Section 3.2 A Fundamental Principle of Counting

Math 141

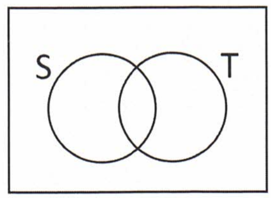
Main ideas

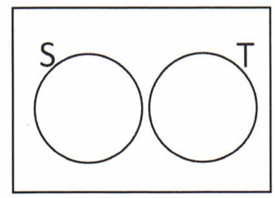
Recall: $x \in S \cup T$ means $x \in S$ or $x \in T$ (or both) while $x \in S \cap T$ means $x \in S$ and $x \in T$

Number of elements in set S $n(S)$

Mutually exclusive $S \cap T = \emptyset$; that is, $n(S \cap T) = 0$

Inclusion-exclusion Principle $n(S \cup T) = n(S) + n(T) - n(S \cap T)$ $n(S \cup T) = n(S) + n(T)$ if $n(S \cap T) = 0$

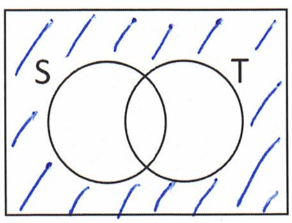




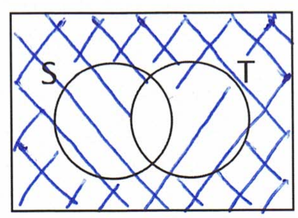
DeMorgan's Laws $(S \cup T)' = S' \cap T'$, $(R \cup S \cup T)' = R' \cap S' \cap T'$, etc.

$(S \cap T)' = S' \cup T'$, $(R \cap S \cap T)' = R' \cup S' \cup T'$, etc.

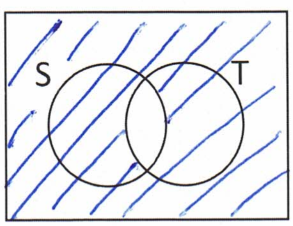
$(S \cup T)'$



S' //// T' \\\\



$(S \cap T)'$



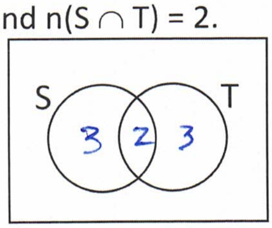
$(S \cup T)' = S' \cap T'$ in words: not (S or T) = not S and not T

$(S \cap T)' = S' \cup T'$ in words: not (S and T) = not S or not T

Another example: $(R \cup (S \cap T))' =$

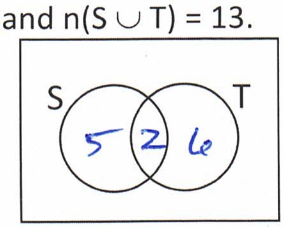
Problems Recall that $\boxed{n(S \cup T) = n(S) + n(T) - n(S \cap T).}$

1. Find $n(S \cup T)$ given that $n(S) = 5$, $n(T) = 5$, and $n(S \cap T) = 2$.



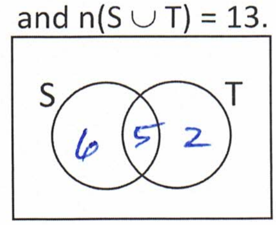
$n(S \cup T) = 5 + 5 - 2$
$= 8$

2. Find $n(S \cap T)$ given that $n(S) = 7$, $n(T) = 8$ and $n(S \cup T) = 13$.



$13 = 7 + 8 - n(S \cap T)$
$\Rightarrow n(S \cap T)$
$= 7 + 8 - 13 = 2$

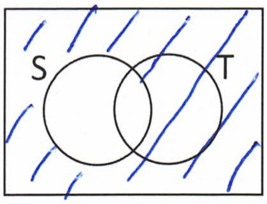
3. Find $n(S)$ given that $n(T) = 7$, $n(S \cap T) = 5$ and $n(S \cup T) = 13$.



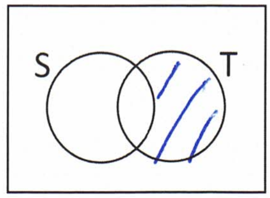
$13 = n(S) + 7 - 5$
$\Rightarrow n(S)$
$= 13 - 7 + 5 = 11$

4. Draw Venn Diagrams that correspond to the following:

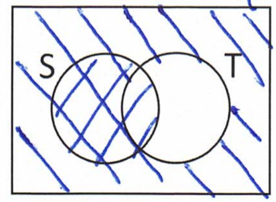
(a) S'



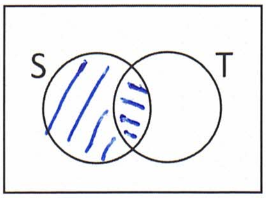
(b) $S' \cap T$



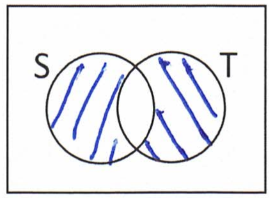
(c) $S \cup T' = (S' \cap T)'$



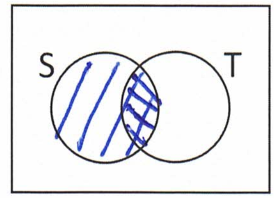
(d) $(S \cap T) \cup (S \cap T')$



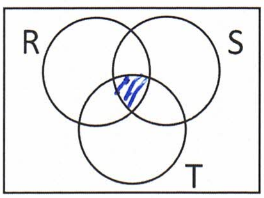
(e) $(S \cap T') \cup (S' \cap T)$



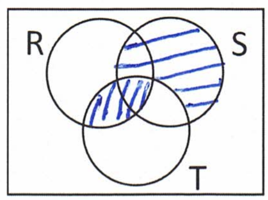
(f) $S \cup (S \cap T)$



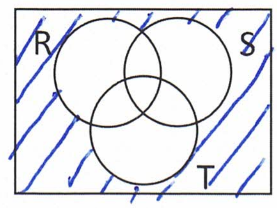
(g) $R \cap S \cap T$



(h) $(R \cap T) \cup (S \cap T')$

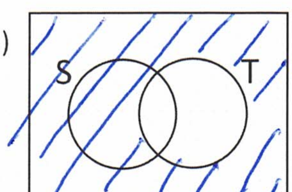


(i) $R' \cap S' \cap T' = (R \cup S \cup T)'$



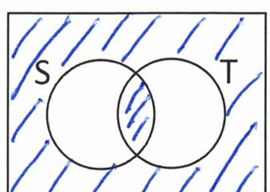
5. Give set theoretic expressions that describe each of the following Venn Diagrams:

(a)



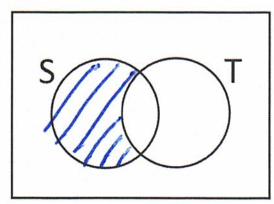
T'

(b)



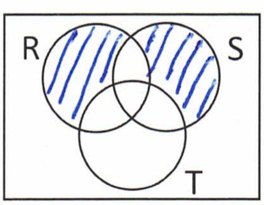
$(S \cap T) \cup (S' \cap T')$

(c)



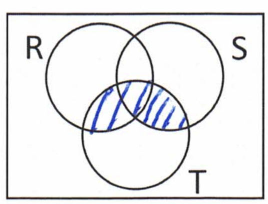
$S \cap T'$

(d)



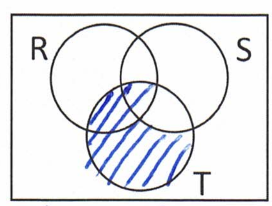
$(R \cap S' \cap T') \cup (R' \cap S \cap T')$

(e)



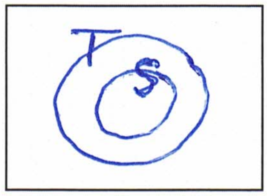
$(R \cup S) \cap T$

(f)



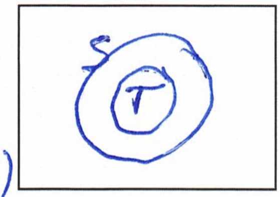
$(R \cap T) \cup (T \cap S')$

6. If $n(S \cap T) = n(S)$:



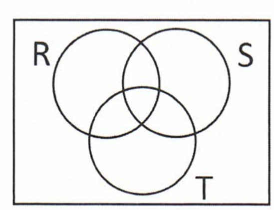
In general, $n(S \cap T) \leq n(S)$

If $n(S \cup T) = n(S)$:



In general, $n(S) \leq n(S \cup T)$

7. Inclusion-exclusion principle for three sets:



$n(R \cup S \cup T) = n(R) + n(S) + n(T)$
$- n(R \cap S) - n(R \cap T) - n(S \cap T)$
$+ n(R \cap S \cap T)$