Section 4.2 Assignments of Probabilities
Math 141

Main ideas

For event E and sample space S:
n(E) is the number of outcomes in event E
n(S) is the number of outcomes in the sample space, i.e. the total number of outcomes
Pr(E) is the probability of event E.

Properties of probabilities:
$\Pr(E) = \frac{\mathrm{n(E)}}{\mathrm{n(S)}}$
$0 \leq \Pr(E) \leq 1$ (why decimal values rather than percentages? "per cent" means "per 100")
If E can occur in multiple ways {s, t, u, ..., z}, then Pr(E) = Pr(s) + Pr(t) + Pr(u) + ... + Pr(z)
For $S = \{s_1, s_2, \dots, s_N\}$ with probabilities $\{p_1, p_2, \dots, p_N\}$, then $p_1 + p_2 + \cdots + p_N = 1$.
Pr(E) + Pr(E') = 1 (more on this in Section 4.3). So $Pr(E) = 1 - Pr(E')$

Inclusion-exclusion principle:
Sets: $n(E \cup F) = n(E) + n(F) - n(E \cap F)$
Probabilities: $Pr(E \cup F) = Pr(E) + Pr(F) - Pr(E \cap F)$

Mutually exclusive:
Sets: $E \cap F = \varnothing$, $n(E \cap F) = 0$
Probabilities: $Pr(E \cap F) = 0$

Probability distribution: the possible outcomes and the probability of each.

Problems

1. Roll 2 dice. We are interested in the sum.
   Event E is "sum is ≥ 7"
   Event F is "sum is odd"
   Event G is "doubles"

Sum of 2 dice

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

Pr(E) = $21/36$

Pr(F) = $18/36$

$Pr(E \cap F)$ = $12/36$ which $\leq 21/36, 18/36$

$Pr(E \cup F)$ = $27/36 = 21/36 + 18/36 - 12/36$ which $\geq \frac{21}{36}, \frac{18}{36}$

Pr(E') = $15/36 = 1 - 21/36$

Pr(G) = $6/36$

$Pr(F \cup G)$ = $18/36 + 6/36$

Probability distributions:

Sum of 2 dice

Outcome	Probability
2	1/36
3	2/36
4	3/36
5	4/36
6	5/36
7	6/36
8	5/36
9	4/36
10	3/36
11	2/36
12	1/36

Sum 36/36

Difference of 2 dice

Outcome	Probability
0	6/36
1	10/36
2	8/36
3	6/36
4	4/36
5	2/36

Sum 36/36

Sum of 2 dice

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

Difference between 2 dice

	1	2	3	4	5	6
1	0	1	2	3	4	5
2	1	0	1	2	3	4
3	2	1	0	1	2	3
4	3	2	1	0	1	2
5	4	3	2	1	0	1
6	5	4	3	2	1	0

2. Use the probability distribution at right.
   Let E = $\{s_1, s_2\}$, F = $\{s_2, s_3, s_4\}$.

Outcome	Probability
s_1	.10
s_2	.20
s_3	.40
s_4	.05
s_5	.25

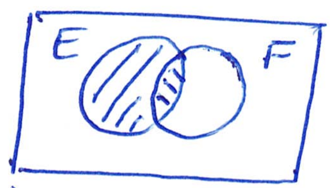
   Pr(E) = .30

   Pr(F) = .65

   Pr(E $\cap$ F) = .20

   Pr(E $\cup$ F) = .75 (= .30 + .65 − .20)

   Pr(E $\cap$ F') = $Pr(s_1)$ = .10



   Notice: $Pr(E) = Pr(E \cap F) + Pr(E \cap F')$

3. Flip 6 coins. We are interested in how many are heads. Find the probability distribution.

$2^6 = 64$ possible outcomes

# of heads	Probability
0	C(6,0)/64 = 1/64
1	C(6,1)/64 = 6/64
2	C(6,2)/64 = 15/64
3	C(6,3)/64 = 20/64
4	C(6,4)/64 = 15/64
5	C(6,5)/64 = 6/64
6	C(6,6)/64 = 1/64

Preview of how probabilities are affected by additional information

4. Two coins are tossed. What is the probability both coins are heads? 1/4
5. Two coins are tossed. We are told that at least one of the coins is heads. 1/3
   What is the probability both coins are heads?
6. Two coins are tossed. We can see one of the coins, and we see that it is heads. 1/2
   What is the probability both coins are heads?