Sections 4.5 and 4.6 Continued for Medical Testing

Math 141

Main ideas

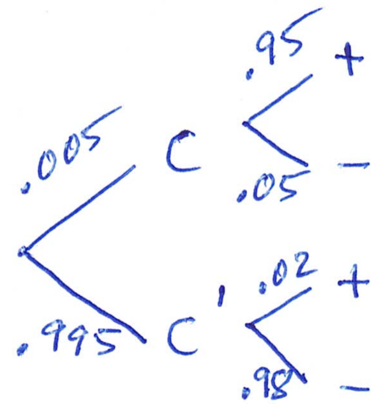
Sensitivity is $\Pr(+|C)$.

Specificity is $\Pr(-|C')$.

Positive predictive value is $\Pr(C|+)$.

Negative predictive value is $\Pr(C'|-)$.

Problems



1. A medical test checks for a certain condition.
   95% of those with condition test positive
   5% of those with condition test negative
   2% of those without condition test positive
   98% of those without condition test negative.

   In the general population, from past experience:
   0.5% of the population has the condition
   99.5% of the population does not have the condition.

$$\Pr(C|+) = \frac{\Pr(C \text{ and } +)}{\Pr(+)} = \frac{\Pr(C \text{ and } +)}{\Pr(C \text{ and } +)+\Pr(\text{not } C \text{ and } +)} = \frac{(.005)(.95)}{(.005)(.95)+(.995)(.02)} = \frac{.00475}{.02465} \approx .1927$$

$$\Pr(\text{not } C|+) = \frac{\Pr(\text{not } C \text{ and } +)}{\Pr(+)} = \frac{\Pr(\text{not } C \text{ and } +)}{\Pr(C \text{ and } +)+\Pr(\text{not } C \text{ and } +)} = \frac{(.995)(.02)}{(.005)(.95)+(.995)(.02)} = \frac{.01990}{.02465} \approx .8073$$

$$\Pr(C|-) = \frac{\Pr(C \text{ and } -)}{\Pr(-)} = \frac{\Pr(C \text{ and } -)}{\Pr(C \text{ and } -)+\Pr(\text{not } C \text{ and } -)} = \frac{(.005)(.05)}{(.005)(.05)+(.995)(.98)} = \frac{.00025}{.97535} \approx .0003$$

$$\Pr(\text{not } C|-) = \frac{\Pr(\text{not } C \text{ and } -)}{\Pr(-)} = \frac{\Pr(\text{not } C \text{ and } -)}{\Pr(C \text{ and } -)+\Pr(\text{not } C \text{ and } -)} = \frac{(.995)(.98)}{(.005)(.05)+(.995)(.98)} = \frac{.97510}{.97535} \approx .9997$$

Results of test

	No Test	Positive +	Negative −
$Pr(C)$	.005	.1927	.0003
$Pr(\text{not } C)$	.995	.8073	.9997

Notice sum of $\Pr(C)$ and $\Pr(\text{not } C)$ in each case.

Another view of why this occurs: "Natural Frequencies." Same info as before:

95% of those with the condition test positive

5% of those with condition test negative

2% of those not with the condition test positive

98% of those not with the condition test negative.

In the general population:

0.5% of the population has the condition

99.5% of the population does not have the condition.

Condition	Test	Persons of this type ("Natural Frequencies")
Yes	+	(.005)(1,000,000)(.95) = (.95)(5000) = 4750
Yes	–	(.005)(1,000,000)(.05) = (.05)(5000) = 250
No	+	(.995)(1,000,000)(.02) = (.02)(995000) = 19900
No	–	(.995)(1,000,000)(.98) = (.98)(995000) = 975000

$$\Pr(C) = \frac{\text{\# with condition}}{\text{total \#}} = \frac{5000}{1000000} = .005$$

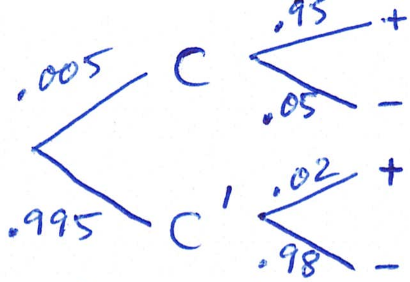
$$\Pr(C|+) = \frac{\text{\# with condition who tested +}}{\text{total \# who tested +}}$$

$$= \frac{(.005)(1000000)(.95)}{(.005)(1000000)(.95) + (.995)(1000000)(.02)}$$

$$= \frac{4750}{4750 + 19900}$$

$$= \frac{4750}{24650}$$

$$= .1927$$



2. Effects of changing values:

Pr(C) = .005
Pr(C') = .995
Pr(+|C) = .95
Pr(−|C) = .05
Pr(+|C') = .02
Pr(−|C') = .98

$$\Pr(C|+) = \frac{(.005)(.95)}{(.005)(.95)+(.995)(.02)} = .1927$$

Pr(C) = .05
Pr(C') = .95
Pr(+|C) = .95
Pr(−|C) = .05
Pr(+|C') = .02
Pr(−|C') = .98

$$\Pr(C|+) = \frac{(.05)(.95)}{(.05)(.95)+(.95)(.02)} = .7143$$

Pr(C) = .005
Pr(C') = .995
Pr(+|C) = .5
Pr(−|C) = .5
Pr(+|C') = .5
Pr(−|C') = .5

$$\Pr(C|+) = \frac{(.005)(.5)}{(.005)(.5)+(.995)(.5)} = .005 = \Pr(C)$$

Pr(C) = .005
Pr(C') = .995
Pr(+|C) = 1
Pr(−|C) = 0
Pr(+|C') = .02
Pr(−|C') = .98

$$\Pr(C|+) = \frac{(.005)(1)}{(.005)(1)+(.995)(.02)} = .2008$$

Pr(C) = .005
Pr(C') = .995
Pr(+|C) = .95
Pr(−|C) = .05
Pr(+|C') = 0
Pr(−|C') = 1

$$\Pr(C|+) = \frac{(.005)(.95)}{(.005)(.95)+(.995)(0)} = 1$$

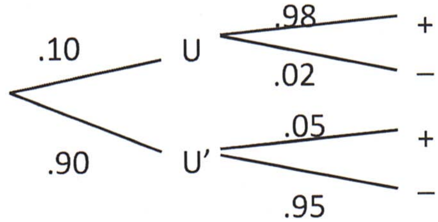
Pr(C) = .005
Pr(C') = .995
Pr(+|C) = .95
Pr(−|C) = .05
Pr(+|C') = .02
Pr(−|C') = .98

$$\Pr(C|+,-) = \frac{(.005)(.95)(.05)}{(.005)(.95)(.05)+(.995)(.02)(.98)} = .012$$

Pr(C) = .005
Pr(C') = .995
Pr(+|C) = .95
Pr(−|C) = .05
Pr(+|C') = .05
Pr(−|C') = .95

$$\Pr(C|+,-) = \frac{(.005)(.95)(.05)}{(.005)(.95)(.05)+(.995)(.05)(.95)} = .005 = \Pr(C)$$

3. A drug-testing lab produces false negative results 2% of the time and false positives 5% of the time. Suppose that the laboratory has been hired by a company at which they estimate that 10% of the employees use drugs. Let U be "is drug user," + be "tests positive," and – be "tests negative."



$\Pr(+) = (.10)(.98) + (.90)(.05) = .1430$

$\Pr(U \text{ and } ++) = (.10)(.98)^2 = .0960$

$\Pr(++) = (.10)(.98)^2 + (.90)(.05)^2 = .0983$

$\Pr(+++) = (.10)(.98)^3 + (.90)(.05)^3 = .0942$

$\Pr(+\, n \text{ times}) = (.10)(.98)^n + (.90)(.05)^n \to 0 \quad as\, n \to \infty.$

$$\Pr(U|+) = \frac{(.10)(.98)}{(.10)(.98) + (.90)(.05)} = .6853$$

$$\Pr(U|++) = \frac{(.10)(.98)^2}{(.10)(.98)^2 + (.90)(.05)^2} = .9766$$

$$\Pr(U|+\, n\, times) = \frac{\Pr(U \text{ and } +\, n \text{ times})}{\Pr(+\, n \text{ times})} = \frac{(.10)(.98)^n}{(.10)(.98)^n + (.90)(.05)^n} \to 1 \quad as\, n \to \infty.$$

$$\Pr(U|+,-) = \frac{(.10)(.98)(.02)}{(.10)(.98)(.02) + (.90)(.05)(.95)} = .0438$$

$$\Pr(++|+) = \frac{\Pr(++ \text{ and } +)}{\Pr(+)} = \frac{\Pr(++)}{\Pr(+)} = \frac{.0983}{.1430} = .6874$$

$$\Pr(+++|++) = \frac{\Pr(+++)}{\Pr(++)} = \frac{.0942}{.0983} = .9587$$

$$\Pr(+\ again\,|+\ n \text{ times}) = \frac{\Pr(+\ \ n+1 \text{ times})}{\Pr(+\ \ n \text{ times})} = \frac{(.10)(.98)^{n+1} + (.90)(.05)^{n+1}}{(.10)(.98)^n + (.90)(.05)^n} \to .98 \quad as\, n \to \infty.$$