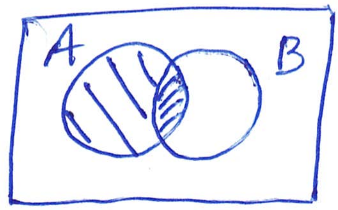
Section 4.5 Tree Diagrams

Math 141

Main ideas



Since

$$A = \quad (A \cap B) \cup \quad (A \cap B')$$

then

$$\Pr(A) = Pr(A \cap B) + Pr(A \cap B') = \Pr(B)\Pr(A|B) + \Pr(B')\Pr(A|B')$$

and similarly

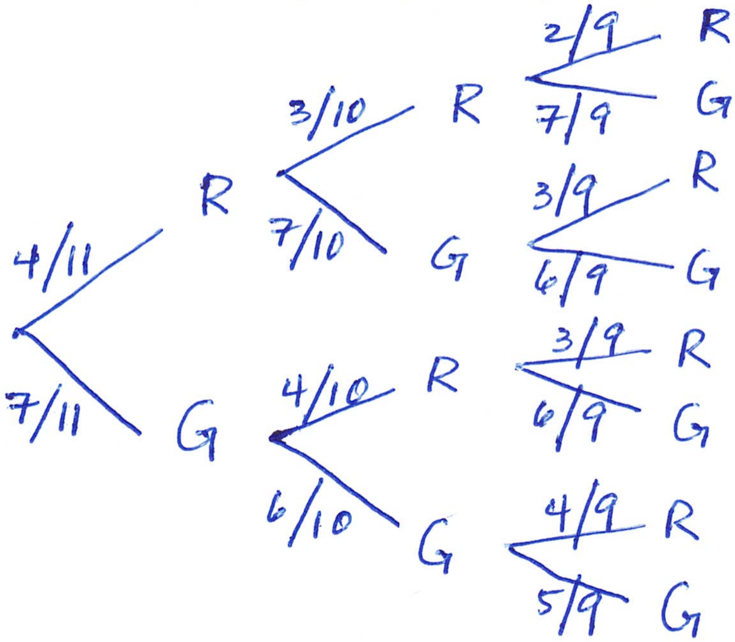
$$\Pr(B) = Pr(A \cap B) + Pr(A' \cap B) = \Pr(A)\Pr(B|A) + \Pr(A')\Pr(B|A').$$

Tree diagrams help us organize and visualize and better work with information.

Problems

1. A basket contains 11 balls: 4 red and 7 green. We will select three balls.

   Create a tree diagram representing the 8 possible outcomes.



One ball. Select one ball. What is the probability it is red?

$\frac{4}{11}$

Two balls. Select one ball, and then a second ball.

Pr(R2 | R1) = $\frac{3}{10}$

Pr(R2 | G1) = $\frac{4}{10}$

What is the probability that the second ball is red if we don't know the color of the first ball (i.e. we didn't look at the first ball when we selected it)?

$$\Pr(R2) = \Pr(R1 \cap R2) + \Pr(G1 \cap R2)$$
$$= \Pr(R1)\Pr(R2|R1) + \Pr(G1)\Pr(R2|G1)$$
$$= \frac{4}{11}\cdot\frac{3}{10} + \frac{7}{11}\cdot\frac{4}{10} = \frac{40}{110} = \frac{4}{11} = \Pr(R1)$$

What is the probability the third ball is red if we don't know the color of the first two balls?

$$\Pr(R3) = \Pr(R1 \cap R2 \cap R3) + \cdots$$
$$= \Pr(R1)\Pr(R2|R1)\Pr(R3|R1, R2) + \cdots$$
$$= \frac{4}{11}\cdot\frac{3}{10}\cdot\frac{2}{9} + \frac{4}{11}\cdot\frac{7}{10}\cdot\frac{3}{9} + \frac{7}{11}\cdot\frac{4}{10}\cdot\frac{3}{9} + \frac{7}{11}\cdot\frac{6}{10}\cdot\frac{4}{9}$$
$$= \cdots = \frac{360}{990} = \frac{4}{11}$$

2. What are the best possible tennis serve combos, given three serve types? (See online handout on tennis serve for more detail.)

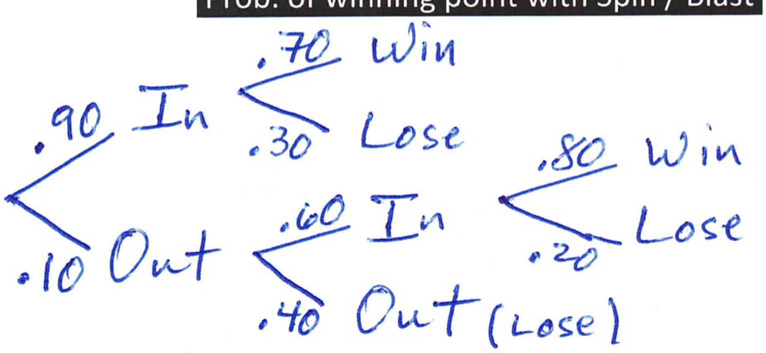
Three possible serves:

Serve option	Prob. serve will be in	Prob. you will win the point
Gentle	1.00	0.55
Spin	0.90	0.70
Blast	0.60	0.80

Seven possible serve combinations:

First / second serve	Probability of winning point	Rank
Gentle	.550	7
Spin / Gentle	.685	4
Spin / Spin	.693	3
Spin / Blast	.678	5
Blast / Gentle	.700	2
Blast / Spin	.732	1
Blast / Blast	.672	6

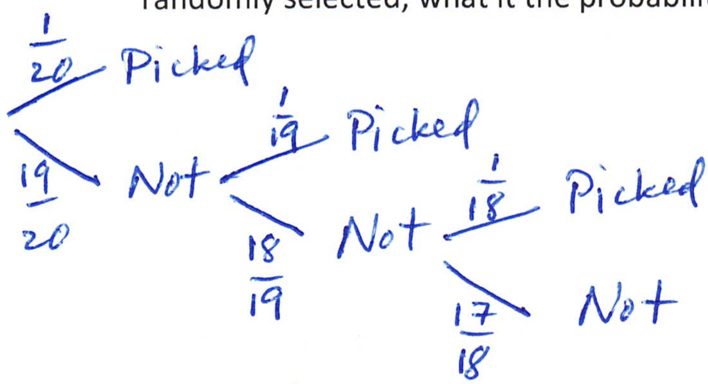
Prob. of winning point with Spin / Blast



$$\Pr(\text{win}) = (.9)(.7) + (.1)(.6)(.8) = .678$$

See more results online at class homepage "Section 4.5 Tennis serves."

3. In a group of 20 people, there is one named Marcus Aurelius. If the three people are randomly selected, what it the probability that Marcus is chosen as one of the three?

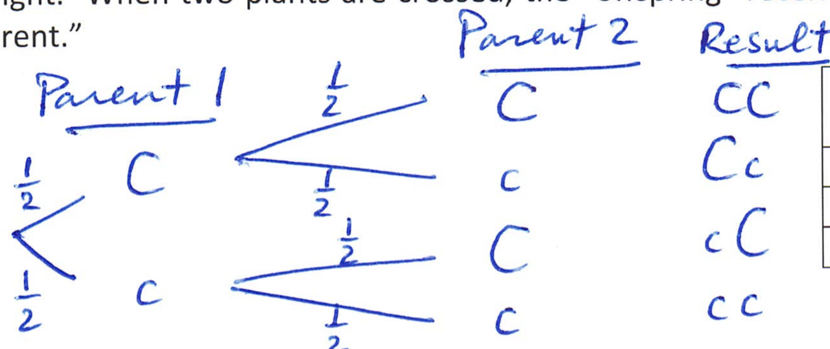


Compare to:

$$\frac{C(19,2)}{C(20,3)} = \dots = \frac{3}{20}.$$

$$\frac{1}{20} + \frac{19}{20}\cdot\frac{1}{19} + \frac{19}{20}\cdot\frac{18}{19}\cdot\frac{1}{18} = \frac{1}{20} + \frac{1}{20} + \frac{1}{20} = \frac{3}{20}$$

4. For a certain type of pea plant, the color of the flower produced by the plant—either red (the dominant color) or white (recessive)—is determined by a pair of genes. Each gene is either *C* (dominant) or *c* (recessive). The three possible gene pairs are shown at right. When two plants are crossed, the "offspring" receives one gene from each "parent."

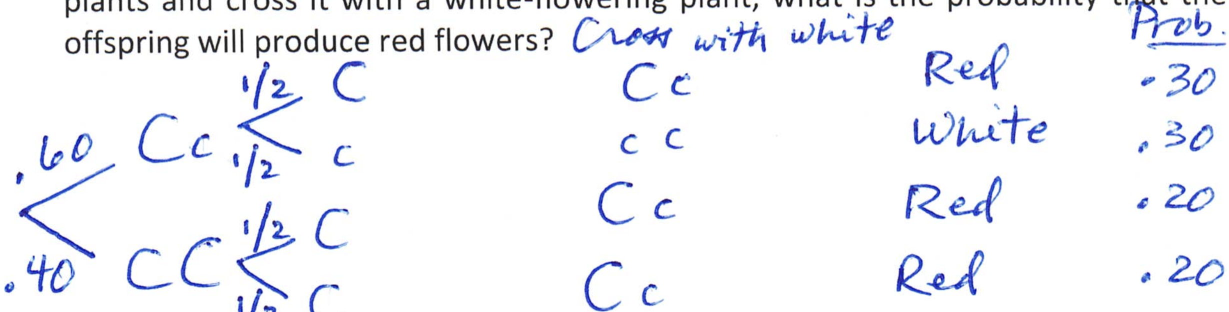


Gene pair	Flower color	
CC	Red	1/4
Cc or cC	Red	2/4
cc	White	1/4

a. Suppose you cross two pea plants of type Cc. What is the probability that the offspring has white flowers? Red flowers?

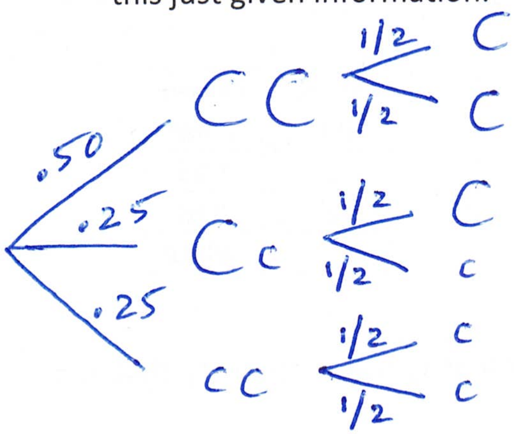
$\frac{1}{4}$ $\frac{3}{4}$

b. Suppose you have a batch of red-flowering plants, of which 60% have genotype Cc and 40% CC. (Why would none be of type cc?) If you randomly select one of these plants and cross it with a white-flowering plant, what is the probability that the offspring will produce red flowers?



$$(.60)(\tfrac{1}{2}) + (.40)(\tfrac{1}{2}) + (.40)(\tfrac{1}{2}) = .70$$

Suppose now that there are 100 plants of type *CC*, 50 of type *Cc* and 50 of type *cc*. Suppose you randomly select one of these 200 plants and cross it with a white-flowering (*cc*) plant. To help answer the next two questions, create a tree diagram for this just given information.



Cross with white		Of 200
Cc	Red	50
Cc	Red	50
Cc	Red	25
cc	White	25
cc	White	25
cc	White	25

c. What is the probability that the offspring will produce white flowers? Red flowers?

White : $(.25)(\frac{1}{2}) + (.25)(\frac{1}{2}) + (.25)(\frac{1}{2}) = .375$ $\quad \frac{75}{200}$

Red : $(.50)(\frac{1}{2}) + (.50)(\frac{1}{2}) + (.25)(\frac{1}{2}) = .625$ $\quad \frac{125}{200}$

d. If the offspring produces red flowers, what is the probability that the selected plant was type *Cc*?

$$Pr(Cc \mid Red) = \frac{Pr(Cc \text{ and } Red)}{Pr(Red)}$$

$$= \frac{(.25)(\frac{1}{2})}{.625}$$

$$= .20 \leftarrow \frac{25 \text{ Cc and Red}}{125 \text{ Red}}$$

If time, work HW 4.5.25.