^2 + x + 1 > 0$ for all x, the domain of f and f' is $\mathbb{R}$. $f(-1) = \ln 1 = 0$, $f\left(-\frac{1}{2}\right) = \ln \frac{3}{4} \approx -0.29$, and $f(1) = \ln 3 \approx 1.10$. So $f(1) = \ln 3 \approx 1.10$ is the absolute maximum value and $f\left(-\frac{1}{2}\right) = \ln \frac{3}{4} \approx -0.29$ is the absolute minimum value.