1. $\binom{5}{3}(0.3)^3(0.7)^2 = (10)(0.027)(0.49) = 0.1323$

2. $\binom{6}{1}(0.4)^1(0.6)^5 = (6)(0.4)(0.07776) = 0.1866$

3. $\binom{4}{3}\left(\frac{1}{3}\right)^3\left(\frac{2}{3}\right)^1 = (4)\left(\frac{1}{27}\right)\left(\frac{2}{3}\right) = \frac{8}{81} \approx 0.0988$

4. $\binom{3}{2}\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^1 = (3)\left(\frac{1}{36}\right)\left(\frac{5}{6}\right) = \frac{5}{72} \approx 0.06944$

5. $\Pr(X=3) = \binom{10}{3}\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^7 = \frac{15}{128} \approx 0.1172$

6. $\Pr(X=0) = \binom{10}{0}\left(\frac{1}{2}\right)^0\left(\frac{1}{2}\right)^{10} = \frac{1}{1024} \approx .0009766$

7. $\Pr(X=7) = \binom{10}{7}\left(\frac{1}{2}\right)^7\left(\frac{1}{2}\right)^3 = \frac{15}{128} \approx 0.1171875$

$\Pr(X=8) = \binom{10}{8}\left(\frac{1}{2}\right)^8\left(\frac{1}{2}\right)^2 = \frac{45}{1024} \approx 0.043945$

$\Pr(X = 7 \text{ or } 8) = 0.117188 + 0.04395 = 0.1611$

8. $\Pr(X=3) = \binom{10}{3}\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^7 = \frac{15}{128} \approx 0.1171875$

$\Pr(X=2) = \binom{10}{2}\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^8 = \frac{45}{1024} \approx 0.043945$

$\Pr(X = 3 \text{ or } 2) = 0.117188 + 0.04395 = 0.1611$

9. $$\begin{aligned}\Pr(\text{At least } 1) &= 1 - \Pr(X=0)\\ &= 1 - 0.0009766\\ &= 0.9990\end{aligned}$$

10. $\Pr(X=9) = \binom{10}{9}\left(\frac{1}{2}\right)^9\left(\frac{1}{2}\right)^1 = \frac{5}{512} \approx 0.009766$

$\Pr(X=10) = \binom{10}{10}\left(\frac{1}{2}\right)^{10}\left(\frac{1}{2}\right)^0 = \frac{1}{1024} \approx 0.0009766$

$$\begin{aligned}\Pr(\text{At most } 7) &= 1 - \Pr(X = 8, 9, 10)\\ &= 1 - 0.043945 - 0.009766\\ &\qquad - 0.0009766\\ &= 0.9453\end{aligned}$$

11. $\Pr(X=4) = \binom{4}{4}\left(\frac{1}{6}\right)^4\left(\frac{5}{6}\right)^0 = \frac{1}{1296} \approx 0.000772$

12. $\Pr(X=2) = \binom{4}{2}\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^2 = \frac{25}{216} \approx 0.1157$

13. $\Pr(X=3) = \binom{4}{3}\left(\frac{1}{6}\right)^3\left(\frac{5}{6}\right)^1 = \frac{5}{324} \approx 0.015432$

$\Pr(X = 2 \text{ or } 3) = 0.11574 + 0.01543 = 0.1312$

14. $\Pr(X=1) = \binom{4}{1}\left(\frac{1}{6}\right)^1\left(\frac{5}{6}\right)^3 = \frac{125}{324} \approx 0.3858$

$\Pr(X = 2 \text{ or } 1) = 0.1157 + 0.3858 = 0.5015$

15. $$\begin{aligned}&\Pr(X = 2 \text{ or } 1 \text{ or } 0)\\ &\quad = 0.1157 + 0.3858 + 0.4823 = 0.9838\end{aligned}$$

16. $\Pr(X=4) = \binom{4}{4}\left(\frac{1}{6}\right)^4\left(\frac{5}{6}\right)^0 = \frac{1}{1296} \approx 0.000772$

$$\begin{aligned}&\Pr(X = 2 \text{ or } 3 \text{ or } 4)\\ &\quad = 0.1157 + 0.0154 + 0.00077 = 0.1319\end{aligned}$$

17. $\Pr(X=2) = \binom{8}{2}(0.14)^2(0.86)^6 \approx 0.2220$

18. $\Pr(X=0) = \binom{8}{0}(0.14)^0(0.86)^8 \approx 0.2992$

19. $\Pr(X=4)=\binom{8}{4}(0.14)^4(0.86)^4 \approx 0.01471$

$\Pr(X=5)=\binom{8}{5}(0.14)^5(0.86)^3 \approx 0.00192$

$\Pr(X=4 \text{ or } 5)=0.01471+0.00192=0.01663$

20. $\Pr(X=1)=\binom{8}{1}(0.14)^1(0.86)^7 \approx 0.3897$

$\Pr(X=2)=\binom{8}{2}(0.14)^2(0.86)^6 \approx 0.2220$

$\Pr(X=1 \text{ or } 2)=0.3897+0.2220=0.6117$

21. $\Pr(\text{At least } 3)=1-\Pr(X=0 \text{ or } 1 \text{ or } 2)$
$=1-0.2992-0.3897-0.2220$
$=0.08908$

22. $\Pr(X=7)=\binom{8}{7}(0.14)^7(0.86)^1 \approx 0.00000725$

$\Pr(X=8)=\binom{8}{8}(0.14)^8(0.86)^0 \approx 0.000000148$

$\Pr(\text{At most } 6)=1-\Pr(X=7 \text{ or } 8)$
$=1-0.00000725-0.000000148$
$=0.999993$

23. $\Pr(X=7)=\binom{7}{7}(0.761)^7(0.239)^0 \approx 0.1478$

24. $\Pr(X=3)=\binom{7}{3}(0.761)^3(0.239)^4 \approx 0.0503$

25. $\Pr(X=6)=\binom{7}{6}(0.761)^6(0.239)^1 \approx 0.3249$

$\Pr(X=6 \text{ or } 7)=0.3249+0.1478=0.4727$

26. $\Pr(X=0)=\binom{7}{0}(0.761)^0(0.239)^7 \approx 0.0000445$

$\Pr(X=1)=\binom{7}{1}(0.761)^1(0.239)^6 \approx 0.0009928$

$\Pr(X=2)=\binom{7}{2}(0.761)^2(0.239)^5 \approx 0.0094837$

$\Pr(X=0 \text{ or } 1 \text{ or } 2)$
$=0.0000445+0.0009928+0.0094837$
$=0.01052$

27. **a.** $\Pr(X=2)=\binom{4}{2}\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^2=\frac{3}{8}=0.375$

b. $\Pr(X=3)=2\binom{4}{3}\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^1=\frac{8}{16}=0.5$

c. $\Pr(X=4)=2\binom{4}{4}\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)^0=\frac{2}{16}=0.125$

28. **a.** $\Pr(X=15)=\binom{20}{15}(0.6)^{15}(0.4)^5 \approx 0.07465$

b. $\Pr(X=19)=\binom{20}{19}(0.6)^{19}(0.4)^1 \approx 0.000487$

$\Pr(X=20)=\binom{20}{20}(0.6)^{20}(0.4)^0 \approx 0.0000366$

$\Pr(\text{Fewer than } 19)=1-\Pr(X=19 \text{ or } 20)$
$=1-0.000487-0.0000366$
$=0.9995$

29. $\Pr(X=3)=\binom{5}{3}(0.76)^3(0.24)^2 \approx 0.2529$

$\Pr(X=5)=\binom{5}{5}(0.76)^5(0.24)^0 \approx 0.2536$

The probability of getting 5 successes is higher.

30. Based on the histogram, the probability of getting 4 successes is higher. Out of a group of five students, it is more likely for exactly four of them to have been accepted to their first choice than it is for exactly three of them to have been accepted to their first choice.

31. Based on the histogram, the probability of getting 10 successes is higher. Out of a group of

40 cattle, it is more likely that exactly 10 recover than exactly 9 recover.

32. $\Pr(X=5)=\binom{40}{5}(0.25)^5(0.75)^{35}\approx 0.0272$

$\Pr(X=15)=\binom{40}{15}(0.25)^{15}(0.75)^{25}\approx 0.0282$

The probability of getting 15 successes is higher.

33. The probability that the salesman sells cars to three or four of the customers.

34. The proability that at least one of the 10 policyholders files a claim.

35. **a.** $1-0.4602-0.0796=0.4602$

b. $0.4602+0.0796=0.5398$

36. **a.** $1-0.9845-0.01302=0.00248$

b. $0.9845+0.01302=0.99752$

37. The probability of success is .5 therefore the histogram will be symmetrical about the middle value, or 5.

38. 0.5; because the histogram is symmetrical, half of the values will be greater than 5 and half will be 5 or less.

39. 1; because the sum of all individual probabilities is 1.

40. 1; because the sum of all individual probabilities is 1.

41. Let "success" = "lives to 100." Then $p = 0.075$, $q = 0.925$, $n = 77$.

$$\begin{aligned}\Pr(X\geq 2)&=1-\Pr(X=0)-\Pr(X=1)\\&=1-(0.925)^{77}\\&\quad-77(0.075)^1(0.925)^{76}\\&\approx 0.9821\end{aligned}$$

42. Let "success" = "left-handed." Then $p = 0.1$, $q = 0.9$, $n = 10$.

$$\begin{aligned}\Pr(X\geq 2)&=1-\Pr(X=0)-\Pr(X=1)\\&=1-(0.9)^{10}-10(0.1)^1(0.9)^9\\&\approx 0.2639\end{aligned}$$

43. Let "success" = "adverse reaction." Then p = 0.02, $q = 0.98$, $n = 56$.

$$\begin{aligned}\Pr(X\geq 3)&=1-\Pr(X=0)-\Pr(X=1)-\Pr(X=2)\\&=1-(0.98)^{56}-\binom{56}{1}(0.02)^1(0.98)^{55}\\&\quad-\binom{56}{2}(0.02)^2(0.98)^{54}\\&\approx 0.1018\end{aligned}$$

44. Let "success" be "brand X." Then $p = 0.3$, $q = 0.7$, $n = 9$.

$$\begin{aligned}&\Pr(X > 2)\\&= 1 - \Pr(X \leq 2)\\&= 1 - [\Pr(X = 0) + \Pr(X = 1) + \Pr(X = 2)]\\&=1-\binom{9}{0}(0.3)^0(0.7)^9-\binom{9}{1}(0.3)^1(0.7)^8\\&\quad-\binom{9}{2}(0.3)^2(0.7)^7\\&=1-0.04035-0.15565-0.26683\\&=0.5372\end{aligned}$$

45. Let "success" be "a vote for" the candidate. Then $p = 0.6$, $q = 0.4$, $n = 5$.

$$\begin{aligned}&\Pr(X \leq 2)\\&= \Pr(X = 0) + \Pr(X = 1) + \Pr(X = 2)\\&=\binom{5}{0}(0.6)^0(0.4)^5+\binom{5}{1}(0.6)^1(0.4)^4+\binom{5}{2}(0.6)^2(0.4)^3\\&= 0.01024 + 0.0768 + 0.2304\\&\approx 0.3174\end{aligned}$$

46. Let "success" be "a guilty vote."
Then $p = 0.76$, $q = 0.24$, $n = 12$.

$$\begin{aligned}&\Pr(X \geq 10) = \Pr(X = 10) + \Pr(X = 11) + \Pr(X = 12)\\&=\binom{12}{10}(0.7)^{10}(0.3)^2+\binom{12}{11}(0.7)^{11}(0.3)^1\\&\quad+\binom{12}{12}(0.7)^{12}(0.3)^0\\&=0.16779+0.07118+0.01384\\&=0.2528\end{aligned}$$

47. Let "success" = "defective." Then $p = 0.03$, $q = 0.97$, $n = 20$.

$$\Pr(X \ge 2) = 1 - \Pr(X = 0) - \Pr(X = 1)$$
$$= 1 - (0.97)^{20} - 20(0.03)^1(0.97)^{19}$$
$$\approx 0.1198$$

48. Let "success" be "defective." Then $p = 0.02$, $q = 0.98$, $n = 300$.

$$\Pr(X \le 2) = \Pr(X = 0) + \Pr(X = 1) + \Pr(X = 2)$$
$$= \binom{300}{0}(0.02)^0(0.98)^{300} + \binom{300}{1}(0.02)^1(0.98)^{299} + \binom{300}{2}(0.02)^2(0.98)^{298}$$
$$\approx 0.06018$$

49.

		Mother A	Mother a
Father	A	AA	Aa
	a	Aa	aa

Child's genes	Probability
AA	$\frac{1}{4}$
Aa	$\frac{2}{4}$
aa	$\frac{1}{4}$

Let "success" be "aa." Then

$p = \frac{1}{4}$, $q = \frac{3}{4}$, $n = 3$.

$$\Pr(X \ge 1) = 1 - \Pr(X = 0)$$
$$= 1 - \binom{3}{0}\left(\frac{1}{4}\right)^0\left(\frac{3}{4}\right)^3$$
$$\approx 1 - 0.4219$$
$$= 0.5781$$

50. The first method has probability $1 - \left(\frac{99}{100}\right)^{100} \approx 0.6340$ of detecting the theft. In the second method, the probability of selecting 4 ingots from a single bin is $\frac{\binom{99}{4}}{\binom{100}{4}} = 0.96$, so this method has probability $1 - (0.96)^{25} \approx 0.6396$ of detecting the theft, which is slightly better.

51. Let "success" be "gets a hit." Then $p = 0.3$, $q = 0.7$, $n = 4$.

$$\Pr(X = 0) = \binom{4}{0}(0.3)^0(0.7)^4 = 0.2401$$
$$\Pr(X = 3) = \binom{4}{3}(0.3)^3(0.7)^1 = 0.0756$$

52. Let "success" be "make the free throw." Then $p = 0.9$, $q = 0.1$, $n = 5$.

$$\Pr(X = 2) = \binom{5}{2}(0.9)^2(0.1)^3 = 0.0081$$

53. Here $p = 0.82$. The expected value is $\mu = np = 10(0.82) = 8.2$

$$\Pr(X = 8) = C(10,8)\,(0.82)^8(0.18)^2 = 0.2980$$
$$\Pr(X = 9) = C(10,9)\,(0.82)^9\,(0.18)^1 = 0.3017$$

So 9 is the most likely number.

54. Let "success" be a bull's-eye. Then $p = 0.64$, $q = 0.36$, $n = 10$.

$$\Pr(X = 6) = \binom{10}{6}(0.64)^6(0.36)^4 \approx 0.2424$$
$$\Pr(X = 7) = \binom{10}{7}(0.64)^7(0.36)^3 \approx 0.2462$$

So 7 bull's-eyes is the most probable number.

55. **a.** For the underdog to win the series in five sets, the underdog must win two of the first four sets and win the fifth set. The probability that this occurs is

$$\binom{4}{2}(1-p)^2 p^2 (1-p) = \binom{4}{2}(1-p)^3 p^2.$$

b. Using the same reasoning,

$\Pr(\text{underdog wins in 3 sets}) = (1-p)^3$

$\Pr(\text{underdog wins in 4 sets}) = \binom{3}{2}(1-p)^2 p(1-p)$

$= \binom{3}{2}(1-p)^3 p$

c.

$$\Pr(\text{underdog wins set}) = (1-p)^3\left(1+\binom{3}{1}p+\binom{4}{2}p^2\right)$$
$$= (1-p)^3(1+3p+6p^2)$$
$$= (1-.7)^3(1+3(.7)+6(.7)^2)$$
$$= .16308$$

d. Suppose all five sets of the match are played even if one of the players wins three sets before the fifth set. Then the probability that the underdog wins at least three sets is

$$\binom{5}{0}(1-p)^5+\binom{5}{1}p(1-p)^4+\binom{5}{2}p^2(1-p)^3.$$

Substituting $p = .7$ gives a probability ≈ 0.16308.

56. a. For the underdog to win the series in five games, the underdog must win three of the first four games and win the fifth game. The probability that this occurs is

$$\binom{4}{3}(1-p)^3 p(1-p) = \binom{4}{3}(1-p)^4 p.$$

b. Using the same reasoning,

$\Pr(\text{underdog wins in 4 games}) = (1-p)^4$

$\Pr(\text{underdog wins in 6 games})$

$= \binom{5}{3}(1-p)^3 p^2(1-p)$

$= \binom{5}{3}(1-p)^4 p^2$

$\Pr(\text{underdog wins in 7 games})$

$= \binom{6}{3}(1-p)^3 p^3(1-p)$

$= \binom{6}{3}(1-p)^4 p^3$

c. $\Pr(\text{underdog wins series})$

$$= (1-p)^4\left(1+\binom{4}{3}p+\binom{5}{3}p^2+\binom{6}{3}p^3\right)$$
$$= (1-p)^4(1+4p+10p^2+20p^3)$$
$$= (1-0.6)^4(1+4(0.6)+10(0.6)^2+20(0.6)^3)$$
$$= 0.289792$$

d. Suppose all seven games of the World Series are played even if one of the teams wins four games before the seventh game. Then the probability that the underdog wins at least four games is

$$\binom{7}{0}(1-p)^7+\binom{7}{1}p(1-p)^6+\binom{7}{2}p^2(1-p)^5+\binom{7}{3}p^3(1-p)^4.$$

Substituting $p = .6$ gives a probability $\approx .289792$.

57. 17; because $1-\binom{17}{0}\left(\frac{1}{6}\right)^0\left(\frac{5}{6}\right)^{17} = 0.9549$

58. 37; because

$$1-\binom{37}{0}\left(\frac{1}{6}\right)^0\left(\frac{5}{6}\right)^{37}-\binom{37}{1}\left(\frac{1}{6}\right)^1\left(\frac{5}{6}\right)^{36} = 0.9901$$

59. 114; because

$$1-\binom{114}{0}(0.00045)^0(0.99955)^{114} = 0.0500$$

60. Let "success" = "becomes centenarian." Then $p = 0.075$, $q = 0.925$, n is unknown.

$$\Pr(X \ge 2) = 1-\Pr(X=0)-\Pr(X=1)$$
$$= 1-(0.925)^n-\binom{n}{1}(0.075)^1(0.925)^{n-1}$$
$$= 1-(0.925)^n-n(0.075)(0.925)^{n-1}$$

$\Pr(X \ge 2) > 0.9 \Leftrightarrow$

$(0.925)^n + n(0.075)(0.925)^{n-1} < 0.1 \Leftrightarrow$

$(0.925)^{n-1}(0.925+n(0.075)) < 0.1$

The smallest value of n for which this occurs is $n = 51$.

61. Using a TI83 graphing calculator input the following steps:

binomcdf(100, .5, 60) – binomcdf(100, .5, 39)

the answer will be approximately 0.9648.