Section 2.4 Math 141
The Inverse of a Matrix

Main ideas

The inverse of a matrix A^{-1} is the matrix such that $AA^{-1} = I$ and $A^{-1}A = I$.

For A to have an inverse it must be square (# rows = # columns), and its inverse A^{-1} will be the same size.

The identity matrix I is useful: $AI = IA = A$.

Given the system of equations $AX = B$ (for square A), the solution X is $X = A^{-1}B$.

For $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, $A^{-1} = \frac{1}{D}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ where determinant $D = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$.

In class

1. Use a matrix inverse to find the solution to $\begin{matrix} x + 2y = 5 \\ 3x + 4y = 6 \end{matrix}$.

   ___x + ___ y + ___z = ___

2. ___x + ___ y + ___z = ___ (Numbers from students.)

   ___x + ___ y + ___z = ___

   Use Excel or some other technology to find the inverse of $\begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}$,

   and use it to find the solution to the system of 3 equations and 3 unknowns.

In groups

3. HW 2.4.12. Use a matrix inverse to find the solution to $\begin{matrix} 5x + 3y = 1 \\ 7x + 4y = 2 \end{matrix}$.

4. The system $\begin{matrix} x + 2y = 1 \\ 3x + 6y = k \end{matrix}$ does not have a unique solution. Why not?

   For what value(s) of k are there infinite solutions?

   For what value(s) of k is there no solution?

   Notice the matrix $\begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$ does not have an inverse. Why not?

   If it did have an inverse, the system of equations $AX = B$ would have the unique solution $X = A^{-1}B$.

5. $\begin{matrix} x + 2y + 4z = 5 \\ 3x + 6y \quad - z = 6 \end{matrix}$ does not have a unique solution. Why not?

   The matrix $\begin{bmatrix} 1 & 2 & 4 \\ 3 & 6 & -1 \end{bmatrix}$ (or any other matrix with more rows than columns) does not have an inverse. Otherwise (i.e. if A had an inverse), the system of equations $AX = B$ would have a unique solution $X = A^{-1}B$.

6. The system $\begin{matrix} x + 2y = 5 \\ 3x + 6y = 6 \\ 4x - \ y = 7 \end{matrix}$ does not have a unique solution. Why not?

   The matrix $\begin{bmatrix} 1 & 2 \\ 3 & 6 \\ 4 & -1 \end{bmatrix}$ (or any matrix with more columns than rows) does not have an inverse. Otherwise (i.e. if A had an inverse), the system of equations $AX = B$ would have a unique solution $X = A^{-1}B$.