Section 4.3 Calculating Probabilities of Events
Math 141

Main idea

Pr(E) + Pr(E′) = 1 which means that **Pr(E) = 1 – Pr(E′)**.

Problems

1. Three persons with birthdays.

   How many ways for them to have their birthdays? $365 \cdot 365 \cdot 365$

   Find the probabilities that

   All 3 have the same birthday: $365 \cdot 1 \cdot 1 / 365^3$

   2 are the same, 1 is different: $C(3,2) \cdot 365 \cdot 364 / 365^3$

   All 3 have different birthdays: $365 \cdot 364 \cdot 363 / 365^3$

   Notice that these three probabilities add up to 1: $365^3 / 365^3$

$$365 \cdot 1 \cdot 1 + 3 \cdot 365 \cdot 364 + 365 \cdot 364 \cdot 363 = \cdots = 365^3$$

2. Given four persons, what are the probabilities that

   All 4 have the same birthday (AAAA):
   $365 \cdot 1 \cdot 1 \cdot 1 / 365^4$

   3 are the same, 1 is different (AAAB):
   $C(4,3) \cdot 365 \cdot 364 / 365^4$

   2 are the same, the other 2 are different (AABC):
   $C(4,2) \cdot 365 \cdot 364 \cdot 363 / 365^4$

   2 are the same, the other 2 are different and same as each other (AABB):
   $C(4,2)/2 \cdot 365 \cdot 364 / 365^4$

   All 4 are different (ABCD):
   $365 \cdot 364 \cdot 363 \cdot 362 / 365^4$

   Notice:

   Sum of above probabilities is 1

3. What is the probability that of 50 persons, all have *different* birthdays?

$$\frac{365 \cdot 364 \cdot 363 \cdot \cdots \cdot 316}{365 \cdot 365 \cdot 365 \cdot \cdots \cdot 365}$$

4. What is the probability that of 50 persons, 2 or more of them share the *same* birthday?

$1 -$

5. A politician knows that a 9-person committee vote is stacked against her 6 to 3. However, she has the option of letting a randomly selected subcommittee decide the issue. Find the probability that she wins the vote with subcommittee sizes of

1: $\dfrac{C(3,1) \cdot C(6,0)}{C(9,1)} = \dfrac{3}{9} \approx .33$

3: $\dfrac{C(3,2) \cdot C(6,1) + C(3,3) \cdot C(6,0)}{C(9,3)} = \dfrac{19}{84} \approx .23$

5: $\dfrac{C(3,3) \cdot C(6,2)}{C(9,5)} = \dfrac{15}{126} \approx .12$

7: 0

6. What is the likelihood that 2 or more of 15 random numbers between 1 and 100 are the same?

$$1 - \frac{100 \cdot 99 \cdot 98 \cdot \cdots \cdot 86}{100 \cdot 100 \cdot 100 \cdot \cdots \cdot 100} = 1 - .3313 = .6687$$

7. Jimmy and Jonny are driving along on a road. Jonny suggests the following game. They will look at license plates of oncoming cars and focus on the last two digits. For example, the license plate ABC512 or 7412BG would both yield 12, and the license plate XY406T would yield 06 (i.e. 6). Jonny bets Jimmy that at least two of the next fifteen cars will yield the same number. What is the probability that Jonny wins? Assume each of the 100 possible numbers is equally likely to occur.

Same

8. One die is rolled twelve times. What is the probability of getting exactly:

No 3s: $C(12,0) \cdot 1^0 \cdot 5^{12} / 6^{12} = 5^{12}/6^{12} \approx .1122$

One 3s: $C(12,1) \cdot 1^1 \cdot 5^{11} / 6^{12} =$ $.2692$

Two 3s: $C(12,2) \cdot 1^2 \cdot 5^{10} / 6^{12} =$ $.2961$

Three 3s: $C(12,3) \cdot 1^3 \cdot 5^9 / 6^{12} =$ $.1974$

All twelve 3s: $C(12,12) \cdot 1^{12} \cdot 5^{12}/6^{12} =$ (tiny)

At least one 3: $1 - 5^{12}/6^{12}$

Notice which outcome is most likely, second most likely, etc.

9. Select 4 balls from a bin with 5 blue and 6 orange balls.

How many ways to select any 4 from the 11? $C(11,4) = 330$

How many ways to select 1 blue, 3 orange? $C(5,1) \cdot C(6,3) = 100$

Find the probabilities of selecting

0 blue, 4 orange: $C(5,0) \cdot C(6,4) / C(11,4) = 15/330$

1 blue, 3 orange: $C(5,1) \cdot C(6,3) / C(11,4) = 100/330$

2 blue, 2 orange: $C(5,2) \cdot C(6,2) / C(11,4) = 150/330$

3 blue, 1 orange: $C(5,3) \cdot C(6,1) / C(11,4) = 60/330$

4 blue, 0 orange: $C(5,4) \cdot C(6,0) / C(11,4) = 5/330$

At least one orange: $1 - \frac{5}{330}$

Notice which outcome is most likely, second most likely, etc.

10. Compare the previous problem to the following problem.

Select 4 different numbers between 1 to 11. Find the probability of selecting

0 even, 4 odd numbers:

1 even, 3 odd numbers:

Etc.

Same!