Section 5.7 Normal Approximation to the Binomial Distribution

Math 141

Main ideas

We can approximate the binomial distribution with the normal approximation. The larger n is, the better the approximation.

Recall for the binomial distribution that $\mu = np$ and $\sigma = \sqrt{np(1-p)}$. Also recall the notation that $\binom{n}{k} = C(n,k)$.

Problems

First, recall Problems 3 and 4 from the previous handout. Notice how as n is larger, the distribution looks more and more like a normal distribution. Also see Figures 5.7.1 –4. Finally, recall that when we have normal distribution we need the mean and standard deviation. This is one reason we were interested in μ and σ for binomial distribution problems (like shooting free throws).

1. For a 70% free throw shooter who takes 100 shots, what is the probability of making between 60 and 80 shots? Approximate this answer using a normal distribution. (I've also included a zoomed-in view around 60 and 80.)

$\mu = 100(.70) = 70 \quad \sigma = \sqrt{100(.70)(.30)} \approx 4.58$

$$\Pr(\underset{59.5}{60} \le X \le \underset{80.5}{80})$$

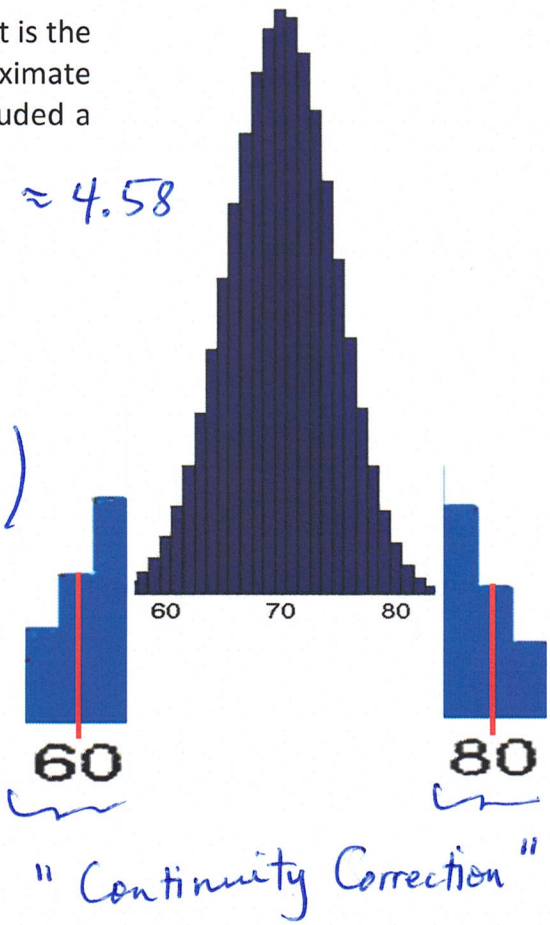
$$= \Pr\left(\frac{59.5-70}{4.58} \le z \le \frac{80.5-70}{4.58}\right)$$

$$\approx \Pr(-2.30 \le z \le 2.30)$$

$$= A(2.30) - A(-2.30)$$

$$= .9893 - .0107$$

$$= .9786$$



"Continuity Correction"

2. The incidence rate of color blindness among men in a certain country is 20%. A sample of 70 men is taken. What is the probability that 14 or more of the men are color blind? Find this value exactly.

$$Pr(X = 14) + \cdots + Pr(X = 70)$$
$$= C(70,14)(.20)^{14}(.80)^{56} + \cdots + C(70,70)(.20)^{70}(.80)^{0}$$
$$= .5476 \text{ using Excel}$$

3. Use the normal curve to approximate the probability of the Problem 2.

$$\mu = 70(.20) = 14 \qquad \sigma = \sqrt{70(.20)(.80)} \approx 3.35$$

$$Pr\left(\frac{13.5-14}{3.35} \le z \le \frac{70.5-14}{3.35}\right) \approx Pr(-.15 \le z \le 16.88)$$

$$\approx Pr(-.15 \le z) = 1 - Pr(z \le -.15) = 1 - .4404$$
$$= .5596$$

4. For Problem 2, find the probability that exactly 14 of the 70 men are color blind. Find this value exactly.

$$C(70,14)(.20)^{14}(.80)^{56} = .1185$$

5. Use the normal curve to approximate the probability of Problem 4.

$$Pr\left(\frac{13.5-14}{3.35} \le z \le \frac{14.5-14}{3.35}\right) = Pr(-.15 \le z \le .15)$$
$$= .5596 - .4404 = .1192$$

6. An airline accepts 150 reservations for a flight on an airplane that holds 140 passengers. If the probability of a passenger for this flight cancelling is .14, estimate the probability that one or more passengers will be bumped. Let X be the number who do not show up for the flight.

X	# bumped
0	10
1	9
⋮	⋮
9	1
10	0

$$Pr(X < 10) = Pr\left(z \le \frac{9.5-(150)(.14)}{\sqrt{150(.14)(.86)}}\right) \quad 9.5$$

$$\approx Pr(z \le -2.70) = .0035$$