Section 5.4 The Mean

Math 141

Main ideas

Sample mean $\bar{x} = x_1\left(\frac{f_1}{n}\right) + x_2\left(\frac{f_2}{n}\right) + \cdots + x_r\left(\frac{f_r}{n}\right)$ where $\frac{f_i}{n}$ is the fraction of the time each outcome x_i did occur.

Population mean $\mu = x_1\left(\frac{f_1}{N}\right) + x_2\left(\frac{f_2}{N}\right) + \cdots + x_r\left(\frac{f_r}{N}\right)$ where $\frac{f_i}{N}$ is the fraction of the time each outcome x_i did occur.

Expected value is $E(X) = x_1p_1 + x_2p_2 + \cdots + x_Np_N$ where p_i is the fraction of the time each outcome x_i should occur, and:

- Is the average outcome that will occur if the experiment is repeated multiple times and what happens follows what should happen, i.e. it matches the probability distribution.
- Is generally not actually one of the possible values that could occur.
- Is always between the minimum and maximum possible values that could occur.

Given n binomial trials with probability of success p, where X is the number of successes, then the expected value of X is $\boldsymbol{E(X) = np}$.

Problems

1. Suppose I am interested in the number of years each student in class has been at Pepperdine, and I get the following results:

   1: 6 2: 7 3: 2 4: 0

   Sample mean. The average number of years each student in our class has been at Pepperdine is:

   $\bar{x} = \frac{1\cdot 6 + 2\cdot 7 + 3\cdot 2 + 4\cdot 0}{15} = 1\left(\frac{6}{15}\right) + 2\left(\frac{7}{15}\right) + 3\left(\frac{2}{15}\right) + 4\left(\frac{0}{15}\right)$

2. Suppose you flip four coins 111 times:

Number of heads	Number of times outcome occurred	Fraction of time outcome *did* occur
0	9	9 / 111
1	23	23 / 111
2	37	37 / 111
3	32	32 / 111
4	10	10 / 111

What should have happened:

Number of heads	Fraction of time outcome *should* occur
0	1/16
1	4/16
2	C(4,2)/16 = 6/16
3	4/16
4	1/16

Sample mean: the average number of heads that actually occured was:

$\bar{x} = 0\left(\frac{9}{111}\right) + 1\left(\frac{23}{111}\right) + \cdots + 4\left(\frac{10}{111}\right)$

$= 2.10$

Expected value: The average number of heads that should have occured is:

$E(X) = 0\left(\frac{1}{16}\right) + 1\left(\frac{4}{16}\right) + 2\left(\frac{6}{16}\right) + 3\left(\frac{4}{16}\right) + 4\left(\frac{1}{16}\right) = 2$

3. Pay \$1 to play a game: flip a coin until you get heads or until you flip the coin four times. You win \$.50 for each flip (you are guaranteed at least one flip: the first flip).

Suppose the following did occur:

Outcome	Winnings x_i	Occurrences f_i
H	-.50	7
TH	0	6
TTH	.50	5
TTTH	1.00	1
TTTT	1.00	3

Sample mean: the average winnings was

$\bar{x} = (-.50)\left(\frac{7}{22}\right) + \cdots + (1.00)\left(\frac{3}{22}\right)$
$= .1364$

What should occur, on average:

Outcome	Winnings x_i	Probability p_i
H	-.50	1/2
TH	0	1/4
TTH	.50	1/8
TTTH	1.00	1/16
TTTT	1.00	1/16

Expected value: the expected average winnings

$E(X) = (-.50)\left(\frac{1}{2}\right) + \cdots + (1.00)\left(\frac{1}{16}\right)$
$= -.0625$

4. Expected return on investments.

Let X_A = return for investment A

k	$\Pr(X_A = k)$
\$ 1000	.20
\$ 2000	.50
\$ 3000	.30

Let X_B = return for investment B

k	$\Pr(X_B = k)$
-\$ 1000	.30
\$ 0	.10
\$ 4000	.60

$E(X_A) = 1000(.20) + 2000(.50) + 3000(.30) = 2100$

$E(X_B) = -1000(.30) + 0(.10) + 4000(.60) = 2100$

5. Life insurance for couple. Policy is for 5 years.
Payout: \$ 0 if both are still alive.
\$ 10,000 if one dies, the other lives.
\$ 15,000 if both die.
Pr(Man lives ≥ 5 years) = .90.
Pr(Woman lives ≥ 5 years) = .95.
Let X be payout to couple.

Man	Woman	Payout	Probability
Live	Live	0	(.90)(.95) = .855
Live	Not	10 000	(.90)(.05) = .045
Not	Live	10 000	(.10)(.95) = .095
Not	Not	15 000	(.10)(.05) = .005

$E(X) = 0(.855) + \cdots + 15000(.005)$
$= 1475$

6. If you are a 70% free throw shooter ($p = .70$) free throw shooter and you shoot 10 shots ($n = 10$), and where X is the number of shots made, then (using our work from Class Handout 5.3) we have

$E(X) = 0(.00006) + 1(.0001) + \cdots + 10(.0282) = 7$

Or (much more simply) $E(X) = 10(.70) = 7.$