Section 3.4 The Multiplication Principle

Math 141

Main ideas

The multiplication principle: if there are t tasks with $m_1, m_2, \ldots, m_t$ choices for those tasks, then there are $m_1 \cdot m_2 \cdot \cdots \cdot m_t$ ways to accomplish all t tasks.

There are $r! = r \cdot (r-1) \cdot (r-2) \cdot \cdots \cdot 2 \cdot 1$ ways to order (i.e. put in a particular order) r items.

Problems

1. You have 3 shirts, 4 pairs of pants and 7 hats. How many clothing combinations if:
   You need 1 shirt and 1 pair of pants? $3 \cdot 4$

   You need 1 shirt, 1 pair of pants and 1 hat? $3 \cdot 4 \cdot 7$

2. How many license plates are there with:
   3 letters and 3 numbers (California plates until 1982)? $26 \cdot 26 \cdot 26 \cdot 10 \cdot 10 \cdot 10$

   1 number, 3 letters, 3 numbers (California plates after 1982)? $10 \cdot$

3. How many 8-letter passwords are there? $26 \cdot 26 \cdot \cdots \cdot 26 = 26^8$

4. Before 1995, 3-digit area codes XXX in the United States had the following restrictions:
   Neither 0 nor 1 could be used as the first digit.
   0 or 1 had to be in the second digit.
   The third digit could be anything.
   How many different area codes were possible? $8 \cdot 2 \cdot 10$

5. After 1995, any number could be the second digit of 3-digit area codes.
   How many different area codes are now possible? $8 \cdot 10 \cdot 10$

6. How many 7-digit phone numbers XXX-XXXX are there which don't begin with 0?

$9 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10$

7. How many different social security numbers are available if the only restriction is that the number 000-00-0000 cannot be assigned?

$10^9 - 1$

8. A palindrome is a number that reads the same forwards or backwards, e.g. 58485.
   How many 7-digit palindromes are there? $10 \cdot 10 \cdot 10 \cdot 10 \cdot 1 \cdot 1 \cdot 1$

   How many 8-digit palindromes are there? $10 \cdot 10 \cdot 10 \cdot 10 \cdot 1 \cdot 1 \cdot 1 \cdot 1$

9. How many ways are there to order (that is, place in a particular order):
   Three people—Alice, Bob and Chris? $3 \cdot 2 \cdot 1$

   Three of anything? $3 \cdot 2 \cdot 1 = 3!$

   Six of anything? $6!$

10. How many different three digit numbers from 000 to 999 are there if:
    Any number is allowed? $10 \cdot 10 \cdot 10 = 1000$

    All digits are different? $10 \cdot 9 \cdot 8 = 720$

    Two digits are the same? $10 \cdot 1 \cdot 9 + 10 \cdot 9 \cdot 1 + 10 \cdot 9 \cdot 1 = 270$

    All three digits are the same? $10 \cdot 1 \cdot 1 = 10$

    Notice: $1000 = 720 + 270 + 10$

11. In how many ways can three couples be seated if:
    Anyone can sit next to anyone else? $6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 6! = 720$

    Each couple must be seated together? $6 \cdot 1 \cdot 4 \cdot 1 \cdot 2 \cdot 1 = 48$

    Or an alternative way to compute this: $3 \cdot 2 \cdot 1 \cdot 2 \cdot 2 \cdot 2 = 48$

12. A five-digit ZIP code is said to be detour-prone if it looks like a valid but different ZIP code when read upside down. For instance, **68091** is detour-prone, but **84604** and **98086** are not. How many of the 10^5 ZIP codes are detour prone?

$5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 - 5 \cdot 5 \cdot 3 \cdot 1 \cdot 1$