1. Population mean

2. Sample mean

3. Expected value

4. Expected value

5. $E(X) = 0(0.25) + 1(0.2) + 2(0.1) + 3(0.25) + 4(0.2) = 1.95$

6. $$E(X) = -1(0.1) - \frac{1}{2}(0.4) + 0(0.25) + \frac{1}{2}(0.2) + 1(0.05) = -0.15$$

7. **a.** $$\text{GPA} = \frac{4+4+4+4+3+3+2+2+2+1}{10} = \frac{29}{10} = 2.9$$

b.

Grade	Relative Frequency
4	$\frac{4}{10} = 0.4$
3	$\frac{2}{10} = 0.2$
2	$\frac{3}{10} = 0.3$
1	$\frac{1}{10} = 0.1$

c. $E(X) = 4(0.4) + 3(0.2) + 2(0.3) + 1(0.1)$
$= 2.9$

8. a. $\bar{x} = \dfrac{9.8 + 9.8 + 9.4 + 9.2 + 9.2 + 9.0}{6}$
$= \dfrac{56.4}{6}$
$= 9.4$

b.

Score	Relative Frequency
9.8	$\frac{2}{6}$
9.4	$\frac{1}{6}$
9.2	$\frac{2}{6}$
9.0	$\frac{1}{6}$

c. $\bar{x} = 9.8\left(\dfrac{2}{6}\right) + 9.4\left(\dfrac{1}{6}\right) + 9.2\left(\dfrac{2}{6}\right) + 9.0\left(\dfrac{1}{6}\right)$
$= 9.4$

9. $\bar{x}_A = 0(0.3) + 1(0.3) + 2(0.2) + 3(0.1) + 4(0) + 5(0.1)$
$= 1.5$
$\bar{x}_B = 0(0.2) + 1(0.3) + 2(0.3) + 3(0.1) + 4(0.1) + 5(0)$
$= 1.6$
Group A had fewer cavities.

10. $\bar{x}_A = 1000(0.2) + 2000(0.6) + 3000(0.2) = 2000$
$\bar{x}_B = -2000(0.2) + 0(0.2) + 4000(0.6) = 2000$
Both investments have the same expected returns.

11. $E(X) = 1(0.2) + 2(0.4) + 3(0.3) + 4(0.1)$
$= 2.3$

12. $E(X) = 3(0.2) + 4(0.1) + 5(0.4) + 6(0.2) + 7(0.1)$
$= 4.9$

13. $E(X) = 7(0.25) + 8(0.25) + 9(0.25) + 10(0.25)$
$= 8.5$

14. $E(X) = 2(.05) + 3(.1) + 4(.2) + 5(.3) + 6(.2) + 7(.1) + 8(.05)$
$= 5$

15.

Earnings	Probability
–\$1	$\frac{37}{38}$
\$35	$\frac{1}{38}$

$E(X) = -1\left(\dfrac{37}{38}\right) + 35\left(\dfrac{1}{38}\right) \approx -\$0.0526$

16. Let w be the amount won.
$E(x) = -1\left(\dfrac{36}{38}\right) + w\left(\dfrac{2}{38}\right) = -\dfrac{1}{19}$
$w = \dfrac{-\frac{1}{19} + \frac{36}{38}}{\frac{2}{38}} = \$17$

17.

Earnings	Probability
–50¢	$\frac{2}{6} = \frac{1}{3}$
0¢	$\left(\frac{4}{6}\right)\left(\frac{2}{5}\right) = \frac{4}{15}$
50¢	$\left(\frac{4}{6}\right)\left(\frac{3}{5}\right)\left(\frac{2}{4}\right) = \frac{1}{5}$
\$1	$\left(\frac{4}{6}\right)\left(\frac{3}{5}\right)\left(\frac{2}{4}\right)\left(\frac{2}{3}\right) = \frac{2}{15}$
\$1.50	$\left(\frac{4}{6}\right)\left(\frac{3}{5}\right)\left(\frac{2}{4}\right)\left(\frac{1}{3}\right)\left(\frac{2}{2}\right) = \frac{1}{15}$

$E(X) = -.5\left(\dfrac{1}{3}\right) + 0\left(\dfrac{4}{15}\right) + .5\left(\dfrac{1}{5}\right) + 1\left(\dfrac{2}{15}\right) + 1.5\left(\dfrac{1}{15}\right)$
$\approx \$.1667$

18.

Winnings	Probability
\$2	$\left(\frac{2}{8}\right)\left(\frac{1}{7}\right) = \frac{1}{28}$
\$1	$\left(\frac{2}{8}\right)\left(\frac{6}{8}\right) + \left(\frac{6}{8}\right)\left(\frac{2}{7}\right) = \frac{12}{28}$
\$0	$\left(\frac{6}{8}\right)\left(\frac{5}{7}\right) = \frac{15}{28}$

$E(X) = 2\left(\dfrac{1}{28}\right) + 1\left(\dfrac{12}{28}\right) + 0\left(\dfrac{15}{28}\right) = 0.5$
The player should pay 50¢ per play to break even.

19. Let x be the cost of the policy.
$E(X) = (-x)(0.9) + (10{,}000 - x)(0.1) = -x + 1000$
The expected value is zero if $x = 1000$.
He should be willing to pay up to $1000.

20. The probability the man dies is 0.1.
The probability the woman dies is 0.05.
Assuming independent life spans, the probability both the man and the woman die in the next 5 years is (0.1)(0.05) = 0.005. Therefore, the probability that only the man dies is (0.1)(0.95) = 0.095 and the probability that only the woman dies is (0.9)(0.05) = 0.045. The probability they both live is (0.9)(0.95) = 0.855.
Let x be the cost of the policy.
$E(X) = (-x)(0.855) + (10{,}000 - x)(0.095) + (10{,}000 - x)(0.045) + (15{,}000 - x)(0.005)$
$= -x + 1475.$
The expected value is zero if $x = 1475$. They should be willing to pay up to $1475.

21. $\dfrac{5\times 0.300 + 4\times 0.350}{9} \approx 0.322$

22. $\dfrac{3\times 91 + 5\times 87}{8} = \dfrac{708}{8} = 88.5$

23.

Recorded Value	Probability
1	$\frac{1}{36}$
2	$\frac{3}{36}$
3	$\frac{5}{36}$
4	$\frac{7}{36}$
5	$\frac{9}{36}$
6	$\frac{11}{36}$

$$\mathrm{E}(X) = 1\left(\frac{1}{36}\right) + 2\left(\frac{3}{36}\right) + 3\left(\frac{5}{36}\right) + 4\left(\frac{7}{36}\right) + 5\left(\frac{9}{36}\right) + 6\left(\frac{11}{36}\right) \approx 4.47$$

24.

Smaller Value	Probability
1	$\frac{11}{36}$
2	$\frac{9}{36}$
3	$\frac{7}{36}$
4	$\frac{5}{36}$
5	$\frac{3}{36}$
6	$\frac{1}{36}$

$$E(X)=1\left(\frac{11}{36}\right)+2\left(\frac{9}{36}\right)+3\left(\frac{7}{36}\right)+4\left(\frac{5}{36}\right)+5\left(\frac{3}{36}\right)+6\left(\frac{1}{36}\right)=\frac{91}{36}\approx 2.528$$

25. $E(X)=-1\left(\frac{125}{216}\right)+1\left(\frac{75}{216}\right)+3\left(\frac{15}{216}\right)+5\left(\frac{1}{216}\right)=\frac{0}{216}=0$

26.

House Winnings	Probability
0	$\frac{5}{9}$
1	$\frac{5}{12}$
2	$\frac{1}{36}$

$$E(X)=0\left(\frac{5}{9}\right)+1\left(\frac{5}{12}\right)+2\left(\frac{1}{36}\right)=\frac{17}{36}\approx 47.22 \text{ cents}$$

27. $E(X)=15(0.205)=3.075$

28. $E(X)=30(0.71)=21.3$

29. The expected number of times that a 5 or a 6 will appear is $\mu=np=30\left(\frac{1}{3}\right)=10$

30. The expected number of hits is $\mu=np=40(0.275)=11$

31. The expected value of points when taking one three point shot is $\mu = 3(0.40) = 1.2$ points. The expected value of taking three free-throws when the success probability is 60% is $\mu = 3(0.60) = 1.8$, which is the greater expected value.

32. To calculate the probability of success for a binomial random variable with 20 trials whose expected value is 3, use the formula: $\mu=np$. Substitute 3 for μ and 20 for n: $3 = 20p$. Thus, $p = 0.15$.

33.

Number of Republicans	Probability
0	$\frac{10}{84}$
1	$\frac{40}{84}$
2	$\frac{30}{84}$
3	$\frac{4}{84}$

$$\text{E(Rep.)} = 0\left(\frac{10}{84}\right) + 1\left(\frac{40}{84}\right) + 2\left(\frac{30}{84}\right) + 3\left(\frac{4}{84}\right) = \frac{4}{3} \approx 1.333$$

$$\text{E(Dem.)} = 3 - E(\text{Rep.}) = 3 - \frac{4}{3} = \frac{5}{3} \approx 1.667$$

34.

Number of Red	Probability
0	$\frac{1}{210}$
1	$\frac{24}{210}$
2	$\frac{90}{210}$
3	$\frac{80}{210}$
4	$\frac{15}{210}$

$$\text{E(Red)} = 0\left(\frac{1}{210}\right) + 1\left(\frac{24}{210}\right) + 2\left(\frac{90}{210}\right) + 3\left(\frac{80}{210}\right) + 4\left(\frac{15}{210}\right) = \frac{12}{5} = 2.4$$

$$\text{E(Green)} = 4 - E(\text{Red}) = 4 - \frac{12}{5} = \frac{3}{5} = 1.6$$

35. No Replacing

Number of Red	Probability
0	$\frac{1}{35}$
1	$\frac{12}{35}$
2	$\frac{18}{35}$
3	$\frac{4}{35}$

$$\text{E(Red)} = 0\left(\frac{1}{35}\right) + 1\left(\frac{12}{35}\right) + 2\left(\frac{18}{35}\right) + 3\left(\frac{4}{35}\right) = \frac{12}{7} \approx 1.714$$

Replacing

Number of Red	Probability
0	$\frac{27}{343}$
1	$\frac{108}{343}$
2	$\frac{144}{343}$
3	$\frac{64}{343}$

$$\text{E(Red)} = 0\left(\frac{27}{343}\right) + 1\left(\frac{108}{343}\right) + 2\left(\frac{144}{343}\right) + 3\left(\frac{64}{343}\right) = \frac{12}{7} \approx 1.714$$

36. No Replacing

Number of Hearts	Probability
0	$\frac{741}{1326}$
1	$\frac{507}{1326}$
2	$\frac{78}{1326}$

$$E(\text{Heart}) = 0\left(\frac{741}{1326}\right) + 1\left(\frac{507}{1326}\right) + 2\left(\frac{78}{1326}\right)$$
$$= \frac{1}{2} = 0.5$$

Replacing Number of Hearts	Probability
0	$\frac{9}{16}$
1	$\frac{3}{8}$
2	$\frac{1}{16}$

$$E(\text{Heart}) = 0\left(\frac{9}{16}\right) + 1\left(\frac{3}{8}\right) + 2\left(\frac{1}{16}\right)$$
$$= \frac{1}{2} = 0.5$$

37. Let x = chance of rain.
$$-8000 + 40{,}000x = 0$$
$$x = 0.20 \to 20\%$$

38. Let x = probability stolen.
$$-150 + 75{,}000x = -250 + 100{,}000x$$
$$100 = 25{,}000x$$
$$x = 0.004$$

39. $\frac{7x + 4y}{x + y}$

Answer (b) is correct.

40. $\frac{kx + y}{k + 1}$

Answer (b) is correct.

41. Solve $\frac{16 \cdot 54 + 14x}{30} = 56.1$
$$14x + 864 = 1683$$
$$14x = 819$$
$$x = 58.5°$$

42. Solve $\frac{83 + 92 + 89 + 3x}{6} = 90$
$$3x + 264 = 540$$
$$3x = 276$$
$$x = 92$$

43. $\frac{5 + 6 + x}{3} = \frac{2 + 7 + 9}{3}$
$$11 + x = 18$$
$$x = 7$$

44.
$$-2(0.1) + -1(0.2) + 0(0.6) + x(0.1) = 0$$
$$-0.2 - 0.2 + 0 + 0.1x = 0$$
$$-0.4 + 0.1x = 0$$
$$0.1x = 0.4$$
$$x = 4$$

45. Solve $\frac{12 \cdot 5 + 24x}{5 + x} = 19$
$$19x + 95 = 60 + 24x$$
$$-5x = -35$$
$$x = 7$$

46. Let x = revenue for third week
$2x$ = revenue for first week
$\frac{1}{2}x$ = revenue for second week.
$$\frac{x + 2x + \frac{1}{2}x}{3} = 14{,}000$$
$$\frac{7}{2}x = 42{,}000$$
$$x = \$12{,}000$$
$$2x = \$24{,}000$$

47. Let x = number of cases.
$$20\left(\frac{1}{2}x\right) + 30\left(\frac{1}{2}x\right) = 75{,}000$$
$$25x = 75{,}000$$
$$x = 3000 \text{ cases}$$

48. Let x = number of magazines.
$$2.00\left(\frac{1}{2}x\right) + 2.50\left(\frac{1}{2}x\right) = 135.00$$
$$2.25x = 135.00$$
$$x = 60 \text{ magazines}$$