1. $n = 25,\ p = \frac{1}{5}$

$\mu = np = 25\left(\frac{1}{5}\right) = 5,$

$\sigma = \sqrt{npq} = \sqrt{25\left(\frac{1}{5}\right)\left(\frac{4}{5}\right)} = 2$

a. $\Pr(X = 5) \approx \Pr\left(\frac{4.5-5}{2} \le Z \le \frac{5.5-5}{2}\right)$
$= \Pr(-.25 \le Z \le .25)$
$= 0.5987 - 0.4013$
$= 0.1974$

b. $\Pr(3 \le X \le 7)$
$\approx \Pr\left(\frac{2.5-5}{2} \le Z \le \frac{7.5-5}{2}\right)$
$= \Pr(-1.25 \le Z \le 1.25)$
$= 0.8944 - 0.1056$
$= 0.7888$

c. $\Pr(X < 10) \approx \Pr\left(Z \le \frac{9.5-5}{2}\right)$
$= \Pr(Z \le 2.25)$
$= 0.9878$

2. $n = 18,\ p = \frac{2}{3}$

$\mu = np = 18\left(\frac{2}{3}\right) = 12,$

$\sigma = \sqrt{npq} = \sqrt{18\left(\frac{2}{3}\right)\left(\frac{1}{3}\right)} = 2$

a. $\Pr(X = 10) \approx \Pr\left(\frac{9.5-12}{2} \le Z \le \frac{10.5-12}{2}\right)$
$= \Pr(-1.25 \le Z \le -0.75)$
$= 0.2266 - 0.1056$
$= 0.1210$

b. $\Pr(8 \le X \le 16)$
$\approx \Pr\left(\frac{7.5-12}{2} \le Z \le \frac{16.5-12}{2}\right)$
$= \Pr(-2.25 \le Z \le 2.25)$
$= 0.9878 - 0.0122$
$= 0.9756$

c. $\Pr(X > 12) \approx \Pr\left(Z \ge \frac{12.5-12}{2}\right)$
$= \Pr(Z \ge .25)$
$= 1 - 0.5987$
$= 0.4013$

3. $n = 20,\ p = \frac{1}{6}$

$\mu = 20\left(\frac{1}{6}\right) = \frac{10}{3},\ \sigma = \sqrt{20\left(\frac{1}{6}\right)\left(\frac{5}{6}\right)} = \frac{5}{3}$

$\Pr(X \ge 8) \approx \Pr\left(Z \ge \frac{7.5 - \frac{10}{3}}{\frac{5}{3}}\right)$
$= \Pr(Z \ge 2.5)$
$= 1 - 0.9938$
$= 0.0062$

4. $n = 16,\ p = \frac{1}{2}$

$\mu = 16\left(\frac{1}{2}\right) = 8,\ \sigma = \sqrt{16\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)} = 2$

$\Pr(X \ge 12) \approx \Pr\left(Z \ge \frac{11.5-8}{2}\right)$
$= \Pr(Z \ge 1.75)$
$= 1 - 0.9599$
$= 0.0401$

5. $n = 100,\ p = \frac{1}{2}$

$\mu = 100\left(\frac{1}{2}\right) = 50,\ \sigma = \sqrt{100\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)} = 5$

$\Pr(X \ge 63) \approx \Pr\left(Z \ge \frac{62.5-50}{5}\right)$
$= \Pr(Z \ge 2.5)$
$= 1 - 0.9938$
$= 0.0062$

6. $n=90,\ p=\frac{9}{19}$

$$\mu=90\left(\frac{9}{19}\right)=\frac{810}{19},\ \sigma=\sqrt{90\left(\frac{9}{19}\right)\left(\frac{10}{19}\right)}=\frac{90}{19}$$

$$\begin{aligned}\Pr(X>45)&\approx\Pr\left(Z\geq\frac{45.5-\frac{810}{19}}{\frac{90}{19}}\right)\\&\approx\Pr(Z\geq 0.61)\\&\approx\Pr(Z\geq 0.60)\\&=1-0.7257\\&=0.2743\end{aligned}$$

7. $n=75,\ p=\frac{3}{4}$

$$\mu=75\left(\frac{3}{4}\right)=56.25,\quad \sigma=\sqrt{75\left(\frac{3}{4}\right)\left(\frac{1}{4}\right)}=3.75$$

$$\begin{aligned}\Pr(X\geq 68)&\approx\Pr\left(Z\geq\frac{67.5-56.25}{3.75}\right)\\&=\Pr(Z\geq 3)\\&=1-0.9987\\&=0.0013\end{aligned}$$

8. $n=54,\ p=\frac{2}{5}$

$$\mu=54\left(\frac{2}{5}\right)=21.6,\ \sigma=\sqrt{54\left(\frac{2}{5}\right)\left(\frac{3}{5}\right)}=3.6$$

$$\begin{aligned}\Pr(X<14)&\approx\Pr\left(Z\leq\frac{13.5-21.6}{3.6}\right)\\&=\Pr(Z\leq -2.25)\\&=0.0122\end{aligned}$$

9. $n=20, p=0.310$

$\mu\ =20(0.310)=6.2,$

$\sigma=\sqrt{20(0.31)(0.69)}\approx 2.068$

$$\begin{aligned}\Pr(X\geq 6)&\approx\Pr\left(Z\geq\frac{5.5-6.2}{2.068}\right)\\&\approx\Pr(Z\geq -0.34)\\&\approx\Pr(Z\geq -0.35)\\&=1-0.3632\\&=0.6368\end{aligned}$$

10. $n=1000, p=0.25$

$\mu=1000(0.25)=250,\ \sigma=\sqrt{1000(0.25)(0.75)}\approx 13.693$

$$\begin{aligned}\Pr(X\geq 290)&\approx\Pr\left(Z\geq\frac{289.5-250}{13.693}\right)\\&\approx\Pr(Z\geq 2.88)\\&\approx\Pr(Z\geq 2.90)\\&=1-0.9981\\&=0.0019\end{aligned}$$

Yes, the new campaign reached more of the target audience than would have been expected from the old campaign.

11. $n=1000, p=0.02$

$\mu\ =1000(0.02)=20,$

$\sigma=\sqrt{1000(0.02)(0.98)}\approx 4.427$

$$\begin{aligned}\Pr(X<15)&\approx\Pr\left(Z\leq\frac{14.5-20}{4.427}\right)\\&\approx\Pr(Z\leq -1.24)\\&\approx\Pr(Z\leq -1.25)\\&=0.1056\end{aligned}$$

12. $\text{E}(X)=\ \mu_{.}=np=70(0.20)=14$

$\sigma=\sqrt{70(0.20)(0.80)}\approx 3.347$

$$\begin{aligned}\Pr(X=14)&\approx\Pr\left(\frac{13.5-14}{3.347}\leq Z\leq\frac{14.5-14}{3.347}\right)\\&\approx\Pr(-0.15\leq Z\leq 0.15)\\&=0.5596-0.4404\\&=0.1192\end{aligned}$$

13. probability of failure = (0.01)(0.02)(0.01) = 0.000002

$n=1{,}000{,}000,$

$E(X)=\ \mu=1{,}000{,}000(0.000002)=2$

$\sigma=\sqrt{1{,}000{,}000(0.000002)(0.999998)}\approx 1.414$

$$\begin{aligned}\Pr(X>3)&\approx\Pr\left(Z\geq\frac{3.5-2}{1.414}\right)\\&\approx\Pr(Z\geq 1.06)\\&\approx\Pr(Z\geq 1.05)\\&=1-0.8531\\&=0.1469\end{aligned}$$

14. Estimated probability: $\frac{175}{250} = 0.7$

$n = 250, p = 0.75$
$\mu = 250(0.75) = 187.5,$
$\sigma = \sqrt{250(0.75)(0.25)} \approx 6.847$

$$\Pr(X \le 175) \approx \Pr\left(Z \le \frac{175.5 - 187.5}{6.847}\right)$$
$$\approx \Pr(Z \le -1.75)$$
$$= 0.0401$$

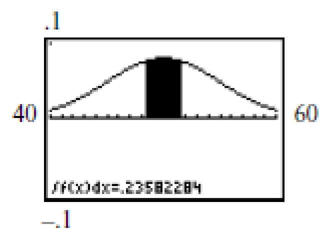
15. $n = 100, p = 0.35$
$\mu = 100(0.35) = 35,$
$\sigma = \sqrt{100(0.35)(0.65)} \approx 4.770$
$\Pr(30 \le X \le 40)$
$$= \Pr\left(\frac{29.5 - 35}{4.77} \le Z \le \frac{40.5 - 35}{4.77}\right)$$
$$\approx \Pr(-1.15 \le Z \le 1.15)$$
$$= 0.8749 - 0.1251$$
$$= 0.7498$$

16. $n = 150, p = 0.14$
$\mu = 150(0.14) = 21,$
$\sigma = \sqrt{150(0.14)(0.86)} \approx 4.250$
Passengers will have to be bumped if fewer than 10 passengers cancel.
$$\Pr(X \le 9) = \Pr\left(Z \le \frac{9.5 - 21}{4.25}\right)$$
$$\approx \Pr(Z \le -2.71)$$
$$\approx \Pr(Z \le -2.70)$$
$$= 0.0035$$

17. $n = 1000, p = 0.03$
$\mu = 1000(0.03) = 30,$
$\sigma = \sqrt{1000(0.03)(0.97)} \approx 5.394$
$$\Pr(X \ge 29) \approx \Pr\left(Z \ge \frac{28.5 - 30}{5.394}\right)$$
$$\approx \Pr(Z \ge -0.278)$$
$$\approx \Pr(Z \ge -0.28)$$
$$= 1 - 0.3897$$
$$= 0.61$$

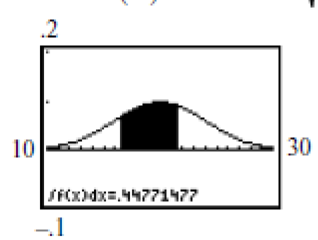
18. $n = 100,\ p = \frac{1}{2}$
$$\mu = 100 \cdot \frac{1}{2} = 50,\ \sigma = \sqrt{100 \cdot \frac{1}{2} \cdot \frac{1}{2}} = 5$$
$$\Pr(X > 65) \approx \Pr\left(Z \ge \frac{65.5 - 50}{5}\right)$$
$$= \Pr(Z \ge 3.1)$$
$$= 1 - 0.9990$$
$$= 0.001$$
The probability that more than 65 tails occur i the same, so the probability that more than 65 heads or tails occur is $2 \times .001 = .002$.

19. $n = 100,\ p = \frac{1}{2}$
Exact: $\Pr(49 \le X \le 51) \approx .2356$
Normal Approximation:
$$\mu = 100\left(\frac{1}{2}\right) = 50,\ \sigma = \sqrt{100\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)} = 5$$



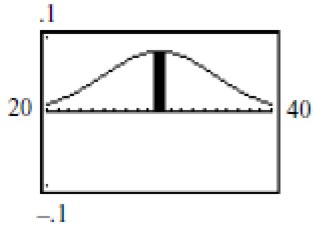
$\Pr(48.5 \le X \le 51.5) \approx 0.2358$

20. $n = 120,\ p = \frac{1}{6}$
Exact: $\Pr(17 \le X \le 21) \approx 0.4544$
Normal Approximation:
$$\mu = 120\left(\frac{1}{6}\right) = 20,\ \sigma = \sqrt{120\left(\frac{1}{6}\right)\left(\frac{5}{6}\right)} \approx 4.082$$



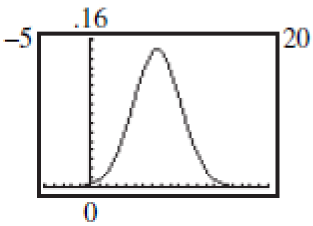
$\Pr(16.5 \le X \le 21.5) \approx 0.4477$

21. $n = 150, p = 0.2$
Exact: $\Pr(X = 30) \approx 0.0812$
Normal Approximation: $\mu = 150(0.2) = 30,$
$\sigma = \sqrt{150(0.2)(0.8)} \approx 4.899$



$\Pr(29.5 \le X \le 30.5) \approx 0.0813$

22. $n = 150, p = 0.05$
Exact: $\Pr(X \le 5) \approx 0.2344$
Normal Approximation:
$\mu = 150(0.05) = 7.5,$
$\sigma = \sqrt{150(0.05)(0.95)} \approx 2.669$



$\Pr(X \le 5) \approx 0.2269$

23. 0.0410

1 – binomcdf (300, 0.02, 10)

1 – BINOMDIST (10, 300, 0.02, 1)

binomial probabilities $n = 300, p = 0.02,$

endpoint = 10

24. 0.0262

1 – binomcdf (40, 0.25, 15)

1 – BINOMDIST (15, 40, 0.25, 1)

binomial probabilities $n = 40, p = 0.25,$

endpoint = 15

25. 0.0796

binompdf (100,0.5,50)

BINOMDIST (50, 100, 0.5, 0)

binomial probabilities $n = 100, p = 0.5,$

endpoint = 50

26. 0.2197

1 - binomcdf (100,1/6,19);

1 – BINOMDIST (19, 100, 1/6, 1)

binomial probabilities $n = 100, p = 1/6,$

endpoint = 19

Chapter 7 Fundamental Concept Check

1. *Bar Charts* and *pie charts* provide graphical ways of displaying qualitative data. A *histogram* is a graphical way of presenting a frequency distribution. A *box plot* is a graphical way of presenting a five–number summary of a collection of data.

2. The following definitions apply to a set of numbers. The *median* is the middle value when the numbers are ordered. The *first quartile* is a number for which roughly 25% of the numbers are less than that number; same as the 25^{th} percentile. The *third quartile* is a number for which roughly 75% of the numbers are less than that that number, same as the 75^{th} percentile. The *interquartile range* is the difference between the third quartile and the first quartile. The five–number summary consist of the minimum value, first quartile, median, third quartile, and maximum value.

3. A *frequency distribution* for a collection of numerical data is a table listing each number in the collection and the number of times it appears. A *relative frequency distribution* for a collection of numerical data is a table listing each number in the collection and the percentage of times it appears. A *probability distribution* is a table displaying the outcomes of an experiment and their probabilities.

4. Consider a table for the distribution. To construct a histogram for the table, draw a coordinate system, write the numbers from the left column of the table below the x–axis, and above each number draw a rectangle having height given in the second column of the table.