Section 2.2 Math 141
General Systems of Linear Equations

Main ideas

For systems of linear equations, there is always 0 or 1 or ∞ solutions. Generally:

- $number\ of\ equations > number\ of\ unknowns \Rightarrow 0\ solution$
- $number\ of\ equations = number\ of\ unknowns \Rightarrow 1\ solution$
- $number\ of\ equations < number\ of\ unknowns \Rightarrow \infty\ solutions$

But there can be exceptions.

How to recognize each case of 0 or 1 or ∞ solutions, after Gaussian elimination:

- A row of *all* 0's can simply be ignored (the equation on that row was a redundant restriction/relationship between the unknowns).
- A row of all 0's on the left (in the coefficient part of the matrix) along with any non-zero value on the right (the "right hand side" part of the matrix) means there is no solution, no matter what other numbers there are in the matrix.
- If we end up with fewer non-zero rows than there are columns, then (assuming that there are no rows with all zeros on the left and something non-zero on the right, that is, assuming that there is a solution) there are ∞ solutions, and the free variables in the solution correspond to the non-pivot columns (the columns without a pivot 1 in them).

In Class

1. Let's look at Book Examples 2 and 3 to see how to recognize when there is no solution (Example 2) and infinite solutions (Example 3). Note: a bottom row of all zeroes (Example 3) doesn't necessarily mean there are infinite solutions. You simply ignore that row when determining the solution. See Figure 3 in the book for how some possible cases might look. (But in general, we don't care how it looks.)

2. In groups, work an example with $\#\ equations < \#\ unknowns$. HW 2.2.38:
$$x + 5y + 3z = 9$$
$$2x + 9y + 7z = 5$$
Typically, $\#\ equations < \#\ unknowns$ results in infinite solutions. Why?

3. In groups, work an example with $\#\ equations > \#\ unknowns$.
$$n + \quad d = 6$$
$$5n - \quad d = 0$$
$$5n + 10d = 75$$
Typically, $\#\ equations > \#\ unknowns$ results in no solution. Why?

4. In groups, as there is time, work HW 2.2.50, then HW 2.2.43, then HW 2.2.44.