

## Homework 23 Solutions

$$18. f(x) = \frac{x^2 - 3x + 1}{x^2} = 1 - \frac{3}{x} + \frac{1}{x^2} = 1 - 3x^{-1} + x^{-2} \Rightarrow$$

$$f'(x) = 0 - 3(-1)x^{-2} + (-2)x^{-3} = 3x^{-2} - 2x^{-3} \quad \text{or} \quad \frac{3}{x^2} - \frac{2}{x^3} \quad \text{or} \quad \frac{3x - 2}{x^3}$$

$$19. y = \frac{x^2 + 4x + 3}{\sqrt{x}} = x^{3/2} + 4x^{1/2} + 3x^{-1/2} \Rightarrow$$

$$y' = \frac{3}{2}x^{1/2} + 4\left(\frac{1}{2}\right)x^{-1/2} + 3\left(-\frac{1}{2}\right)x^{-3/2} = \frac{3}{2}\sqrt{x} + \frac{2}{\sqrt{x}} - \frac{3}{2x\sqrt{x}} \quad \left[\text{note that } x^{3/2} = x^{2/2} \cdot x^{1/2} = x\sqrt{x}\right]$$

$$\text{The last expression can be written as } \frac{3x^2}{2x\sqrt{x}} + \frac{4x}{2x\sqrt{x}} - \frac{3}{2x\sqrt{x}} = \frac{3x^2 + 4x - 3}{2x\sqrt{x}}.$$

$$23. u = \sqrt[5]{t} + 4\sqrt{t^5} = t^{1/5} + 4t^{5/2} \Rightarrow u' = \frac{1}{5}t^{-4/5} + 4\left(\frac{5}{2}t^{3/2}\right) = \frac{1}{5}t^{-4/5} + 10t^{3/2} \quad \text{or} \quad 1/(5\sqrt[5]{t^4}) + 10\sqrt{t^3}$$

$$24. v = \left(\sqrt{x} + \frac{1}{\sqrt[3]{x}}\right)^2 = \left(\sqrt{x}\right)^2 + 2\sqrt{x} \cdot \frac{1}{\sqrt[3]{x}} + \left(\frac{1}{\sqrt[3]{x}}\right)^2 = x + 2x^{1/2-1/3} + 1/x^{2/3} = x + 2x^{1/6} + x^{-2/3} \Rightarrow$$

$$v' = 1 + 2\left(\frac{1}{6}x^{-5/6}\right) - \frac{2}{3}x^{-5/3} = 1 + \frac{1}{3}x^{-5/6} - \frac{2}{3}x^{-5/3} \quad \text{or} \quad 1 + \frac{1}{3\sqrt[6]{x^5}} - \frac{2}{3\sqrt[3]{x^5}}$$

$$45. (a) s = t^3 - 3t \Rightarrow v(t) = s'(t) = 3t^2 - 3 \Rightarrow a(t) = v'(t) = 6t$$

$$(b) a(2) = 6(2) = 12 \text{ m/s}^2$$

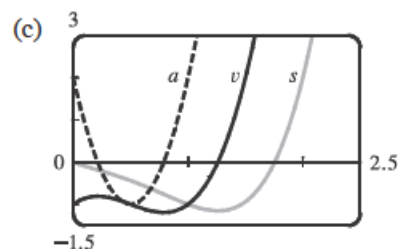
$$(c) v(t) = 3t^2 - 3 = 0 \text{ when } t^2 = 1, \text{ that is, } t = 1 \text{ and } a(1) = 6 \text{ m/s}^2.$$

$$46. (a) s = t^4 - 2t^3 + t^2 - t \Rightarrow$$

$$v(t) = s'(t) = 4t^3 - 6t^2 + 2t - 1 \Rightarrow$$

$$a(t) = v'(t) = 12t^2 - 12t + 2$$

$$(b) a(1) = 12(1)^2 - 12(1) + 2 = 2 \text{ m/s}^2$$



$$48. f(x) = x^3 - 4x^2 + 5x \Rightarrow f'(x) = 3x^2 - 8x + 5 \Rightarrow f''(x) = 6x - 8.$$

$$f''(x) > 0 \Rightarrow 6x - 8 > 0 \Rightarrow x > \frac{4}{3}. f \text{ is concave upward when } f''(x) > 0; \text{ that is, on } \left(\frac{4}{3}, \infty\right).$$

$$5. \text{ By the Quotient Rule, } y = \frac{e^x}{x^2} \Rightarrow y' = \frac{x^2 \frac{d}{dx}(e^x) - e^x \frac{d}{dx}(x^2)}{(x^2)^2} = \frac{x^2(e^x) - e^x(2x)}{x^4} = \frac{xe^x(x-2)}{x^4} = \frac{e^x(x-2)}{x^3}.$$

$$6. \text{ By the Quotient Rule, } y = \frac{e^x}{1+x} \Rightarrow y' = \frac{(1+x)e^x - e^x(1)}{(1+x)^2} = \frac{e^x + xe^x - e^x}{(x+1)^2} = \frac{xe^x}{(x+1)^2}.$$

$$15. y = (r^2 - 2r)e^r \stackrel{\text{PR}}{\Rightarrow} y' = (r^2 - 2r)(e^r) + e^r(2r - 2) = e^r(r^2 - 2r + 2r - 2) = e^r(r^2 - 2)$$

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## Homework 8 Solutions

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$$16. y = \frac{1}{s + ke^s} \stackrel{\text{QR}}{\Rightarrow} y' = \frac{(s + ke^s)(0) - (1)(1 + ke^s)}{(s + ke^s)^2} = -\frac{1 + ke^s}{(s + ke^s)^2}$$

$$23. f(x) = \frac{x}{x + c/x} \Rightarrow f'(x) = \frac{(x + c/x)(1) - x(1 - c/x^2)}{\left(x + \frac{c}{x}\right)^2} = \frac{x + c/x - x + c/x}{\left(\frac{x^2 + c}{x}\right)^2} = \frac{2c/x}{\frac{(x^2 + c)^2}{x^2}} \cdot \frac{x^2}{x^2} = \frac{2cx}{(x^2 + c)^2}$$

$$24. f(x) = \frac{ax + b}{cx + d} \Rightarrow f'(x) = \frac{(cx + d)(a) - (ax + b)(c)}{(cx + d)^2} = \frac{acx + ad - acx - bc}{(cx + d)^2} = \frac{ad - bc}{(cx + d)^2}$$

$$8. F(x) = (4x - x^2)^{100} \Rightarrow F'(x) = 100(4x - x^2)^{99} \cdot \frac{d}{dx}(4x - x^2) = 100(4x - x^2)^{99}(4 - 2x)$$

[or  $200x^{99}(x - 2)(x - 4)^{99}$ ]

$$9. F(x) = \sqrt{1 - 2x} = (1 - 2x)^{1/2} \Rightarrow F'(x) = \frac{1}{2}(1 - 2x)^{-1/2}(-2) = -\frac{1}{\sqrt{1 - 2x}}$$

$$19. y = (2x - 5)^4(8x^2 - 5)^{-3} \Rightarrow$$
$$y' = 4(2x - 5)^3(2)(8x^2 - 5)^{-3} + (2x - 5)^4(-3)(8x^2 - 5)^{-4}(16x)$$
$$= 8(2x - 5)^3(8x^2 - 5)^{-3} - 48x(2x - 5)^4(8x^2 - 5)^{-4}$$

[This simplifies to  $8(2x - 5)^3(8x^2 - 5)^{-4}(-4x^2 + 30x - 5)$ .]

$$20. h(t) = (t^4 - 1)^3(t^3 + 1)^4 \Rightarrow$$
$$h'(t) = (t^4 - 1)^3 \cdot 4(t^3 + 1)^3(3t^2) + (t^3 + 1)^4 \cdot 3(t^4 - 1)^2(4t^3)$$
$$= 12t^2(t^4 - 1)^2(t^3 + 1)^3 [(t^4 - 1) + t(t^3 + 1)] = 12t^2(t^4 - 1)^2(t^3 + 1)^3(2t^4 + t - 1)$$

$$22. \text{Using Formula 5 and the Chain Rule, } y = 10^{1-x^2} \Rightarrow y' = 10^{1-x^2}(\ln 10) \cdot \frac{d}{dx}(1 - x^2) = -2x(\ln 10)10^{1-x^2}.$$

$$23. y = \left(\frac{x^2 + 1}{x^2 - 1}\right)^3 \Rightarrow$$
$$y' = 3\left(\frac{x^2 + 1}{x^2 - 1}\right)^2 \cdot \frac{d}{dx}\left(\frac{x^2 + 1}{x^2 - 1}\right) = 3\left(\frac{x^2 + 1}{x^2 - 1}\right)^2 \cdot \frac{(x^2 - 1)(2x) - (x^2 + 1)(2x)}{(x^2 - 1)^2}$$
$$= 3\left(\frac{x^2 + 1}{x^2 - 1}\right)^2 \cdot \frac{2x[x^2 - 1 - (x^2 + 1)]}{(x^2 - 1)^2} = 3\left(\frac{x^2 + 1}{x^2 - 1}\right)^2 \cdot \frac{2x(-2)}{(x^2 - 1)^2} = \frac{-12x(x^2 + 1)^2}{(x^2 - 1)^4}$$

$$31. \text{Using Formula 5 and the Chain Rule, } y = 2^{\sin \pi x} \Rightarrow$$
$$y' = 2^{\sin \pi x}(\ln 2) \cdot \frac{d}{dx}(\sin \pi x) = 2^{\sin \pi x}(\ln 2) \cdot \cos \pi x \cdot \pi = 2^{\sin \pi x}(\pi \ln 2) \cos \pi x$$

$$32. y = \sin(\sin(\sin x)) \Rightarrow y' = \cos(\sin(\sin x)) \frac{d}{dx}(\sin(\sin x)) = \cos(\sin(\sin x)) \cos(\sin x) \cos x$$

$$40. y = e^{e^x} \Rightarrow y' = e^{e^x} \cdot (e^x)' = e^{e^x} \cdot e^x \Rightarrow$$

$$y'' = e^{e^x} \cdot (e^x)' + e^x \cdot (e^{e^x})' = e^{e^x} \cdot e^x + e^x \cdot e^{e^x} \cdot e^x = e^{e^x} \cdot e^x(1 + e^x) \quad \text{or} \quad e^{e^x+x}(1 + e^x)$$