1. **a.** The set of all possible pairs: {PN, PD, PQ, PH, ND, NQ, NH, DQ, DH, QH}

   b. The set of pairs containing an even number of cents: {PN, PQ, NQ, DH}

2. a. A male junior is elected

   b. A female junior is not elected

   c. A male or a junior is elected

3. $\Pr(E \cap F) = 0.4 + 0.3 - 0.5 = 0.2$

4. $\Pr(E \cup F) = 0.5 + 0.3 = 0.8$

9. $26\% = \frac{13}{50}$; $a = 13$ and $a + b = 50$, so $b = 37$. The odds the person selected is under 18 are 13 to 37; the odds the person selected is 18 or older are 37 to 13.

10. $25\% = \frac{1}{4}$; $a = 1$ and $a + b = 4$, so $b = 3$. The odds are 1 to 3.

11. $\frac{5}{10} \times \frac{4}{9} \times \frac{3}{8} = \frac{1}{12}$

12. There are $C(4, 2) = 6$ ways to choose a "pair" of socks from the drawer; of these, 2 have the same color, so the probability is $\frac{2}{6} = \frac{1}{3}$.

13. $$\begin{aligned} 1 - \frac{C(5,1) \cdot C(95,3) + C(95,4)}{C(100,4)} &= 1 - \frac{5 \cdot 138,415 + 3,183,545}{3,921,225} \\ &= 1 - \frac{3,875,620}{3,921,225} \\ &\approx 0.01163 \end{aligned}$$

14. $\frac{C(5,1) \cdot C(4,2)}{C(9,3)} = \frac{5 \cdot 6}{84} = \frac{30}{84} = \frac{5}{14} \approx 0.3571$

15. **a.** $\Pr(\text{prepared every question}) = \frac{C(8,6)}{C(10,6)} = \frac{28}{210} = \frac{2}{15}$

    b. $\Pr(\text{not prepared on test}) = \frac{C(8,4)}{C(10,6)} = \frac{70}{210} = \frac{1}{3}$

16. a. $$\begin{aligned} \Pr(\text{winning}) &= \Pr(7) + \Pr(11) \\ &= \frac{6}{36} + \frac{2}{36} \\ &= \frac{8}{36} = \frac{2}{9} \end{aligned}$$

$$\begin{aligned}\Pr(\text{losing}) &= \Pr(2)+\Pr(3)+\Pr(12)\\ &= \frac{1}{36}+\frac{2}{36}+\frac{1}{36}\\ &= \frac{4}{36}=\frac{1}{9}\end{aligned}$$

b. $\Pr(\text{winning}) = \Pr(6) = \frac{5}{36}$

$\Pr(\text{losing}) = \Pr(7) = \frac{6}{36} = \frac{1}{6}$

17. $1-\left(\frac{1}{2}\right)^5 = \frac{31}{32}$

18. Pr(no tails) + Pr(1 tail each) + Pr(2 tails each) + Pr(3 tails each) $=\left(\frac{1}{8}\right)^2+\left(\frac{3}{8}\right)^2+\left(\frac{3}{8}\right)^2+\left(\frac{1}{8}\right)^2=\frac{5}{16}$

19. $\frac{2}{7}\times\frac{1}{6}=\frac{1}{21}$

20. $\Pr(\text{select 4 winning teams}) = \left(\frac{1}{17}\right)^4 = \frac{1}{83,521}$

$\text{Odds against} = 1-\frac{1}{83,521}:\frac{1}{83,521} = 83,520:1$

21. **a.** $\left(\frac{1}{36}\right)^3$

b. $\left(\frac{1}{10}\right)^4$

c. $\left(\frac{26}{36}\right)^3\left(\frac{1}{2}\right)^4 = \left(\frac{17,576}{46,656}\right)\left(\frac{1}{16}\right) = \frac{2197}{93,312}$

22. **a.** Pr(all three cards are aces)

$= pr(A_1 \cap A_2 \cap A_3) = \frac{4}{52}\cdot\frac{4}{52}\cdot\frac{4}{52} = \frac{1}{2197}$

b. Pr (at least one ace) = 1 – Pr(no aces)

$= 1-\left(\frac{48}{52}\right)^3 = 1-\left(\frac{12}{13}\right)^3 = \frac{469}{2197}$

23. Record the six consecutive outcomes; there are 6^6 possibilities, of which 6! contain each number exactly once. Hence the probability is $\frac{6!}{6^6}=\frac{5}{324}\approx 0.0154.$

24. $\Pr(4\text{ different numbers})=\frac{6\cdot 5\cdot 4\cdot 3}{6\cdot 6\cdot 6\cdot 6}=\frac{5}{18},$ so the odds in favor of getting four different numbers are

$\frac{5}{18}:1-\frac{5}{18}=5\text{ to }13.$

25. $1-\frac{365\cdot 364\cdot 363\cdot 362\cdot 361}{365^5}\approx 0.0271.$

26. $1-\frac{6\cdot 5\cdot 4}{7\cdot 7\cdot 7}=1-\frac{120}{343}=\frac{223}{343}\approx 0.6501$

27. $\Pr(E\mid F)=\frac{.4+.3-.5}{.3}=\frac{.2}{.3}=\frac{2}{3}$

28. $\Pr(F)=\frac{\frac{1}{10}}{\frac{1}{7}}=\frac{7}{10}$

29. Pr(at least one tail appears in three coins given that at least one head appeared)

1) Look at the sample space: HHH, HHT, HTH, HTT, THH, THT, TTH, TTT.

2) The first seven outcomes have at least one head.

3) Pr (one or more tails in the first seven outcomes) = 6/7

30. $\Pr\left(\text{one }3\middle|\text{no doubles}\right)=\frac{5+5}{30}=\frac{1}{3}$

31. **a.** $\Pr(\text{employed})=\frac{147.48}{154.81}\approx 0.9527$

b. $\Pr(\text{Male})=\frac{80.74}{154.81}\approx 0.5215$

c. $\Pr(\text{Female}|\text{employed})=\frac{70.70}{147.48}\approx 0.4794$

d. $\Pr(\text{employed}|\text{Female})=\frac{70.70}{74.07}\approx 0.9545$

32. **a.** $\Pr(\text{eng})=\frac{15}{50}=\frac{3}{10}$

b. $\Pr(\text{Eng}|\text{public})=\frac{10}{25}=\frac{2}{5}$

c. $\Pr(\text{Private}|\text{eng})=\frac{5}{15}=\frac{1}{3}$

d. $\Pr(\text{Public}|\text{eng}) = \dfrac{10}{15} = \dfrac{2}{3}$

33. $(0.08)(0.5) = 0.04$

34. $$\Pr(R_1 \cap G_2 \cap G_3 \cap R_4) = \frac{10}{30}\cdot\frac{20}{29}\cdot\frac{19}{28}\cdot\frac{9}{27} = \frac{95}{1827} \approx 0.0520$$

35. No; $\Pr\left(F \middle| E\right) = \dfrac{1}{6} > \Pr(F) = \dfrac{5}{36}$

36. No; $\Pr(F|E) = \dfrac{3}{4} \neq \Pr(F) = \dfrac{1}{2}$

37. Yes

38. Yes

39. **a.** $\dfrac{1}{4}\times\dfrac{1}{3} = \dfrac{1}{12}$

b. $\dfrac{1}{4}+\dfrac{1}{3}-\dfrac{1}{12} = \dfrac{1}{2}$

40. a. $(0.4)(0.75) = 0.3$

b. $0.4 + 0.75 - 0.3 = 0.85$

41. $\Pr(A \cup B) = \dfrac{1}{2}$; $\Pr(A' \cap B) = \dfrac{1}{3}$

$\Pr(A) + \Pr(A' \cap B) = \Pr(A \cup B)$

$\Pr(A) = \dfrac{1}{2} - \dfrac{1}{3} = \dfrac{1}{6}$

42. $\Pr(A \text{ and } B) = (0.4)(0.3) = 0.12$

$\Pr(A \text{ only}) = 0.3 - 0.12 = 0.18$

$\Pr(B \text{ only}) = 0.4 - 0.12 = 0.28$

$\Pr(\text{exactly } 1) = 0.18 + 0.28 = 0.46$

43. $\left(\dfrac{1}{3}\times\dfrac{2}{3}\right)+\left(\dfrac{2}{3}\times\dfrac{1}{3}\right) = \dfrac{4}{9}$

44. $\Pr(3 \text{ is drawn on 1st or 2nd draw}) = \dfrac{1}{3} + \left(\dfrac{1}{3}\right)\left(\dfrac{1}{2}\right) + \left(\dfrac{1}{3}\right)\left(\dfrac{1}{2}\right) = \dfrac{2}{3}$

45. You should switch. If you stay with your original choice your probability of winning remains $\frac{1}{3}$, whereas if you switch you lose only if your original choice was correct, so your probability of winning is $\frac{2}{3}$.

46. $(0.60 \times 0.90) + (0.40 \times 0.05) = 0.56 = 56\%$

47. $\Pr\left(\text{both parents left-handed} \middle| \text{child left-handed}\right)$

$$= \frac{\Pr(\text{all three left-handed})}{\Pr(\text{child left-handed})}$$

$$= \frac{0.4 \times 0.25 \times 0.25}{(0.4 \times 0.25 \times 0.25) + (0.2 \times 0.25 \times 0.75) + (0.2 \times 0.75 \times 0.25) + (0.1 \times 0.75 \times 0.75)}$$

$$= \frac{4}{25}$$

48. $$\Pr\left(\text{correct} \middle| \text{rejected}\right) = \frac{\Pr(\text{correct}) \times \Pr\left(\text{rejected} \middle| \text{correct}\right)}{\Pr(\text{correct}) \times \Pr\left(\text{rejected} \middle| \text{correct}\right) + \Pr(\text{incorrect}) \times \Pr\left(\text{rejected} \middle| \text{incorrect}\right)}$$

$$= \frac{0.80 \times 0.05}{(0.80 \times 0.05) + (0.20 \times 0.90)} = \frac{2}{11}$$

49. $$\Pr\left(C \middle| \text{wrong}\right) = \frac{\Pr(C) \times \Pr\left(\text{wrong} \middle| C\right)}{\Pr(C) \times \Pr\left(\text{wrong} \middle| C\right) + \Pr(A) \times \Pr\left(\text{wrong} \middle| A\right) + \Pr(B) \times \Pr\left(\text{wrong} \middle| B\right)}$$

$$= \frac{0.20 \times 0.05}{(0.20 \times 0.05) + (0.40 \times 0.02) + (0.40 \times 0.03)}$$

$$= \frac{0.01}{0.03}$$

$$= \frac{1}{3}$$

50. If n is the number of dragons, then $\frac{\text{\# of heads on 1-headed dragons}}{\text{\# of heads}} = \frac{\frac{n}{3}}{\frac{n}{3} + 2 \cdot \frac{n}{3} + 3 \cdot \frac{n}{3}} = \frac{1}{6}$

51. If E and F are independent events, then the outcome of F does not affect the outcome of E and vice versa. If the outcome of F does not affect the outcome of E, then neither would the outcome of F'.
Example: Let E = event you get a six on a die and F = event you get a H on a coin toss. Now, E and F are independent. F' is the event you get a T on a coin toss. The events E and F' are independent, so whether you get a H or T on the coin toss does not affect the probability of the six on a die.

52. If you know $\Pr(E \cup F)$, then you can compute
$\Pr(E \cap F) = \Pr(E) + \Pr(F) - \Pr(E \cup F)$. Alternatively, if you know that E and F are independent events, then you can compute
$\Pr(E \cap F) = \Pr(E) \cdot \Pr(F)$.